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## Meet, Discuss and Trust each other: large versus small groups

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In this paper we propose a dynamical interpretation of the sociological distinction between large and small groups of interacting individuals. In the former case individual behaviors are largely dominated by the group effect, while in the latter mutual relationships do matter. Numerical and analytical tools are combined to substantiate our claims.

*Keywords:* Opinion Dynamics; complex systems; social dynamics.

### 1. Introduction

Sociophysics is a long standing<sup>1</sup> research field addressing issues related to the characterization of the collective social behavior of individuals, such as culture dissemination, the spreading of linguistic conventions, and the dynamics of opinion formation.<sup>2-6</sup> These are all interdisciplinary applications which sit the interface of different domains. The challenge is in fact to model the dynamical evolution of an ensemble made of interacting, micro-

constituents and infer the emergence of collective, macroscopic behaviors that are then eventually accessible for direct experimental inspection. Agent based computational models are widely employed in sociophysics applications and also adopted in this paper. They provide in fact a suitable setting to define local rules which govern the evolution of the microscopic constituents.

In recent years, much effort has been devoted to the investigation of social networks, emerging from interaction among humans. In the sociological literature a main distinction has been drawn between small<sup>7</sup> and large<sup>8-10</sup> groups, as depending on its intrinsic size, the system apparently exhibits distinct social behaviors. Up to a number of participants of the order of a dozen, a group is considered small. All members have a clear perception of other participants, regarded as individual entities: Information hence flows because of mutual relationships. Above this reference threshold, selected individuals see the vast majority of the group as part of a uniform mass: There is no perception of the individual personality, and only average behaviors matter. This distinction is motivated by the fact that usually humans act following certain prebuilt “schemes”<sup>11,12</sup> resulting from past experiences, which enables for rapid decision making without having to necessarily screen all available data. This innate data analysis process allows one for a dramatic saving of cognitive resources.

These conclusions have been reached on the basis of empirical and qualitative evidences.<sup>7,13-15</sup> We are here interested in detecting the emergence of similar phenomena in a simple model of opinion dynamics.<sup>16,17</sup> As we shall see in our proposed formulation agents possess a continuous opinion on a given subject and possibly modify their beliefs as a consequence of binary encounters.

The paper is organized as follows. In the next section the model is introduced. Forthcoming sections are devoted to characterizing the quantities being inspected and develop the mathematical treatment. In the final section we sum up and comment about future perspectives.

## 2. The model

We shall investigate the aforementioned effects related to the size of the group of interacting individuals (*social group*), within a simple opinion dynamics model, recently introduced in<sup>16</sup>. For the sake of completeness we hereafter recall the main ingredients characterizing the model. The interested reader can refer to<sup>16</sup> for a more detailed account.

We consider a closed group composed by  $N$  individuals, whose opinion on a

given issue is represented by a continuous variable  $O_i$ , scanning the interval  $[0, 1]$ ; moreover each agent  $i$  is also characterized by the so-called *affinity*, a real valued vector of  $N - 1$  elements, labeled  $\alpha_{ij}$ , which measures the quality of the relationship between  $i$  and any other actor  $j$  belonging to the community.

Agents interact via binary encounters possibly updating their opinion and relative affinity, which thus evolve in time. Once agents  $i$  and  $j$  interact, via the mechanism described below, they converge to the mean opinion value, provided their mutual affinity scores falls below a pre-assigned threshold quantified via the parameter  $\alpha_c$ . In formulae:

$$O_i^{t+1} = O_i^t - \frac{1}{2} \Delta O_{ij}^t \Gamma_1(\alpha_{ij}^t) \quad \& \quad O_j^{t+1} = O_j^t - \frac{1}{2} \Delta O_{ji}^t \Gamma_1(\alpha_{ji}^t), \quad (1)$$

where  $\Delta O_{ij}^t = O_i^t - O_j^t$  and  $\Gamma_1(x) = \frac{1}{2} [\tanh(\beta_1(x - \alpha_c)) + 1]$ . The latter works as an *activating function* defining the region of trust for effective social interactions. On the other hand bearing close enough opinions on a selected topic, might induce an enhancement of the relative affinity, an effect which is here modeled as:

