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Response to synchronization in firefly-inspired distance-dependent coupled heterogeneous oscillators

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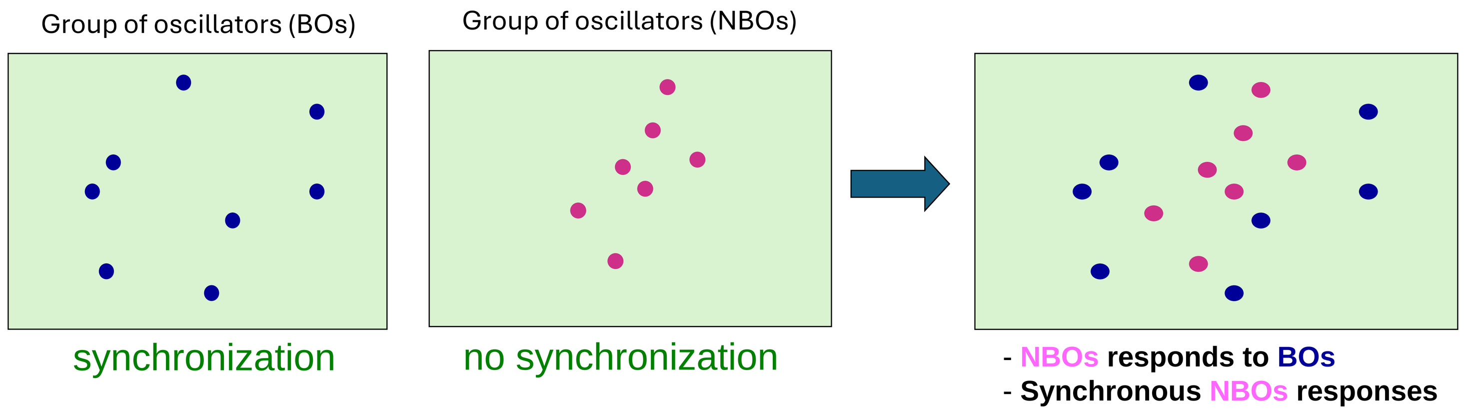
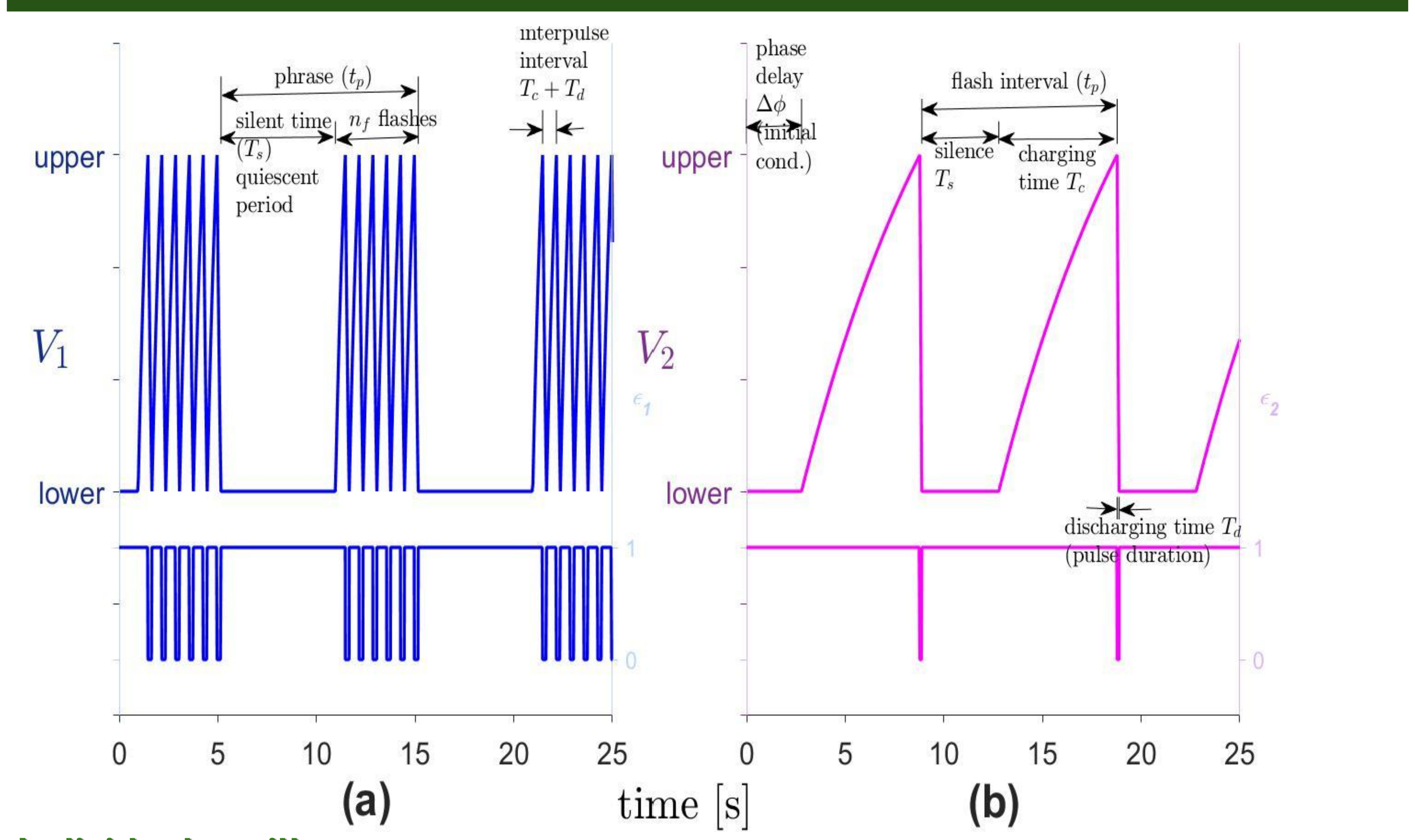
Inspired by a firefly phenomenon, namely the response to synchronization [1], where the firefly females' response is enhanced by the males' synchronization during their courtship, we consider the model based on **bursting (BO) and nonbursting (NBO) oscillators** related, respectively, to males (M) and females (F) [2] to study several pairwise network configurations going from the simplest case in which a male is capable of controlling the simultaneous response of females to the general case of global coupling and when the oscillators are **nonidentical (heterogeneous)** and have **implicitly the capacity to move (variable coupling strength)**. Integrating the model equations, we characterize the response using the flashing rate (FR) and the quality of information (QI), quantities that measures the global response of the females, considering their temporal flashing patterns. Our study includes different kinds of oscillators, each related to specific firefly species.

