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Research paper

The Travelling Schnauzer Problem: Mission planning for heterogeneous vehicles with distance constraints

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ABSTRACT

In many modern tasks involving small exploration submarines or tethered drones connected to a main vehicle, the presence of physical power supply cables or the limitations of wireless communication range require consideration of distance constraints during mission planning. To solve these tasks, in this work we consider the problem where an asset has to visit a set of points of interest while remaining within a certain distance of a mobile base station. First, we consider the case where the points to visit are ordered and we derive the structural properties of the optimal policy. Exploiting the results for ordered points, we consider the general case where the points to visit are not ordered and we mathematically formalize the optimization problem in an efficient way. Since deriving the optimal solution for a high number of points is computationally challenging, we devise a heuristic and we provide the theoretical bounds on optimality gaps. The proposed solution is assessed and validated through simulations. In particular, extensive numerical results on a marine exploration task show the effectiveness of the proposed heuristic both in terms of solution optimality and computation time.

The Travelling Salesman Problem (TSP) is one of the most well-known problems in combinatorial optimization. The basic formulation of the TSP is: *"given a non-ordered set of n points, find the shortest tour that visits* them all". As well known, despite its simple statement, the TSP belongs to the family of NP-Hard problems. Given the high interest in this problem, several algorithms have been proposed, which can solve the TSP (optimally or almost optimally) in a reasonable amount of time, even for quite large instances of the problem (Braun, 1991; Applegate et al., 1998; Tsai et al., 2004; Halim and Ismail, 2019). The TSP has found applications in a wide range of fields, ranging from logistics (Yao, 2019), to warehouse management (Zunic et al., 2017), agriculture (Yun et al., 2019), and ocean monitoring (Sun et al., 2023).

More recently, TSP problems for teams of heterogeneous vehicles have attracted the attention of several researchers. The Multi-Depot Heterogeneous Fleet Routing Vehicle Problem (Yao et al., 2016) considers the use of vehicles with different capacities and speeds to solve a routing problem. In Garone et al. (2011), Garone et al. (2014), the Carrier-Vehicle Travelling Salesman Problem (CVTSP) was introduced. This variant of the TSP considers two different vehicles: a fast, small vehicle with limited autonomy and a slow carrier with unlimited autonomy, which must cooperate to perform rescue missions. This variant of the problem has been widely studied and extended in the field of last-mile delivery problems (Agatz et al., 2018; Poikonen et al., 2017; Poikonen and Golden, 2020), considering the scenario where a drone is responsible for package delivery and needs to be recovered and recharged by a mobile depot truck.

In this paper, we introduce a new variant of the TSP problem for a team of cooperative heterogeneous vehicles, the Travelling Schnauzer Problem (TSchP).[1] The main novelty of this extension lies in how to coordinate two heterogeneous vehicles that must perform a mission where the fast vehicle must visit the points of interest while remaining within a certain distance of the main slow vehicle. This is the case

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[1] The name *Travelling Schnauzer Problem* is derived from the fact that this problem is closely related to the Frechet distance which is classically defined as a dog-at-the-leash problem. Similarly, in our initial formulation of the problem presented in a series of oral workshops was: *A dog (a Schnauzer in our case) has n favourite spots at the park. Every time it goes for a walk on the leash of its owner, it will refuse to go home before visiting all its favourite spots. The dog runs faster than the owner and the leash has a limited length. Which is the optimal trajectory for the dog and the owner that minimizes the time of their walk?*

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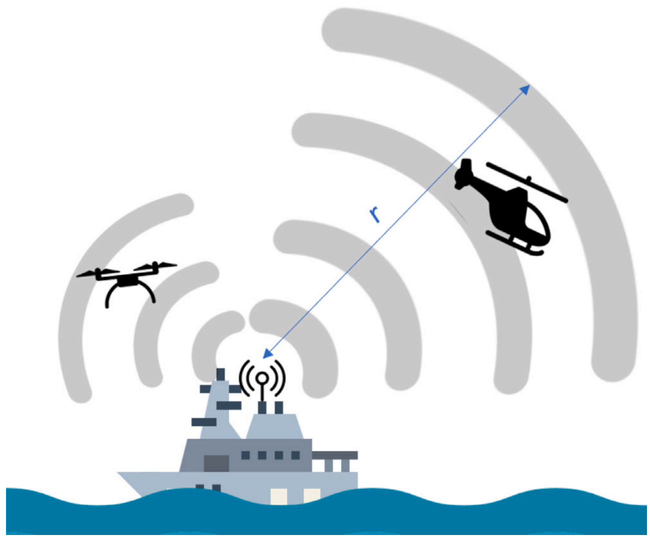


Fig. 1. Schematic of a ship linked with two vehicles by a communication constraint.

where the main vehicle must provide some kind of support, e.g. supply power or communication, to the small and fast vehicle without requiring direct interaction. The distance limitation may be due to the length of a physical cable in tethered drones (Nicotra et al., 2014) and small exploration submarines (Wu and Chen, 2019) or to the communication range in wireless connections, e.g in cooperative ocean search missions (Ke and Chen, 2022).

The TSchP can be seen also as an extension of the problem of constrained communication between vehicles and fixed base stations (Ortiz-Pena et al., 2013; Mozaffari et al., 2019), where some works already address the problem of finding the optimal path given a collection of fixed communication sites (Sabo et al., 2014; Zhang et al., 2018; Minelli et al., 2017). In the TSchP case, instead, the base stations are to be mobile as in the case of air-ground multirobot teams (Chaimowicz et al., 2005) or cooperative maneuvers between ships and aerial vehicles (Ma et al., 2018), as shown in Fig. 1.

The fact that the base vehicle must remain within a certain distance of the points of interest relates the TSchP to other variants of the TSP, such as the Close-Enough Travelling Salesman Problem (CE-TSP) (Coutinho et al., 2016; Di Placido et al., 2023). In the CE-TSP, rather than visiting a series of points, the agent can visit a covering region, similarly to the region formed by the range of connection between both vehicles of the TSchP. Compared to the CE-TSP, the TSchP adds further complexity to the routing scenario, as both vehicles will need to combine their movements, constraining the movements of the base to the speed of the fast vehicle and its visiting time at each point of interest. The TSchP is also related to problems where areas of interest must be reached by a gimballed sensor rather than the main vehicle itself. Accordingly, the movements of the gimbal can be computed such that the information obtained by the mobile sensor is maximized (Ross et al., 2016, 2019). This makes the TSchP a new class of problem that combines properties of heterogeneous vehicle routing problems (e.g., CV-TSP) and variants of the TSP (e.g., CE-TSP).

