Hamiltonian control to desynchronize Kuramoto oscillators with higher-order interactions

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Hamiltonian control to desynchronize Kuramoto oscillators with higher-order interactions

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Many complex systems, as brain dynamics, have to avoid strong synchronization to function correctly.

There are increasing evidences that many complex systems, e.g. brain dynamics, can be better modelled with higher-order interactions.

We propose a control method [7], which generalizes [1,2], that is able to desynchronize the Kumaroto model with higher-order interactions.

Higher-order Kuramoto model

Kuramoto oscillators with higher order interactions

$$\dot{\theta}_i = \omega_i + \frac{K_1}{N}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i) + \frac{K_2}{N^2}\sum_{j,k=1}^{N} B_{ijk}\left[\sin(\theta_j + \theta_k - 2\theta_i) + \sin(2\theta_k - \theta_j - \theta_i)\right], \quad (1)$$

where $\mathbf{A}$ and $\mathbf{B}$ are resp. adjacency matrix and tensors, $\boldsymbol{\omega}$ the natural frequencies.

Measure of synchronization: the order parameter

$$R(t) := \left| \frac{1}{N}\sum_{j=1}^{N} e^{\imath\theta_j(t)} \right|$$

and

$$\hat{R} := \langle R(t) \rangle_{t\in[t_i,t_f]}.$$

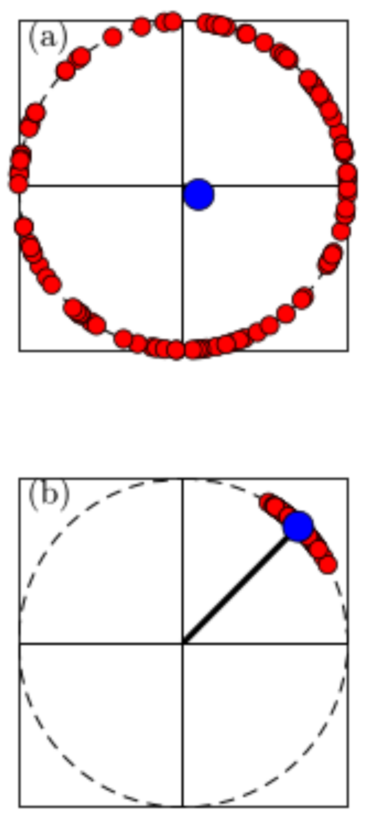

Fig. 1: R illustrations: (a): $R \approx 0$, ; (b): $R \approx 1$.

Pinning control using Hamiltonian theory

Hamiltonian from the function

$$H(\mathbf{I},\boldsymbol{\theta}) := H_0 + V_1 + V_2 := \sum_{i=1}^{N} I_i\omega_i - \frac{K_1}{N}\sum_{i,j=1}^{N} A_{ij}\sqrt{I_iI_j}(I_j - I_i)\sin(\theta_j - \theta_i) - \frac{K_2}{N^2}\sum_{i,j,k=1}^{N} B_{ijk}\sqrt[3]{I_iI_jI_k}(I_j + I_k - 2I_i)\sin(\theta_j + \theta_k - 2\theta_i), \quad (2)$$

such that

- On the manifold $\mathcal{T}_c := \{(\mathbf{I},\boldsymbol{\theta}) | \forall i : I_i = c\}$, $\dot{\theta}_i = \frac{\partial H}{\partial I_i} = (1)$,
- One can remark that (1) synchronizes $\Rightarrow$ $\mathcal{T}_c$ is unstable in (2).

We stabilize $\mathcal{T}_c$ by perturbing H with a suitable function $f(V)$ ([4,5]), where $V = V_1 + V_2$. For a subset of M $(\leq N)$ observed-controlled nodes, we modify (1) by adding

$$h_i^{full} := \left[\frac{\partial f(\tilde{V})}{\partial I_i}\right]\Big|_{\mathbf{I}=\mathbf{c}} \quad (3) \qquad \text{or} \qquad h_i^{pairwise} := \left[\frac{\partial f(\tilde{V}_1)}{\partial I_i}\right]\Big|_{\mathbf{I}=\mathbf{c}} \quad (4)$$

where $\tilde{V}$ (resp. $\tilde{V}_1$) is the restriction of V (resp. V_1) to sums on those M pinned nodes.