Response to synchronization

Considering two groups of dissimilar oscillators (BOs and NBOs), the BOs can synchronize while the NBOs cannot. Nevertheless, NBOs can respond to the BOs synchronization when both groups of oscillators are together.



Firefly-inspired model: BO (M) & NBO (F)



Individual oscillator

$$\frac{dV_i}{dt} = \frac{\ln 2}{T_{c,i}}\left(V_{Mi} - V_i(t)\right)\varepsilon_i(t) - \frac{\ln 2}{T_{d,i}} V_i(t)\left(1-\varepsilon_i(t)\right), \text{ with } T_{c,i} = T_{c0} + \xi_i\sqrt{D} \text{ and } T_{d,i} = T_{d0} + \xi_i\sqrt{D}$$

$$V_i(t) = \left(V_i(t) - V_i^{\text{lower}}\right)\varepsilon_i(t) + V_i^{\text{lower}}$$

ξ_i is a random number coming from a Gaussian distribution; D is an intensity factor

Binary variable ε(t)

$\varepsilon_i(t) = 1$: extinguished oscillator (charging and silent stage)

$\varepsilon_i(t) = 0$: fired oscillator (discharging stage)

Thresholds

$V_i^{\text{lower}} = \dfrac{V_{Mi}}{3}$: lower threshold $V_i^{\text{upper}} = \dfrac{2V_{Mi}}{3}$: upper threshold

The transition between the states determined by ε is described by the following relation:

If	$V_i(t) = V_i^{\text{lower}}$	and	$\varepsilon_i(t) = 0$	then	$\varepsilon_i(t_+) = 1;$
If	$V_i(t) = V_i^{\text{upper}}$	and	$\varepsilon_i(t) = 1$	then	$\varepsilon_i(t_+) = 0;$
If	$V_i(t) = V_i^{\text{lower}}$	and	$\varepsilon_i(t) = 1$	then	$\varepsilon_i(t_+) = 1,$

where t_+ in the condition given by the third relation and is defined in the interval

$$t = [t_+ \quad (k-1)(T_p + n_f(T_c + T_d)) + \Delta\phi]$$

for every k interburst period or phrase, i.e., for every complete cycle comprising the active phase and the silent time.