This paper is organized as follows. In Section 1, the problem is defined and formally stated. In Section 2, the simplified case where the agents have to visit a set of points with a given visit order is studied. In Section 3, a Mixed-Integer Second Order Conic Programming (MISOCP) formulation for the TSchP is provided, where the order of visit and the trajectories for both agents are computed simultaneously. In Section 4, a heuristic algorithm with guaranteed bounds is introduced. In Section 5, several numerical simulations are performed to prove the effectiveness of the heuristic method. Finally, in Section 6, we present some conclusion and discuss future works.

1. Problem formulation

In this paper, we introduce the following problem:

The Travelling Schnauzer Problem. Consider two point-mass agents, the *mobile base station* and the *vehicle*, whose positions in the 2D plane at time t are denoted by $\mathbf{x_b}(t)$ and $\mathbf{x_v}(t)$, respectively. The two agents are connected by a *leash* of length $r>0$, which imposes that the distance between the two agents must be always less than or equal to r,

$$\left\|\mathbf{x_v}(t)-\mathbf{x_b}(t)\right\| \leq r, \quad \forall t \geq 0. \tag{1}$$

It is assumed that the velocity of both agents can be changed instantaneously, which is equivalent to neglecting the dynamics of the agents. We assume their velocities are bounded by the maximal speeds v_b and v_v, where $v_b < v_v$. We assume that no obstacles are present and both agents have unlimited autonomy, i.e., they do not need to stop for recharging or refuelling, and the entire route can be completed without stops. The vehicle needs to visit a set of target points $\mathcal{T} = \{\mathbf{p_1}, \mathbf{p_2}, \dots, \mathbf{p_i}, \dots, \mathbf{p_n}\}$. Whenever the vehicle reaches a point $\mathbf{p_i}$, it will spend an amount of time $t_{p,i}$ at that point. Given the initial position of the two agents $(\mathbf{x_{b,0}}, \mathbf{x_{v,0}})$, find the minimum-time trajectories for the two agents so that the vehicle visits all the target points in $\mathcal{T}$, and, once all points are visited, the two agents return to the initial configuration $(\mathbf{x_{b,0}}, \mathbf{x_{v,0}})$.

2. The case of ordered points

Before considering the TSchP problem, where the order of the points is not defined, we focus on the simpler problem in which the order of visit of the points is fixed a priori. This approach allows us to derive the structural properties of the optimal policy, which will be crucial to formalize the TSchP problem in a computationally convenient way.

We consider the problem of finding the minimum-time trajectory that starts from the initial configuration $(\mathbf{x_{b,0}}, \mathbf{x_{v,0}})$, visits the sequence of points $\Pi = [\mathbf{p_1}, \mathbf{p_2}, \dots, \mathbf{p_i}, \dots, \mathbf{p_n}]$, and finishes at the final configuration $(\mathbf{x_{b,f}}, \mathbf{x_{v,f}})$. The trajectory of the two agents must ensure that, at each instant, the "leash constraint" is satisfied, that is, $\left\|\mathbf{x_v}(t)-\mathbf{x_b}(t)\right\| \leq r$. Clearly, for the problem to be feasible, the desired initial and final configurations must satisfy $\left\|\mathbf{x_{b,0}}-\mathbf{x_{v,0}}\right\| \leq r$ and $\left\|\mathbf{x_{b,f}}-\mathbf{x_{v,f}}\right\| \leq r$.

The first step to solve this problem is to characterize the minimum-time trajectory of the two agents for a generic couple of initial and final configurations (i.e., the case where $n = 0$). The following lemma characterizes this trajectory.

Lemma 1. *Let an initial configuration* $(\mathbf{x_{b,0}}, \mathbf{x_{v,0}})$ *and a final configuration* $(\mathbf{x_{b,f}}, \mathbf{x_{v,f}})$ *such that* $\left\|\mathbf{x_{b,0}}-\mathbf{x_{v,0}}\right\| \leq r$ *and* $\left\|\mathbf{x_{b,f}}-\mathbf{x_{v,f}}\right\| \leq r$. *The trajectory*

$$\begin{cases} \mathbf{x_b}(t) = \mathbf{x_{b,0}} + (\mathbf{x_{b,f}} - \mathbf{x_{b_0}})\dfrac{t}{t_{0,\mathrm{f}}} \\ \mathbf{x_v}(t) = \mathbf{x_{v,0}} + (\mathbf{x_{v,f}} - \mathbf{x_{v,0}})\dfrac{t}{t_{0,\mathrm{f}}} \end{cases} \tag{2}$$

for $t \in [0, t_{0,\mathrm{f}}]$ *where* $t_{0,\mathrm{f}} = max\left\{\dfrac{\left\|\mathbf{x_{v,f}}-\mathbf{x_{v,0}}\right\|}{v_v}, \dfrac{\left\|\mathbf{x_{b,f}}-\mathbf{x_{b,0}}\right\|}{v_b}\right\}$ *is*

1. *feasible, i.e., satisfies constraint (1) at all time* $t \in [0, t_{0,\mathrm{f}}]$;
2. *optimal, i.e., among all the possible feasible trajectories, is the minimal-time one.*

Proof. To prove feasibility, consider the change of variable $\lambda = t/t_f$. Eq. (2) becomes

$$\begin{cases} \mathbf{x_b}(\lambda) = (1-\lambda)\mathbf{x_{b,0}} + \lambda\mathbf{x_{b,f}} \\ \mathbf{x_v}(\lambda) = (1-\lambda)\mathbf{x_{v,0}} + \lambda\mathbf{x_{v,f}} \end{cases} \tag{3}$$

for all $\lambda \in [0, 1]$. At each instant, the distance between the base and the vehicle is

$$\left\|\mathbf{x_v}(\lambda)-\mathbf{x_b}(\lambda)\right\| = \left\|(\mathbf{x_{v,0}}-\mathbf{x_{b,0}})(1-\lambda) + (\mathbf{x_{v,f}}-\mathbf{x_{b,f}})\lambda\right\|. \tag{4}$$

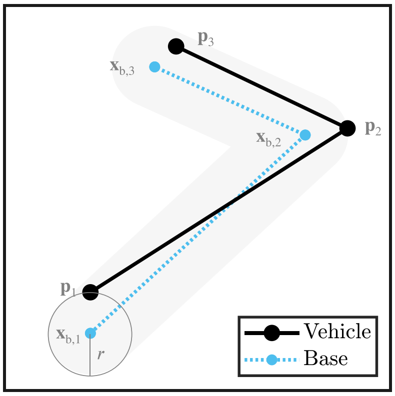


Fig. 2. Example of a feasible path for both vehicles. The grey area represents the admissible region for the vehicle based on the path of the base station. Blue dots in the base station path represent the position of the base when the vehicle arrives and departs from each point of interest.