$$\alpha_{ij}^{t+1} = \alpha_{ij}^t + \alpha_{ij}^t (1 - \alpha_{ij}^t) \Gamma_2(\Delta O_{ij}^t) \quad \& \quad \alpha_{ji}^{t+1} = \alpha_{ji}^t + \alpha_{ji}^t (1 - \alpha_{ji}^t) \Gamma_2(\Delta O_{ji}^t), \quad (2)$$

being  $\Gamma_2(x) = -\tanh(\beta_2(|x| - \Delta O_c))$ . This additional *activating function* quantifies in  $\Delta O_c$  the largest difference in opinion ( $\Delta O_{ij}^t$ ) which yields to a positive increase of the affinity amount  $\alpha_{ij}^t$ . The parameters  $\beta_1$  and  $\beta_2$  are chosen large enough so that  $\Gamma_1$  and  $\Gamma_2$  are virtually behaving as step functions. Within this working assumption, the function  $\Gamma_1$  assumes value 0 or 1, while  $\Gamma_2$  is alternatively  $-1$  or  $+1$ , depending on the value of their arguments <sup>a</sup>.

The affinity variable,  $\alpha_{ij}^t$ , schematically accounts for a large number of hidden traits (diversity, personality, attitudes, beliefs...), which are nevertheless non trivially integrated as an abstract simplified form into the model. Note also that the affinity accounts for a *memory* mechanism: indeed once two agents meet, the outcome of the interaction in part depends on their history via the affinity scores.

To complete the description of the model let us review the selection rule

<sup>a</sup>We shall also emphasize that the logistic contribution entering Eq. (2) maximizes the change in affinity when  $\alpha_{ij}^t \approx 0.5$ , corresponding to agents  $i$  which have not yet build a definite judgment on the selected interlocutor  $j$ . Conversely, when the affinity is close to the boundaries of the allowed domain, marking a clear view on the worth of the interlocutor, the value of  $\alpha_{ij}^t$  is more resistant to subsequent adjustments.

here implemented. Each time step a first agent  $i$ , is randomly extracted, with uniform probability. Then a second agent  $j$  is selected, which minimizes the *social metric*  $D_{ij}^t$  and time  $t$ . This is a quantity defined as:

$$D_{ij}^t = d_{ij}^t + \mathcal{N}_j(0, \sigma), \quad (3)$$

where  $d_{ij}^t = |\Delta O_{ij}^t|(1 - \alpha_{ij}^t)$  is the so-called *social distance* and  $\mathcal{N}_j(0, \sigma)$  represents a normally distributed noise with zero mean and variance  $\sigma$ , that can be termed *social temperature*.<sup>16</sup> The rationale inspiring the mechanisms here postulated goes as follows: The natural tendency for agent  $i$  to pair with her/his closest homologous belonging to the community (higher affinity, smaller opinion distance), is perturbed by a stochastic disturbance, which is intrinsic to the social environment (degree of mixing of the population).

The model exhibits an highly non linear dependence on the involved parameters,  $\alpha_c$ ,  $\Delta O_c$  and  $\sigma$ . In a previous work<sup>16</sup> the asymptotic behavior of the opinions dynamics was studied and the existence of a phase transition between a consensus state and a polarized one demonstrated. It should be remarked however that the fragmented case might be metastable; in fact if the mean separation between the adjacent opinion peaks is smaller than the opinion interaction threshold,  $\Delta O_c$ , there always exists a finite, though small, probability of selecting two individuals belonging to different clusters, hence producing a gradual increase in the mutual affinities, which eventually lead to a merging of the, previously, separated clusters. This final state will be achieved on extremely long time scales, diverging with the group size: socially relevant dynamics are hence likely to correspond to the metastable regimes.

A typical run for  $N = 100$  agents is reported in the main panel of Fig. 1, for a choice of the parameters which yields to a monoclustered phase. This is the setting that we shall be focusing on in the forthcoming discussion: Initial opinions are uniformly distributed in the interval  $[0, 1]$ , while  $\alpha_{ij}^0$  are randomly assigned in  $[0, 1/2]$  with uniform distribution, parameters have been fixed to  $\alpha_c = 0.5$ ,  $\Delta O_c = 0.5$  and  $\sigma = 0.01$ .