References

[1] Moiseff, A., & Copeland, J. (2010). Firefly Synchrony: A Behavioral Strategy to Minimize Visual Clutter. *Science*, **329**(5988), 181.
[2] Ramírez-Ávila, G. M., & Kurths, J. (2016). Unraveling the primary mechanisms leading to synchronization response in dissimilar oscillators. *The European Physical Journal Special Topics*, **225**(13), 2487-2506.
[3] Tomaselli, C., Ramírez-Ávila, G. M., Gambuzza, L. V., Frasca, M., Tuci, E., & Carletti, T. (2024). Implementation of the response to synchronization in e-puck robots. XVIII International Workshop on Artificial Life and Evolutionary Computation (WIVACE 2024), Namur, Belgium.

Model and coupling schemes

COUPLED OSCILLATORS

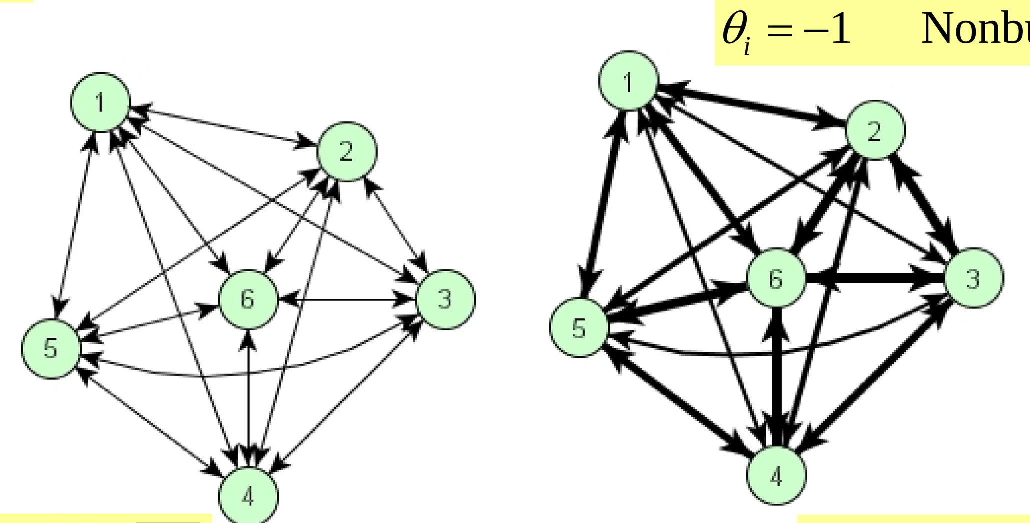
$$\frac{dV_i}{dt} = \frac{\ln 2}{T_{c,i}}\left(V_{Mi} - V_i(t)\right)\varepsilon_i(t) - \frac{\ln 2}{T_{d,i}} V_i(t)\left(1-\varepsilon_i(t)\right) + \theta_i \sum_{j=1}^{N} \beta_{ij}\left(1-\varepsilon_j(t)\right)$$

$$N = N_M + N_F$$

$\theta_i = +1$ Bursting oscillator (BO)

$\theta_i = -1$ Nonbursting oscillator (NBO)