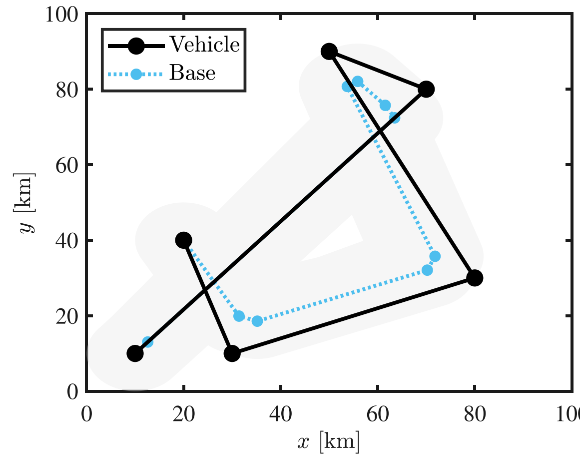


Fig. 3. Optimal path for a case with 6 ordered points of interest.

Using the triangular inequality, we obtain

$$\|\mathbf{x_v}(\lambda)-\mathbf{x_b}(\lambda)\| \leq (1-\lambda)\|\mathbf{x_{v,0}}-\mathbf{x_{b,0}}\|+\lambda\|\mathbf{x_{v,f}}-\mathbf{x_{b,f}}\|, \tag{5}$$

since $\|\mathbf{x_{b,0}}-\mathbf{x_{v,0}}\| \leq r$ and $\|\mathbf{x_{b,f}}-\mathbf{x_{v,f}}\| \leq r$ by hypothesis, we have

$$\|\mathbf{x_v}(\lambda)-\mathbf{x_b}(\lambda)\| \leq (1-\lambda)r+\lambda r = r, \tag{6}$$

which ensures the feasibility of the proposed trajectory. To prove optimality, it is enough to note that the total completion time is equal to the time required for either the base or the vehicle to go at its maximum speed along the segment from $\mathbf{x_{b,0}}$ to $\mathbf{x_{b,f}}$ or from $\mathbf{x_{v,0}}$ to $\mathbf{x_{v,f}}$, respectively. Consequently, no other trajectory for the two vehicles can have a smaller completion time, which concludes the proof.

The above Lemma dramatically simplifies our problem. In fact, to find the fastest trajectory that makes the vehicle visit the sequence Π while always satisfying the leash constraint, it is enough to determine the positions of the *base* when the *vehicle* reaches and departs from each point in the sequence, rather than searching for arbitrary paths on the plane. The trajectory between two successive visits can then be determined using the above lemma. A visual interpretation of this result is shown in Fig. 2.

We recall that when the vehicle reaches the point $\mathbf{p_i}$, it must stay on it for a time $t_{p,i} \geq 0$. Accordingly, let us introduce the points $\mathbf{x_{b1,i}}$ and $\mathbf{x_{b2,i}}$ denoting the positions of the *base* at the *vehicle*'s arrival and departure from the ith point, respectively. Let us also denote as t_i the travel time between the instant the vehicle's departure from ith point and the instant it reaches the $(i+1)$-th point. This time will always be greater than or equal to the time required for the vehicle to travel from $\mathbf{p_i}$ to $\mathbf{p_{i+1}}$,

$$\|\mathbf{p_i}-\mathbf{p_{i+1}}\| \leq v_v t_i \quad \forall i = 0\dots n, \tag{7}$$

and the minimum time for the base to go from $\mathbf{x_{b2,i}}$ to $\mathbf{x_{b1,i+1}}$,

$$\|\mathbf{x_{b2,i}}-\mathbf{x_{b1,i+1}}\| \leq v_b t_i \quad \forall i = 0\dots n, \tag{8}$$

where, to account for the initial and final configuration, we define $\mathbf{p_{n+1}} = \mathbf{x_{v,f}}$, $\mathbf{x_{b1,n+1}} = \mathbf{x_{b2,n+1}} = \mathbf{x_{b,f}}$ and $\mathbf{p_0} = \mathbf{x_{v,0}}$, $\mathbf{x_{b1,0}} = \mathbf{x_{b2,0}} = \mathbf{x_{b,0}}$, respectively.

Similarly, the distance between $\mathbf{x_{b1,i}}$ and $\mathbf{x_{b2,i}}$ must be less than or equal to the maximal distance the base can cover in the time $t_{p,i}$

$$\|\mathbf{x_{b2,i}}-\mathbf{x_{b1,i}}\| \leq v_b t_{p,i} \quad \forall i = 1\dots n. \tag{9}$$

Additionally, the leash constraint must hold for both $\mathbf{x_{b1,i}}$ and $\mathbf{x_{b2,i}}$,

$$\begin{cases} \|\mathbf{p_i}-\mathbf{x_{b1,i}}\| \leq r & \forall i = 1\dots n, \\ \|\mathbf{p_i}-\mathbf{x_{b2,i}}\| \leq r & \forall i = 1\dots n. \end{cases} \tag{10}$$

At this point, the problem of ordered points can be stated as follows

$$\min_{x,t} \sum_{i=1}^{n} t_i \tag{11}$$

subject to (7)–(10).

Such a formulation provides a Second-Order Conic Program (SOCP) problem, which is a specific type of convex problem that can be solved very efficiently. Fig. 3 depicts an example of the optimal path for a given set of 6 points. The visiting time is $t_{p,i} = 10$ min, the maximum distance is $r = 10$ km, and the vehicles have maximum speeds of $v_v = 60$ km/h and $v_b = 24$ km/h, respectively.

3. The travelling Schnauzer problem

In the previous section, we considered the simpler case where the order of the points to be visited was given a priori. In this section, we will deal with the TSchP problem, where the vehicle needs to visit the set of points $\mathcal{T} = \{\mathbf{p_1}, \mathbf{p_2}, \dots, \mathbf{p_i}, \dots, \mathbf{p_n}\}$ and the sequence of visiting points is unknown. In the following, for sake of simplicity, we assume that $\mathbf{p_1} = \mathbf{p_n} = \mathbf{x_{v,0}}$. However, the general case can be addressed by adding two fictitious points $\mathbf{p_0} = \mathbf{p_{n+1}} = \mathbf{x_{v,0}}$ similarly to the approach used in the previous section.

Building on top of the theoretical results for the ordered case, and exploiting the fact that the optimal trajectories are characterized only by the positions of the base when the vehicle reaches and departs from each point, we are able to mathematically formalize the TSchP problem using a minimal number of decision variables. Thanks to this convenient formulation, we can obtain the exact optimal solution to the TSchP problem.