Once the cluster is formed, one can define the *opinion convergence time*,  $T_c$ , i.e. time needed to aggregate all the agents to the main opinion group<sup>b</sup>. A second quantity  $T_\alpha$  can be introduced, which measures the time scale for the convergence of the *mean group affinity* to its asymptotic value 1. The latter will be rigorously established in the next section.

<sup>b</sup>We assume that a group is formed, i.e. aggregated, once the largest difference between opinions of agents inside the group is smaller than a threshold, here  $10^{-4}$ .

Such quantities are monitored as function of time and results are schematically reported in Fig. 1. As clearly depicted, the evolution occurs on sensibly different time scales, the opinion converging much faster for the set of parameters here employed.

In the remaining part of this paper, we will be concerned with analyzing the detail of this phenomenon highlighting the existence of different regimes as function of the amount of simulated individuals. More specifically, we shall argue that in small groups, the mean affinity converges faster than opinions, while the opposite holds in a large community setting. Our findings are to be benchmarked with the empirical evidences, as reported in the psychological literature. It is in fact widely recognized that the dynamics of a small group (workgroup) proceeds in a two stages fashion: First one learns about colleagues to evaluate their trustability, and only subsequently weight their input to form the basis for decision making. At variance, in large communities, only a few binary interactions are possible among selected participants within a reasonable time span. It is hence highly inefficient to wait accumulating the large number of information that would eventually enable to assess the reliability of the interlocutors. The optimal strategy in this latter case is necessarily (and solely) bound to estimating the difference in opinion, on the topic being debated.

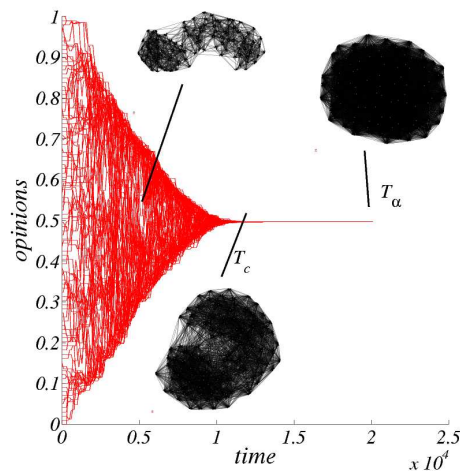


Fig. 1. Opinions as function of time. The underlying network is displayed at different times, testifying on the natural tendency to evolve towards a coherent ensemble of affine individuals.  $T_c$  and  $T_\alpha$  are measured according to our conventions.

### 3. The social network of affinities

In our model the affinity enters the selection mechanism that makes agents to interact. We can restate this fact by postulating the existence of an underlying *social network*, which drives the interactions and thus the opinion flow. In this perspective, the affinity can be seen as the *adjacency matrix* of a *weighted*<sup>c</sup> graph. In such a way we are formally dealing with an *adaptive social network*<sup>18,19</sup>: The network topology influences the opinion dynamics, the latter providing a feedback on the network itself. In other words, the evolution of the topology is inherent to the dynamics of the model because of the proposed self-consistent formulation and not imposed a priori as an additional, external ingredient, i.e. rewire and/or add/remove links according to a given probability<sup>20,21</sup> once the state variables have been updated. From this point of view, the mean group affinity is the *averaged outgoing degree* – called for short ”the degree” in the following – of the network:

$$\langle k \rangle (t) = \frac{1}{N} \sum_i k_i^t, \quad (4)$$

where the degree of the  $i$ -th node is  $k_i^t = \sum_j \alpha_{ij}^t / (N - 1)$ . The normalizing factor  $N - 1$  allows for a direct comparison of networks made of a different number of agents. Let us observe that we chose to normalize with respect to  $N - 1$  because no self-interaction is allowed for.

In the left panel of Fig. 2 we report the probability distribution function for the degree, as a function of time. Let us observe that the initial distribution is correctly centered around the value  $1/4$ , due to the specificity of the chosen initial condition. The approximate Gaussian shape results from a straightforward application of the Central Limit Theorem to the variables  $(k_i^t)_{i=1,\dots,N}$ . In the right panel of Fig. 2 the time evolution of the mean degree  $\langle k \rangle (t)$  is reported. Starting from the initial value  $1/4$ , the mean degree increases and eventually reaches the value  $1$ , characteristic of a complete graph. As previously mentioned this corresponds to a social network where agents are (highly) affine to each other. In the same panel we also plot the analytical curve for  $\langle k \rangle (t)$ , as it is determined hereafter.