Results on complete hypergraphs

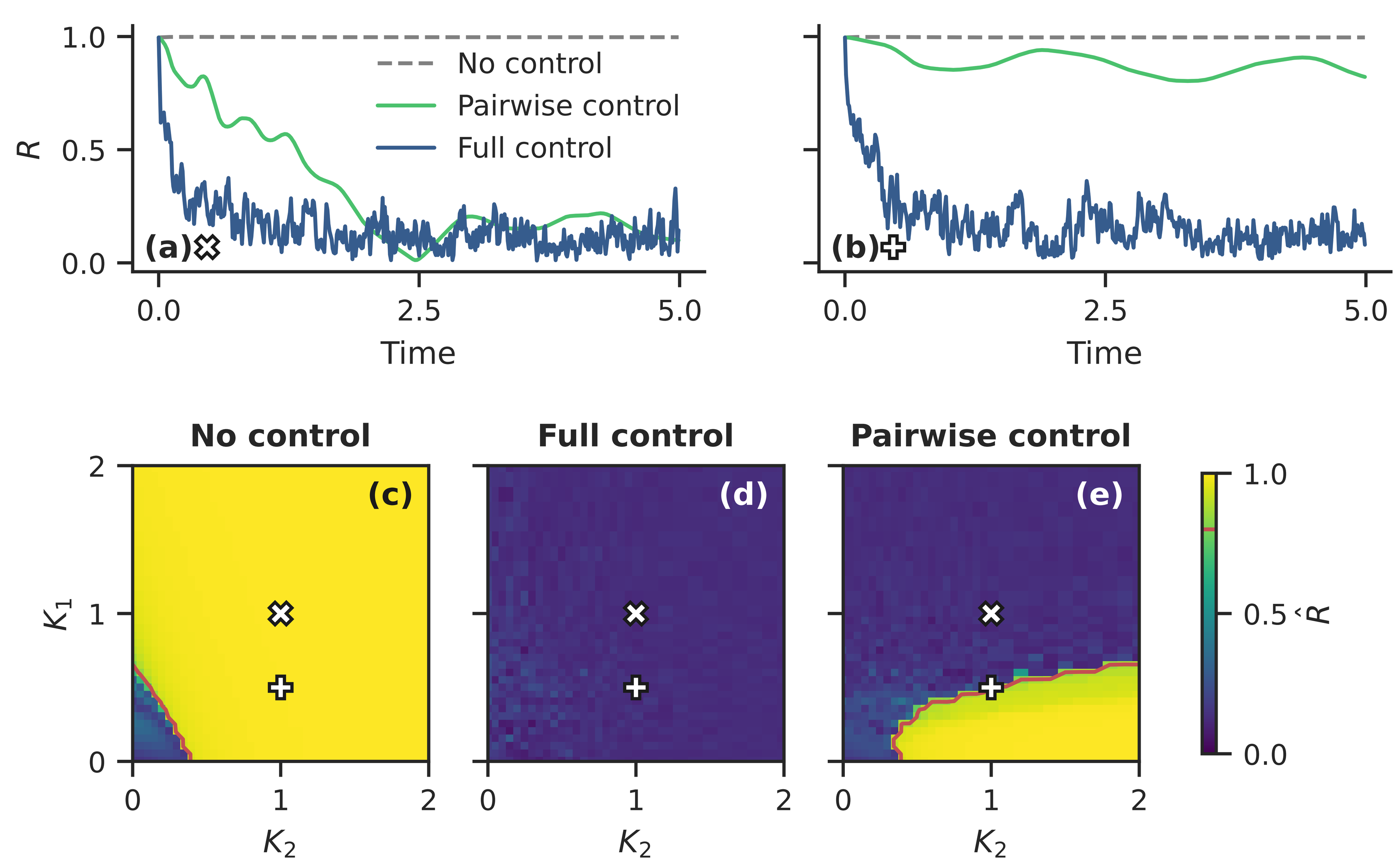

Fig. 2: Illustration of the control on a complete hypergraph with $N = 50$. (a) and (b): $R(t)$ with (resp.) $(K_1, K_2) = (1, 1)$ and $(K_1, K_2) = (0.5, 1)$; (c-d): $\hat{R}$ in function of (K_1, K_2).