First, we will start reformulating the TSchP problem using an approach inspired by the Miller–Tucker–Zemlin (MTZ) formulation of the TSP (Miller et al., 1960). Let us introduce two decision variables: (i) the binary variable q_{ij}, which is 1 if the point $\mathbf{p_j}$ is visited after the point $\mathbf{p_i}$ and 0 otherwise, and (ii) the integer variable u_i, which describes the visiting order of the point i. Since the initial and final configurations are known and fixed a priori, it must hold that

$$\sum_{i=1}^{n} q_{1i} = \sum_{j=1}^{n} q_{jn} = 1, \tag{12}$$

$$\sum_{i=1}^{n} q_{i1} = \sum_{j=1}^{n} q_{nj} = 0. \tag{13}$$

For the target points, we must ensure that each point is visited once and only once, which can be enforced through the following constraint

$$\sum_{i=1}^{n} q_{ik} = \sum_{j=1}^{n} q_{kj} = 1, \quad \forall k = 2,\dots,n-1. \tag{14}$$

Additionally, to avoid possible subtours in the chosen path and to ensure continuity, the following constraints must be added:

$$2 \leq u_i \leq n; \quad \forall i = 2, \dots, n, \tag{15}$$

$$u_i - u_j + 1 \leq (n-1)(1 - q_{ij}); \quad \forall i, j = 2, \dots, n. \tag{16}$$

These constraints ensure that the order of visit u_i increases by one in each visited point based on the order defined by q_{ij} (see (16)), and that the initial point is the first and last of the tour (see (15)).

With respect to the previous case, since the order of points is not known a priori, we must also modify the constraints concerning the travelling time t_i of the vehicle from point i to the following point as

$$(1 - q_{ij}) \left\| \mathbf{p_i} - \mathbf{p_j} \right\| \leq v_v t_i \quad \forall i, j = 1 \dots n, \tag{17}$$

$$(1 - q_{ij}) \left\| \mathbf{x_{b2,i}} - \mathbf{x_{b1,j}} \right\| \leq v_b t_i \quad \forall i, j = 1 \dots n, \tag{18}$$

where the decision variable q_{ij} ensures that the constraint is only active when the point j is visited after the point i, i.e., $q_{ij} = 1$. We recall that, thanks to Lemma 1, to define the fastest trajectory of the base station, we only need to compute the positions $\mathbf{x_{b2,i}}$ and $\mathbf{x_{b1,j}}$ of the base at the departure and arrival of the vehicle for points $\mathbf{p_i}$ and $\mathbf{p_j}$.

To account for the initial and final configuration of the base, we set $\mathbf{x_{b1,1}} = \mathbf{x_{b,0}}$ and $\mathbf{x_{b2,n}} = \mathbf{x_{b,0}}$, respectively.

Finally, the TSchP can be formulated as follows

$$\min_{x,q,t,u} \sum_{i=1}^{n-1} t_i \tag{19}$$

s.t. (9)–(10) (12)–(18).

This formulation results in a Mixed-Integer Non-Linear Programming (MINLP) problem. Interestingly enough, by manipulating the constraint (18) it is possible to rewrite (19) into a Mixed-Integer Second Order Conic Programming (MISOCP) problem. The main idea is to rewrite constraints (18) in logic form as

$$(q_{ij} = 1) \Rightarrow \left\| \mathbf{x_{b,i}} - \mathbf{x_{b,j}} \right\| \leq v_b t_i \tag{20}$$

which, as shown in Williams (2013), by means of the *big-M* method it can be reformulated as

$$\left\| \mathbf{x_{b2,i}} - \mathbf{x_{b2,j}} \right\| \leq v_b t_i + (1 - q_{ij}) M \quad \forall i, j = 1 \dots n, \tag{21}$$

where M is a sufficiently large constant. This reformulation allows us to rewrite (19) as the following MISOCP problem

$$\min_{x,q,t,u} \sum_{i=1}^{n-1} t_i, \tag{22}$$

s.t. (9)–(10), (12)–(17), (21).

The main merit of this formulation is that, although still NP-Hard, efficient solvers for MISOCP exist (Benson and Saglam, 2013) that, for moderately large instances of the problem, can solve optimally this kind of problem in a reasonable time.

Remark 1. The complexity of the resulting optimization problem depends on the number of points. The optimization problem consists of a quadratic number of binary optimization variables, a linear number of integer variables, and a linear number of real variables. It has a linear number of integer equality constraints, a linear number of integer inequality constraints and of second-order conic constraints. Additionally, it includes a quadratic number of mixed-integer linear inequality constraints and of mixed-integer second-order conic constraints.

Remark 2. The parameter M must be a constant large enough to ensure equivalence with Eq. (20). However, the choice of a non-tight value for the constant M can drastically increase the computation time required to solve the problem. By observing (20), the tightest value of M in this particular problem is given by

$$M = \max_{\substack{i=1,\dots,n,\\ j=1,\dots,n}} \left(\left\| \mathbf{p_i} - \mathbf{p_j} \right\| + 2r - v_b t_i \right). \tag{23}$$

This expression, based on (17), can be further simplified as

$$M = \max_{\substack{i=1,\dots,n,\\ j=1,\dots,n}} \left(\left\| \mathbf{p_i} - \mathbf{p_j} \right\| \left(1 - \frac{v_b}{v_v} \right) + 2r \right), \tag{24}$$

providing a simple way to compute the value of M beforehand based on the spatial distribution of the visiting points.

4. Heuristics and bounds

In this section, we propose a ρ-approximated heuristic for the TSchP based on the Euclidean TSP (E-TSP). The E-TSP is a particular case of the Travelling Salesman Problem where the points to be visited lie in the Euclidean space and the objective function is the Euclidean path length. Although NP-Hard, very efficient solvers specifically designed for the E-TSP exist (e.g., the solver Concorde (Applegate et al., 1998)), which are able to optimally solve E-TSP with hundreds of target points in some seconds.

The proposed heuristic consists of two stages: first, a possibly approximated solution of the Euclidean E-TSP is computed obtaining the path of the vehicle and the associated visiting order; then, the solution of SOCP problem for the ordered sequence is computed obtaining the path of the base. The proposed heuristic for the TSchP can be summarized as follows.

Algorithm 1 TSchP Heuristic.

1. Solve a (possibly approximated) E-TSP problem and obtain the associated visiting order Π;
2. Solve problem (11) using the order sequence Π.

The idea behind this heuristic comes from the fact that, if the *vehicle* had no leash, its optimal path would be the one of the E-TSP. Therefore, using the E-TSP to determine the visiting order ensures at least the minimality of the path length of the *vehicle*. This observation also allows us to define a lower bound for the optimal solution of the TSchP.