From the previous observation, it is clear that the time of equilibration of  $\langle k \rangle (t)$  provides an indirect measure of the convergence time for  $\alpha_{ij}^t \rightarrow 1$ . The affinity convergence time,  $T_\alpha$  can be thus defined as:

$$T_\alpha = \min\{t > 0 : \langle k \rangle (t) \geq \eta\}, \quad (5)$$

<sup>c</sup>In fact the trustability relation is measured in terms of the ”weights”  $\alpha_{ij}^t \in [0, 1]$ .

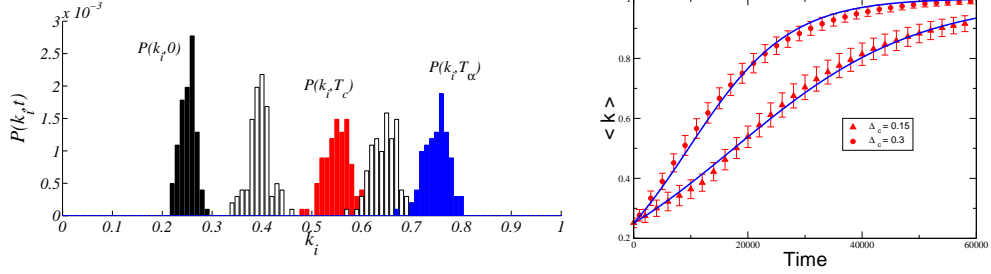


Fig. 2. Time evolution of the degree probability distribution function and the averaged degree. Left panel : Several histograms representing successive snapshots of the dynamics are displayed:  $t = 0$  (black online), two generic intermediate times (wight),  $t = T_c$  (red online) and  $t = T_\alpha$  (blue online); histograms are normalized to unity. Right panel :  $\langle k \rangle$  versus time. Symbols refer to direct simulations. The solid lines are obtained as a one parameter fit of the theoretical expression Eq. (10).

where  $\eta \in (0, 1)$  is a threshold quantity. The closer  $\eta$  is to 1 the stronger the condition on the value of  $\alpha_{ij}^t$  (the larger the value for  $T_\alpha$ ) which identifies an affine unit. In the following we shall assume  $\eta = 3/4$ .

#### 4. Results

The aim of this section is to analyze the behavior of  $T_\alpha$  and of  $\langle k \rangle (t)$  so to provide an analytical interpretation for the results presented in the previous section.

From the definition of mean degree Eq. (4) one can compute the time evolution of  $\langle k \rangle (t)$  in a continuous time setting:

$$\frac{d \langle k \rangle}{dt} = \frac{1}{N(N-1)} \sum_{i,j} \frac{d\alpha_{ij}^t}{dt}. \quad (6)$$

To deal with the update rule for the affinity Eq. (2) we assume that we can decouple the opinions and affinity dynamics by formally replacing the activating function  $\Gamma_2(\Delta O_{ij}^t)$  with a suitably tuned constant. Strictly speaking, this assumption is correct only for  $\Delta O_c = 1$ , in which case the opinions are always close enough so to magnify the mutual affinity scores of the interacting agents, as a results of a self consistent evolution. For the general case,  $\Delta O_c \leq 1$ , what we are here proposing is to replace  $\Gamma_2(\Delta O_{ij}^t)$  by some *effective value* termed  $\gamma_{eff}$ , determined by the dynamics itself. The technical development that yields to the optimal estimate of  $\gamma_{eff}$  will be dealt in the appendix.