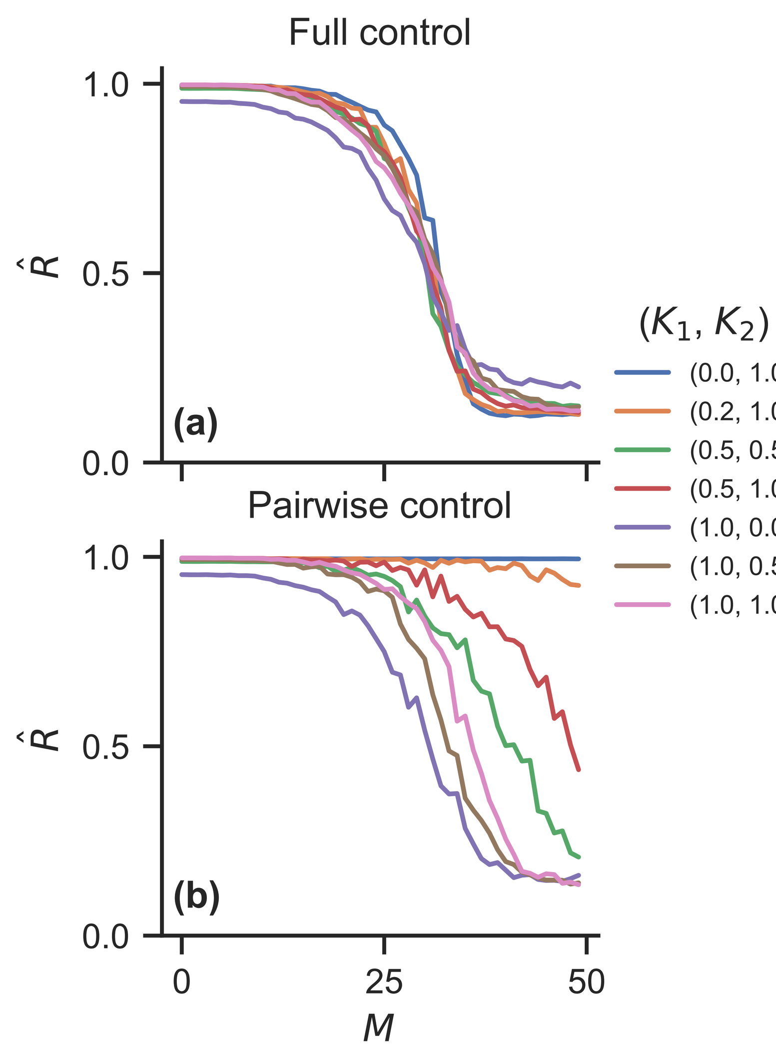

Fig. 3: $\hat{R}$ in function of M for both h^{full} (a) and $h^{pairwise}$ (b) on a complete hypergraph with $N = 50$.

Results on a brain hypergraph [6]

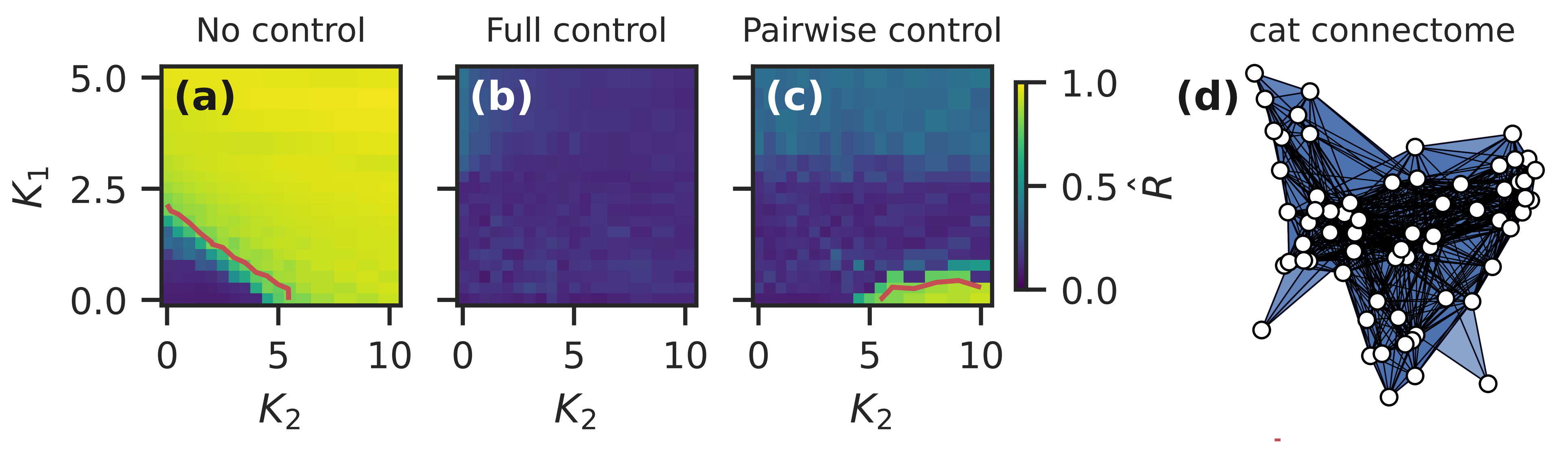

Fig. 4: (a-c): $\hat{R}$ in function of (K_1, K_2) on a hypergraph drawn from a cat connectome (d), with $N = 65$ nodes, 730 links and 3613 triangles.

Discussions

- This method [7], generalizing [1,2], achieves to desynchronize Higher-order Kuramoto model on various hypergraph topologies.
- We get (partial) desynchronization without pinning all the nodes.
- $h^{pairwise}$, equivalent with [1], desynchronizes the system for large K_1; h^{full} is helpful when K_1 is small in comparison to K_2.
- This method requires an exhaustive knowledge of the states and parameters. A more feasible form (like [2]) should be developed in future works.
- This methodology can be extended to system with interactions of any order.

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Check our paper [7] out!

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