Lemma 2. *Let l_{ETSP} be the length of the optimal E-TSP tour for a set of points $\mathcal{T}$. Then*

$$LB = \frac{l_{ETSP}}{v_v} + \sum_{i=1}^{n} t_{p,i} \tag{25}$$

is a lower bound for the optimal cost of the TSchP.

Proof. Being l_{ETSP} the shortest length to complete the tour of the given visiting points, there is no possible combination of points which provides a total distance l such that $l \leq l_{ETSP}$. Since v_v is the maximal speed of the vehicle, no solution to the TSchP can have a completion time smaller than (25).

Using a similar idea, we can provide an upper bound for the proposed heuristic in the case where the visiting order Π is obtained by solving a $(1+\varepsilon)$-approximated E-TSP problem. In this case, we have the following lemma.

Lemma 3. *Let l_{ETSP} be the length of the optimal E-TSP tour for a set of points $\mathcal{T}$. Let Π be obtained by solving a $(1+\varepsilon)$-approximated E-TSP problem. Then*

$$UB = \frac{l_{ETSP}(1+\varepsilon)}{v_b} + \sum_{i=1}^{n} t_{p,i} \tag{26}$$

is an upper bound for the solution provided by Algorithm 1.

Proof. The first observation is that, in the worst case, the ε-approximated algorithm for the E-TSP will give a tour of length $l_{ETSP}(1+\epsilon)$.

At this point, it is enough to note that (26) is always greater or equal than the solution obtained for Algorithm 1 by imposing $\mathbf{x_{b1,i}} = \mathbf{x_{b2,i}} = \mathbf{p_i}$ for $i = 1, \dots, n$.

Using these two bounds, it is possible to prove that the proposed heuristic is a ρ-approximated heuristic for the TSchP.

Proposition 4. *Let l_{ETSP} be the length of the optimal E-TSP tour for a set of points $\mathcal{T}$. Let Π be obtained by solving a $(1+\epsilon)$-approximated E-TSP problem. The solution provided by Algorithm 1 is at most $\rho = \frac{v_v}{v_b}(1+\epsilon)$ times the optimal solution of the TSchP.*

Proof. Using Lemma 2 we can state the following inequality:

$$LB \leq t^{\star}_{TSchP}, \tag{27}$$

where $t^{\star}_{TSchP}$ is the optimal solution to the Travelling Schnauzer problem. Additionally, from Lemma 3 we have that

$$t^{\star}_{TSchP} \leq t_{heuristic} \leq UB, \tag{28}$$

where $t_{heuristic}$ is the completion time associated with the solution of Algorithm 1. Combining (27) and (28) we obtain

$$1 \leq \frac{t_{heuristic}}{t^{\star}_{TSchP}} \leq \frac{UB}{LB} = \frac{v_v(1+\epsilon)+\alpha}{v_b+\alpha}, \tag{29}$$

where $\alpha = \sum_{i=1}^{n} t_{p,i}/l_{ETSP}$. The proof concludes by observing that (29) is maximum for $\alpha = 0$.

Remark 3. Note that whenever $\alpha = \sum_{i=1}^{n} t_{p,i}/l_{ETSP} > 0$ the heuristic provides a better worst-case approximation guarantee than the one provided by Proposition 4, which considers $\alpha = 0$.

Remark 4. The presented heuristic and the associated bounds can be used to help solving the MISOCP problem. For instance, the solution of the heuristic can serve as an initial feasible solution for the MISOCP solver to reduce the computation time. Furthermore, more advanced heuristics can be built based on it. For instance, along with the ruin-and-recreate approach, it is possible to evaluate whether small perturbations in the visit sequence of the vehicle, along with the corresponding optimal trajectory of the base given by the SOCP problem, provide a better overall mission time.

5. Numerical results

In this section, we test the MISOCP formulation and the proposed heuristic on a marine surveillance application. In particular, we compare the performances for different parameters and we numerically evaluate the computational burden. For the first step of the heuristic, we solve the exact E-TSP problem using the MTZ formulation. However, different formulations, E-TSP solvers, or approximated solutions can also be used. The code to solve the MISOCP formulation and the proposed heuristic is available on GitHub at https://github.com/MatthiasPez/TravellingSchnauzerProblem.

5.1. Case study in marine surveillance

We consider a marine surveillance task involving a ship, which plays the role of the slow mobile base station, and a UAV, which plays the role of the fast vehicle. The task requires the UAV to visit a set of locations of interest while maintaining the communication with the base station. Based on realistic cruise speeds for typical patrol vessels and UAVs actually used in marine surveillance operations such as the Comandanti-class patrol vessel (max speed 46.3 km/h) and the UMS Skeldar V200 (max speed 140 km/h), the maximum speeds of the slow vehicle and of the fast vehicle are set to $v_b = 24$ km/h and $v_v = 60$ km/h, respectively. Since communication in an obstacle-free environment is guaranteed as long as the line of sight is maintained, it is required that the UAV remains above the horizon line. Using standard approaches, assuming that the base station antenna is 10 m height and the UAV is at an altitude of 70 m, the maximum distance to guarantee communication is about 41.5 km. For this reason, we set $r = 40$ km. The visiting time at each point is considered invariant, with $t_{p,i} = 1$ h for $i = 1, \dots, n$. The points to visit are located in an area of $400 \times 400\,\mathrm{km}^2$. Point locations are determined by independent and identically distributed uniform random variables, although they may not be representative of real-world instances and could potentially result in easier optimization problems.

5.2. Parameter sensitivity and suboptimality of the heuristic

In this subsection, we compare how the performances of MISOCP formulation and the heuristic are affected by the problem parameters. In particular, we consider different maximum distances r, maximum base speeds v_b, and numbers of points n.

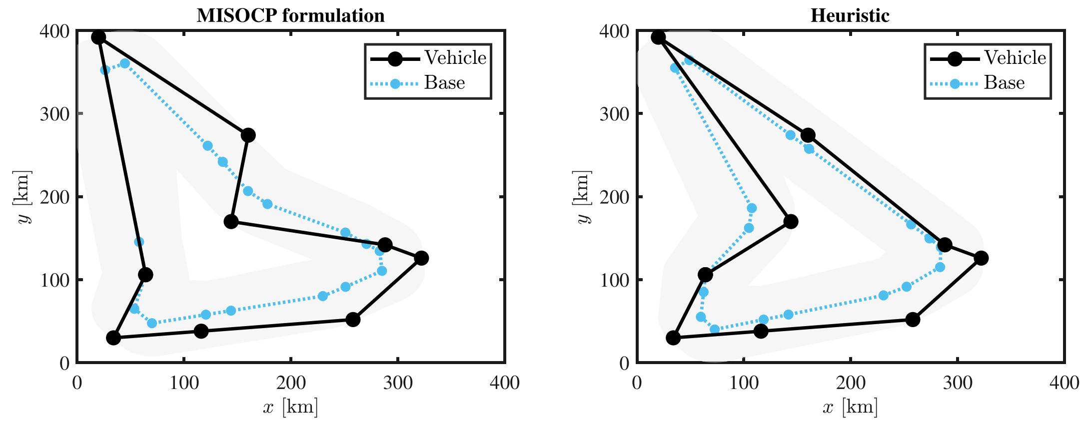
In Fig. 4, we visually compare the paths obtained by solving the exact MISOCP formulation and the proposed heuristic for different parameter combinations, using a fixed point distribution consisting of 9 target points.