Under this assumption, the equation Eq. (2) for the evolution of  $\alpha_{ij}^t$  admits the following continuous version :

$$\frac{d\alpha_{ij}^t}{dt} = \gamma_{eff} \alpha_{ij}^t (1 - \alpha_{ij}^t) , \quad (7)$$

which combined to equation Eq. (6) returns the following equation for the mean degree evolution :

$$\frac{d\langle k \rangle}{dt} = \gamma_{eff} (\langle k \rangle (t) - \langle \alpha_{ij}^t \rangle^2) . \quad (8)$$

Assuming the standard deviation of  $(\alpha_{ij}^t)$  to be small <sup>d</sup> for all  $t$ , implies  $\langle \alpha_{ij}^t \rangle^2 \sim \langle \alpha_{ij}^t \rangle^2 = \langle k \rangle^2$ , which allows us to cast the previous equation for  $\langle k \rangle$  in the closed form :

$$\frac{d\langle k \rangle}{dt} = \gamma_{eff} (\langle k \rangle - \langle k \rangle^2) . \quad (9)$$

This equation can be straightforwardly solved to give:

$$\langle k \rangle = \frac{k_0}{k_0 + (1 - k_0)e^{-\gamma_{eff}t}} , \quad (10)$$

where  $k_0 = \langle k \rangle (0)$ . We can observe that such solution provides the correct asymptotic value for large  $t$ . Moreover  $\gamma_{eff}$  plays the role of a *characteristic* time and in turn enables us to quantify the convergence time  $T_\alpha$  of the affinity via :

$$T_\alpha = \frac{1}{\gamma_{eff}} \log \left( \frac{\eta(1 - k_0)}{k_0(1 - \eta)} \right) = \frac{\eta'}{\gamma_{eff}} . \quad (11)$$

In the appendix we determine <sup>e</sup> the following relation which allows to express  $\gamma_{eff}$  as a function of the relevant variables and parameters of the models, i.e.  $T_c$ ,  $\Delta O_c$  and  $N$ :

$$\gamma_{eff} = \frac{1}{N^2} + \frac{T_c}{T_\alpha N^2} \rho , \quad (12)$$

where  $\rho = -(1 + 2\Delta O_c \log(\Delta O_c) - \Delta O_c^2)$ , thus recalling Eq. (11) we can finally get:

$$T_\alpha = \eta' N^2 \left( 1 - \frac{T_c \rho}{N^2 \eta'} \right) . \quad (13)$$

<sup>d</sup>This assumption is supported by numerical simulations not reported here and by the analytical considerations presented in.<sup>22</sup>

<sup>e</sup>This is case a) of Eq. (A.7). In the second case the result is straightforward:  $T_\alpha = \eta' N^2 / (\rho + 1)$ .

From previous works<sup>16,23</sup> we know that the dependence of  $T_c$  on  $N$ , for large  $N$ , is less than quadratic. Hence, for large  $N$ , the second term in the parenthesis drops, and we hence conclude that, the affinity convergence time grows like  $T_\alpha \sim N^2$ , as clearly shown in the main panel of Fig. 3. The prefactor's estimate is also approximately correct, as discussed in the caption of Fig. 3.

The above results inspire a series of intriguing observation. First, it is implied that the larger the group size the bigger  $T_\alpha$  with respect to  $T_c$ . On the contrary, making  $N$  smaller the gap progressively fades off. Dedicated numerical simulations (see left inset of Fig. 3) allows to indentify a turning point which is reached for small enough values of  $N$ : there the behavior is indeed opposite, and, interestingly,  $T_c > T_\alpha$ . The transition here reproduced could relate to the intrinsic peculiarities of the so called “small group dynamics” to which we made reference in the introductory section<sup>7–10</sup>. Furthermore, it should be stressed that the critical group size determining the switching between the two regimes here identified, is quantified in  $N \simeq 20$ , a value which is surprisingly closed to the one being quoted in social studies.