Fig. 4(a) compares the paths obtained by solving the exact MISOCP formulation and the proposed heuristic when the parameters are set as described in the previous subsection. The main difference between the two resulting paths is in the order of visits of one point in the computed sequence of visits. This indicates that, for the specific point locations considered, it is convenient in terms of the overall mission time to use a longer vehicle path. Nevertheless, despite the visible change in the paths, the cost degradation in terms of total mission time is only about 2.5%.

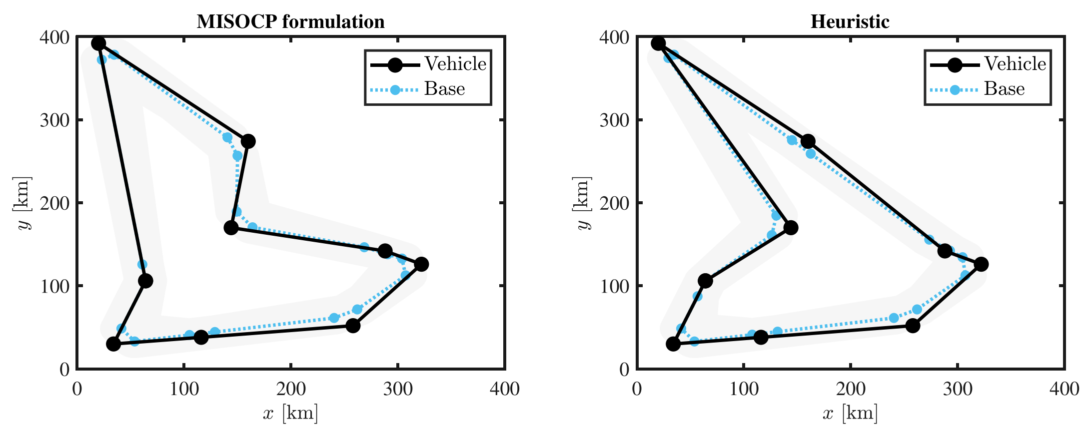
Fig. 4(b) compares the paths given by MISOCP formulation and the proposed heuristic in a different scenario where, for safety reasons, the maximum distance between the vehicle and the base is halved ($r = 20$ km). In this case, the cost degradation is about 1%. Indeed, when the radius is small, the base station is required to pass close to the points to visit, and the optimal cost of the MISOCP is mainly affected by the maximum base speed. Consequently, the optimal solution tends to minimize the distance travelled by the base, and the visit point sequence tends to converge towards the solution of the Euclidean TSP. Remarkably, this is the case considered in Lemma 3, and the optimal cost of the MISOCP is close to the upper bound UB. This suggests that the proposed heuristic is particularly effective when the radius is small, such as in the case of tethered drones (Nicotra et al., 2014) or exploration submarines (Wu and Chen, 2019).

Fig. 4(c) compares the paths given by MISOCP formulation and the proposed heuristic in the case where the maximum vehicle speed is halved ($v_v = 30$ km/h), possibly to increase the autonomy of the UAV. In this case, the cost degradation is null. Indeed, when the speeds are similar, the base station can closely follow the vehicle and, especially when the visit times are high, the MISOCP solution is largely affected by the maximum vehicle speed. The optimal solution tends to minimize the distance travelled by the vehicle and, also in this case, the visit sequence tends to the solution of the Euclidean TSP. This is the case considered in Lemma 2 and the optimal cost of the MISOCP is close to the lower bound LB.

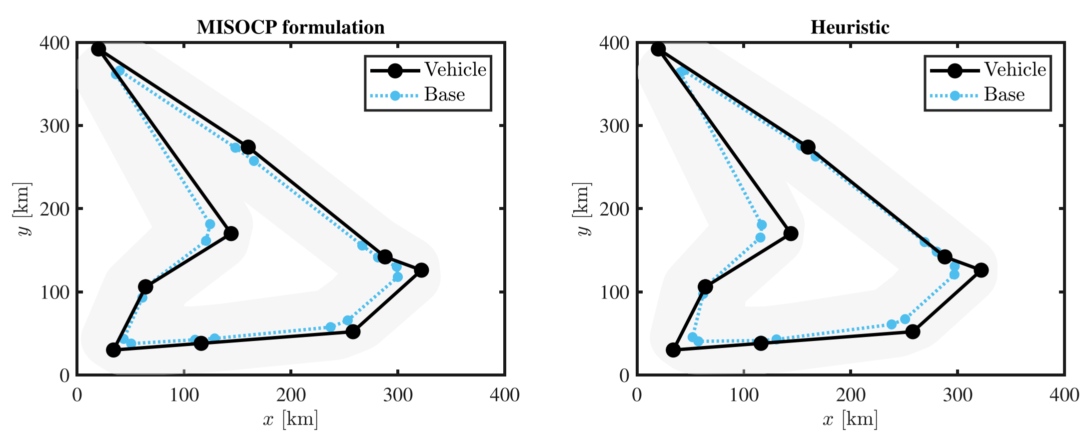
Fig. 5 reports the average cost degradation in terms of total mission time of the heuristic with respect to the exact MISOCP formulation varying the maximum distance r (top) and the maximum base speed v_b (bottom) over 100 random distributions with 9 points. From the figure, we can observe that in limit cases, where r is either small or large, or when v_b is large, the cost degradation is negligible. Indeed, whenever the base and the vehicle are close enough (r small or v_b large) or far enough (r high), the MISOCP is minimized by the minimum length path of the vehicle and the optimal solution coincides with the heuristic. Conversely, in other intermediate cases, the cost degradation



(a) Paths obtained by solving the exact MISOCP formulation (left) and the heuristic (right). The two paths differ mainly in the collocation of a point in the sequence of visits. The cost degradation is 2.5%



(b) Paths obtained by solving the exact MISOCP formulation (left) and the heuristic (right) with smaller communication radius ($r = 20$ km). The cost degradation is 1%.



(c) Paths obtained by solving the exact MISOCP formulation (left) and the heuristic (right) with slower vehicle ($v_v = 30$ km/h). The paths are identical and no cost degradation is incurred with the proposed heuristic.

Fig. 4. Paths obtained for the same point distribution with different parameters.

is larger. In these cases, the solution of the MISOCP formulation tends to minimize the distance travelled by the base station possibly increasing the distance travelled by the vehicle. Consequently, the sequence of points to visit may differ from the solution of the E-TSP if a longer path for the vehicle, while satisfying distance constraints, allows for a shorter path for the base station. This scenario corresponds to Fig. 4(a)–(b). Note that in the case where $v_b = 0$, as in the case of fixed base stations (Mozaffari et al., 2019), not all the point distributions admit a feasible solution of the problem since points further than r from the base station position cannot be visited. If all points are within

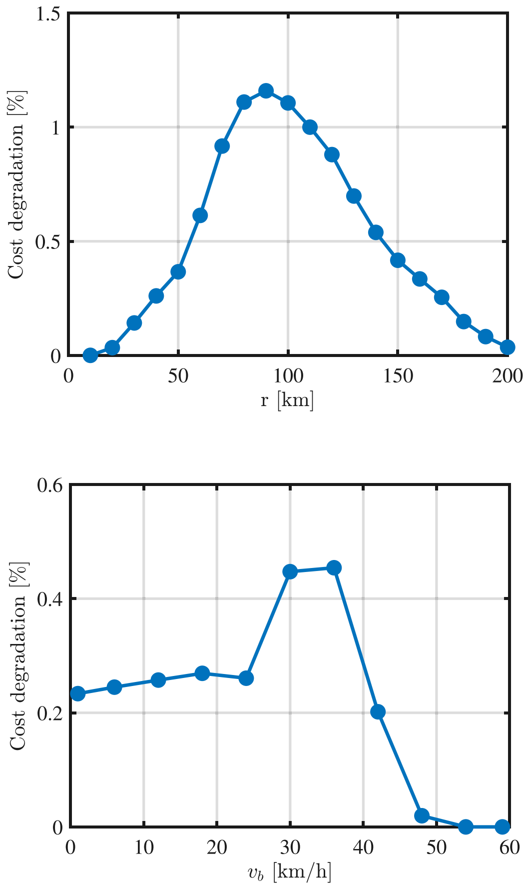


Fig. 5. Average cost degradation varying the maximum distance r (top) and the maximum base speed v_b (bottom).

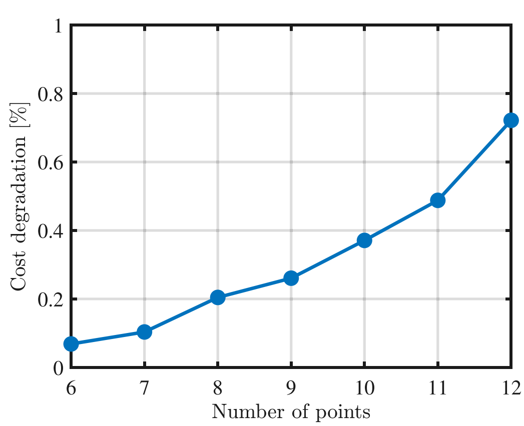


Fig. 6. Average cost degradation varying the number of points.

a distance r to the base station, it can be shown that the proposed heuristic coincides with the solution of the MISOCP.

In order to assess the performance of the heuristic as the number of points varies, we compute the average cost degradation of the heuristic with respect to the exact MISOCP formulation over 100 random point distributions. The results are reported in Fig. 6. It is worth noting that the average degradation remains less than 1%, providing a result very close to the optimal. More details on the cost degradation distribution are shown in Table 1. From this table, it can be observed that 99% of cases fall within the 5% degradation threshold and 100% of cases fall within the 10% degradation threshold.

Table 1
Distribution of cost degradation of the heuristic for different thresholds.

N. of points	$< 1\%$	$< 2.5\%$	$< 5\%$	$< 10\%$
6	97%	99%	100%	100%
7	97%	99%	100%	100%
8	91%	99%	100%	100%
9	92%	98%	99%	100%
10	87%	94%	99%	100%
11	83%	94%	99%	100%
12	71%	92%	99%	100%

We can conclude that, on average, the cost degradation is small. Larger cost degradations typically occur when the minimal vehicle path is not associated to the minimal base station path (as in Fig. 4(a)–(b)). The main limitation of the heuristic in that scenario is due to the two-stage approach in which the visiting order is obtained without considering the base station. This means that a suboptimal E-TSP solution might, in line of principle, outperform the optimal E-TSP solution when considering the whole TSchP problem. As such, there is a clear limitation on the properties that we can infer on the overall problem based on one or another solution to the E-STP, as they could translate into better or worse performance of the TSchP. This limitation could be mitigated by reformulating the first steps of the heuristics to incorporate the dependence on the base station path or solving the exact formulation in a single stage with meta-heuristics such as the Genetic Algorithm (Di Placido et al., 2022) or with methods based on Reinforcement Learning (Wu et al., 2023).

5.3. Computational analysis and intractability of the exact formulation

In this section we compare the computation times of both approaches. For each number of points, results are averaged over 100 configurations with random point locations. The tests are performed on a desktop computer Intel(R) Core(TM) i7-6700U CPU (3.40 GHz) with 24 GB Ram. The code is implemented and executed on Matlab R2024a with the Yalmip suite (version 2023.06.22) and Gurobi solver (version 11.0.0).

Fig. 7 reports the computation time of both approaches for different numbers of points. The computation time required to solve the MISOCP problem formulation dramatically increases with the number of points. In particular, for 12 points, the average computation time is 170 s and up to 3400 s in the worst case (although additional tests have shown that improvements can be obtained by admitting a relative gap of 20% to the Gurobi solver with a marginal average cost degradation). Conversely, the computation time of the heuristic is always below 1 s for instances with up to 12 points. In practice, the exact MISOCP formulation cannot be applied to solve problems with more than 13 points. Conversely, the heuristic can solve larger instances in a modest amount of time. For example, it can solve the ‘berlin52’ instance from TSPLib (Reinelt, 1991) in about 5 s. Additional tests have shown that solving the E-TSP problem based on the MTZ formulation is the main bottleneck in the considered implementation of the heuristic. This can be avoided by using more efficient formulations or specific TSP solvers such as Concorde (Applegate et al., 1998). Specifically, with Concorde, the heuristics can solve the ‘bier127’ and ‘gil262’ instances from TSPLib in about 0.4 s and 9.8 s, respectively. Different solvers for the E-TSP may result in different computation times. Therefore, selecting the most appropriate E-TSP solver for the proposed heuristic may depend on the spatial distribution of points. The reader is referred to McMenemy et al. (2019), Bossek and Trautmann (2016) for a detailed comparison between TSP solvers.