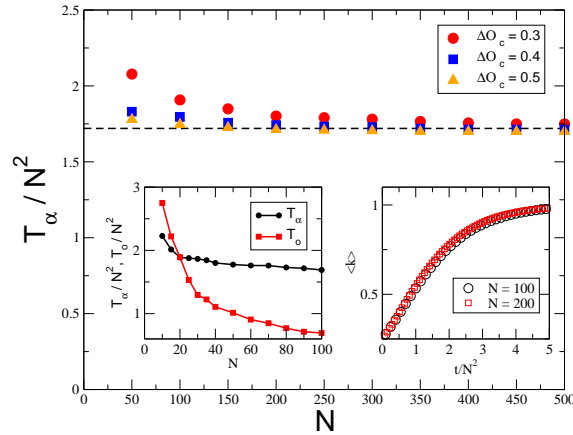


Fig. 3. Main panel:  $T_\alpha/N^2$  vs  $N$  for different values of the parameter  $\Delta O_c$ . The data approach a constant value ( $T_\alpha/N^2 \sim 1.72$ ) clearly indicating that the time of convergence of the affinity matrix scales quadratically with the number of agents, in agreement with the theory. The asymptotic value estimated by our theory is 2.19, the discrepancy being therefore quantified in about 15%. Left inset:  $T_\alpha/N^2$  and  $T_c/N^2$  vs  $N$  for  $\Delta O_c = 0.5$ . As predicted by the theory and the numerics a crossover is found for groups for which opinions converge slower than the affinities: this is the signature of a distinctive difference in the behavior of small and large groups, numerically we found that this difference is effective for  $N \sim 20$ . Right inset:  $\langle k \rangle$  vs  $t/N^2$  is plotted for two different values of  $N$ . As expected the two curves nicely collapse together.

## 5. Conclusion

In this paper we study a model of continuous opinions dynamics already proposed in<sup>16</sup>, which incorporates as main ingredient the affinity between agents both acting on the selection rule for the binary interactions as well entering the postulated mechanism for the update of the individual opinions.

Analyzing the model in the framework of adaptive networks we have been able to show that the sociological distinction between large and small groups can be seen as dynamical effect which spontaneously arises in our system. We have in fact proven that for a set of realistic parameters there exists a critical group size, which is surprisingly similar to the one reported in the psychological literature. Below this critical value agents first converge in mutual affinity and only subsequently achieved a final consensus on the debated issue. At variance, in large groups the opposite holds: The convergence in opinion is the driving force for the aggregation process, affinity converging on larger time scales.

### Appendix A. Computation of $\gamma_{eff}$

The aim of this paragraph is to provide the necessary steps to decouple the opinion and affinity dynamics, by computing an effective value of the activating function  $\Gamma_2(\Delta O_{ij}^t)$ , hereby called  $\gamma_{eff}$ . This will be obtained by first averaging  $\Gamma_2$  with respect to the opinions and then taking the time average of the resulting function:

$$\gamma_{eff} = \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t d\tau \langle \Gamma \rangle_{op}(\tau), \quad (\text{A.1})$$

where the opinion-average is defined by:

$$\langle \Gamma \rangle_{op}(t) = \int_0^1 dx \int_0^1 dy \Gamma_2(|x - y|) f(x) f(y),$$

being  $f(\cdot)$  the opinions probability distribution function. To perform the computation we will assume that for each  $t$ , the opinions are uniformly distributed in the interval  $[a(t), a(t) + L(t)]$  where  $L(t) = 1 - t/T_c$  and  $T_c$  is the opinion convergence time, hence  $f(\cdot) = 1/(NL(t))$ . This assumption is motivated by the “triangle-like” convergence pattern as clearly displayed in the main panel of Fig. 1.

Assuming  $\beta_2$  large enough, we can replace  $\Gamma_2$  by a step function. Hence:

$$\langle \Gamma \rangle_{op}(t) = \frac{1}{N^2 L^2(t)} \int_{a(t)}^{a(t)+L(t)} dx \int_{a(t)}^{a(t)+L(t)} dy \chi(x, y), \quad (\text{A.2})$$

where  $\chi(x, y)$  is defined by (see also Fig. A1)

$$\chi(x, y) = \begin{cases} 1 & \text{if } |x - y| \geq \Delta O_c, \text{ i.e. in the triangles } T_1 \cup T_2 = Q \setminus D \\ -1 & \text{otherwise, i.e. in } D, \end{cases} \quad (\text{A.3})$$

where  $Q$  is the square  $[a, a + L] \times [a, a + L]$ .