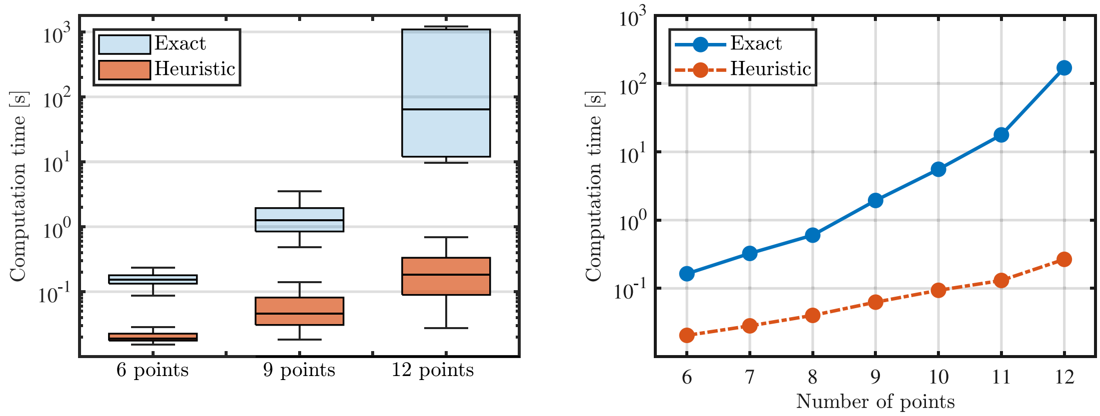


Fig. 7. Computation time with respect to the number of points considered for the exact formulation and the heuristic.

5.4. Discussion and extensions

Section 5.2 showcases the variability of the heuristic's performance based on parameter values and illustrates how the properties of the problem might vary based on these values. Given this, in some situations there may be alternative approaches to the first step of our heuristic once these properties are identified. For instance, in cases where the mobile base is slow enough and the maximum distance is high enough, the point sequence given by the E-TSP might be outperformed by a sequence associated with a longer vehicle path but a shorter base path. In such instances, the Close-Enough Travelling Salesman Problem (CE-TSP) (Coutinho et al., 2016) might be a promising alternative to define the order of visit of the points of interest. The rationale behind this approach is that, given the much faster vehicle, it will likely reach the target points sooner than the base, regardless of the point distribution. This makes the fastest path one in which the base reaches the r distance in the least time possible, aligning with the definition of the CE-TSP. However, an evaluation of the potential benefits of this approach is necessary, as the use of CE-TSP heuristics could impact the heuristic's overall performance and computational time. Alternatively, the framework and methodologies devised to solve the exact CE-TSP, such as the solution based on the branch-and-bound algorithm and SOCP proposed in Coutinho et al. (2016), could be adapted to solve the exact TSchP. Similarly, also heuristics used for the CE-TSP, such as those proposed by Behdani and Smith (2014), Carrabs et al. (2017), Gulczynski et al. (2006), could serve as inspiration for future works of the TSchP.

Beyond the relationships between parameters, certain assumptions of the TSchP also affect the approach to solving this path planning problem. In our problem statement, we assume that both vehicles have unlimited autonomy. That means that there is no need to recharge or refuel them, allowing both to complete the entire mission. This assumption may not be realistic in some marine applications, such as when using small electrically powered drones or submarines. Extensions of the TSchP where this assumption does not hold anymore can draw on inspiration from related problems in the literature. In cases where only the vehicle has limited autonomy, this version of the TSchP could take inspiration from the carrier-vehicle problem (Garone et al., 2008, 2011). In that problem, inspired by marine applications, the UAV must return to the main vehicle (the carrier) to charge. An extended variant of the TSchP would need to account for the added complexity of determining how many target points could be reached before requiring a new landing or recharge. In cases where both vehicles need to be recharged or refuelled, the problem might resemble a TSchP version of the green vehicle routing problem (Felipe et al., 2014) or the recharging vehicle problem (Conrad and Figliozzi, 2011).

The simulation results presented in this section consider a UAV and communications over the air. However, in several oceanic applications, the vehicle might be an Autonomous Underwater Vehicle (AUV) and communications with the base might be underwater. In this case, the proposed framework can still apply as long as communication constraints are still captured by distance constraints.

In the case of disturbances caused by the wind or ocean current, the formulation of the problem becomes challenging. If the disturbances only affect the small vehicle and the fastest path between two points is still given by the straight line, the problem formulation can be adapted to include asymmetric TSP instead of standard TSP. In other cases, defining the optimal path becomes a clear challenge.

6. Conclusions

In this paper, we have presented a novel variant of the TSP called the Travelling Schnauzer Problem (TSchP). This problem defines a minimum-time trajectory planning for two heterogeneous vehicles whose distance must remain lower or equal to a defined constant r at all times. In the first part of the paper, the problem is defined and a formulation for the simplified case of a priori ordered points is given. In the second part, the Travelling Schnauzer Problem is studied and suitably formulated as a mixed-integer second-order conic programming. Additionally, a fast heuristic is provided which is proved to be a ρ-guarantee approximation of the optimal result. Several numerical simulations end the paper to show the effectiveness of the proposed heuristic.

Future research on the TSchP could explore alternative approaches to address the limitations of our MISOCP formulation. blueIn particular, the methods to optimally solve the CE-TSP can be adapted to optimally solve the TSchP. Heuristics can be designed as a one-shot solution, e.g. a Genetic Algorithm, or as enhanced versions of our two-step heuristic, in particular exploiting the solution to the CE-TSP problems as an alternative way to compute the order of visits. Another possibility consists of adding an additional step to the proposed heuristic to test if small modifications of the E-TSP visit sequence achieve lower mission time.

Regarding the problem formulation, the extension to the 3D plane case where the altitude difference becomes part of the formulation is a future line of research. Furthermore, future works will focus on extending the problem to include multiple fast vehicles. The case of several fast vehicles, e.g. a fleet of UAVs, is a common real-life scenario, where each vehicle must be assigned specific points to visit, adding a selection dimension to the current formulation. In this scenario, it is particularly relevant to devise distributed solutions, where fast vehicles compute on board their own trajectories, possibly adapting them to visit new points of interest added online.

CRediT authorship contribution statement

Emanuele Garone: Writing – review & editing, Writing – original draft, Supervision, Methodology, Formal analysis, Conceptualization. **Nicolás Bono Rosselló:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Conceptualization. **Matthias Pezzutto:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology. **Tam W. Nguyen:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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