Let us observe that this applies only when  $L(t) > \Delta O_c$  (see left panel of Fig. A1); while if  $L(t) < \Delta O_c$  the whole integration domain,  $[a, a + L] \times [a, a + L]$ , is contained into the  $|x - y| < \Delta O_c$  (see right panel of Fig. A1). In this latter case, the integration turns out to be simpler. In other words the

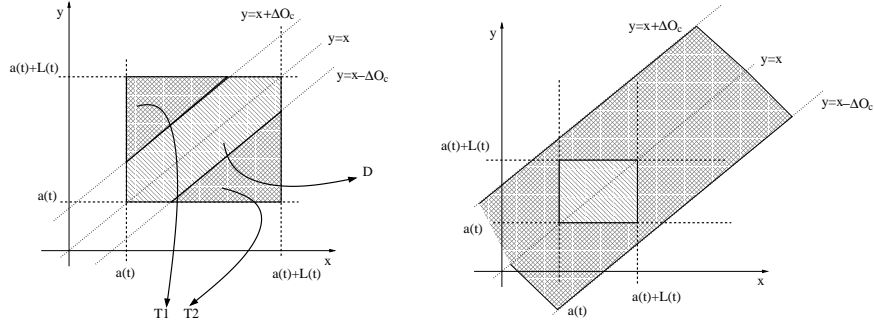


Fig. A1. The geometry of the integration domains. On the left panel the case  $L > \Delta O_c$ , while on the right one the case  $L < \Delta O_c$ .

integral in Eq. (A.2) corresponds to measure the area,  $|D|$ , of the domains shown in Fig. A1 with a sign. Let us perform this computation according to the two cases:  $L > \Delta O_c$  or  $L < \Delta O_c$ .

In the case  $L > \Delta O_c$ , the area of  $D$  is given by  $|D| = |Q| - |T_1| - |T_2|$ , hence

$$\begin{aligned} \langle \Gamma \rangle_{op}(t) &= -\frac{1}{N^2 L^2} (-|D| + |T_1| + |T_2|) = -\frac{1}{N^2 L^2} (-|Q| + 2|T_1| + 2|T_2|) & (\text{A.4}) \\ &= -\frac{1}{N^2 L^2} \left( -L^2 + 4 \frac{(L - \Delta O_c)^2}{2} \right) = \frac{1}{N^2} \left[ 1 - 2 \left( 1 - \frac{\Delta O_c}{L} \right)^2 \right] & (\text{if } L > \Delta O_c). \end{aligned}$$

On the other hand if  $L < \Delta O_c$ , because the square  $Q$  is completely contained into the domain  $|x - y| < \Delta O_c$  where  $\chi$  is equal to  $-1$ , we easily get:  $\langle \Gamma \rangle_{op}(t) = -\frac{1}{N^2 L^2} (-L^2) = \frac{1}{N^2}$ , if  $L < \Delta O_c$ . This last relation together with Eq. (A.4), can be casted in a single formula:

$$\langle \Gamma \rangle_{op}(t) = \frac{1}{N^2} \left[ 1 - 2 \left( 1 - \frac{\Delta O_c}{L} \right)^2 \Theta(L - \Delta O_c) \right]; \quad (\text{A.5})$$

where  $\Theta$  is the Heaviside function,  $\Theta(x) = 1$  if  $x > 0$  and zero otherwise. To conclude we need to compute the time average of  $\langle \Gamma \rangle_{op}(t)$ . Using once again the “triangle-like” convergence assumption for the opinions, i.e.  $L(t) = 1 - t/T_c$ , where  $T_c$  is the opinion convergence time, we get:

$$\gamma_{eff} = \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t d\tau \frac{1}{N^2} \left[ 1 - 2 \left( 1 - 2 \frac{\Delta O_c T_c}{T_c - \tau} + \left( \frac{\Delta O_c T_c}{T_c - \tau} \right)^2 \right) \Theta \left( \frac{T_c - \tau}{T_c} - \Delta O_c \right) \right], \quad (\text{A.6})$$

This integral can be explicitly solved to give:

$$\gamma_{eff} = \begin{cases} \frac{1}{N^2} \left( 1 + \frac{T_c \rho}{T_c} \right) & \text{if } T_\alpha > T_c \\ \frac{\rho+1}{N^2} & \text{if } T_\alpha < T_c, \end{cases} \quad (\text{A.7})$$

where  $\rho = -(1 + 2\Delta O_c \log(\Delta O_c) - \Delta O_c^2)$ .

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