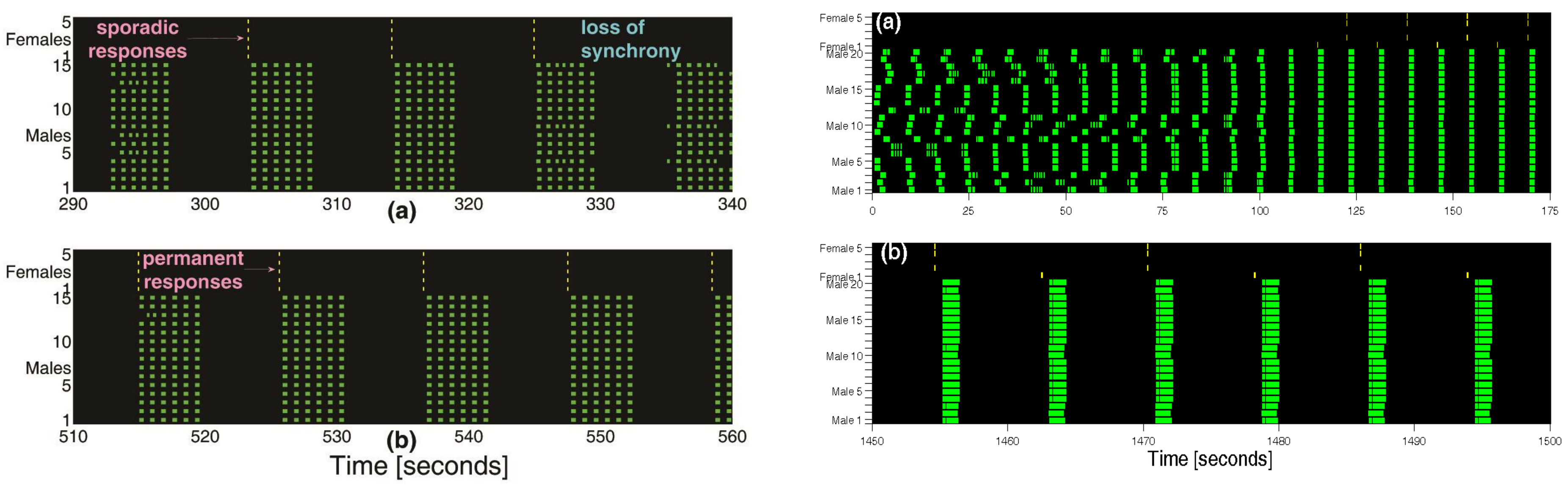
All-to-all coupling **Distance dependent coupling**



$$\beta_{ij} = \frac{\beta + \xi_i\sqrt{D_\beta}}{N}$$

$$\beta_{ij}(r_{ij}) \propto \frac{1}{(r_{ij} + \xi_i\sqrt{D_r})^{\alpha}} \; ; \; \alpha \approx 2.11$$

Examples of response to synchronization



Flashing Rate (FR) and Quality of Information (QI)

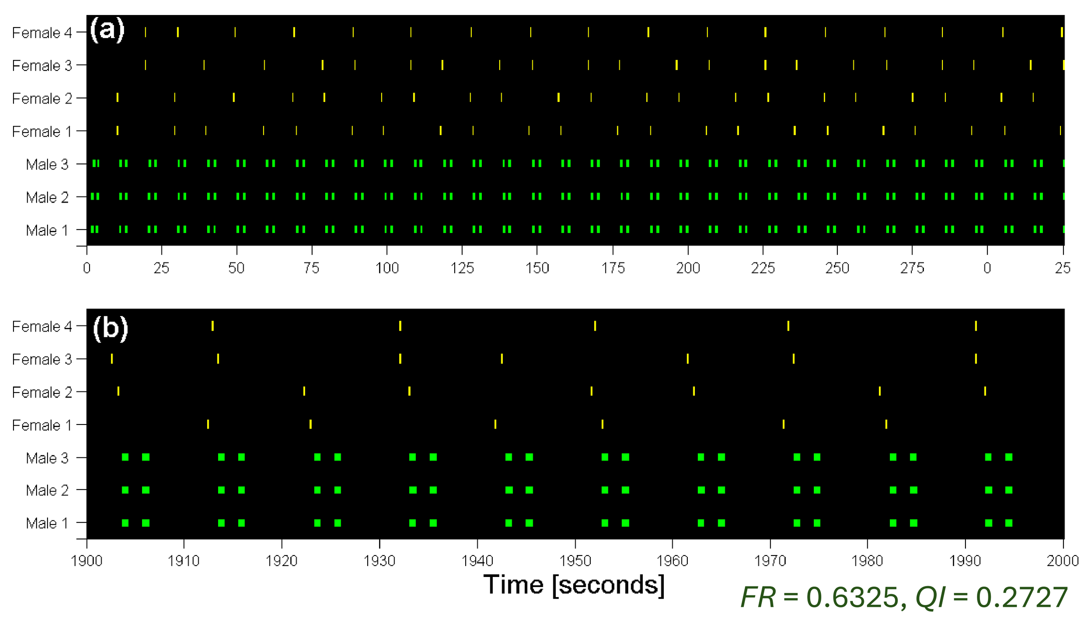
$$FR = \frac{\sum_{i=1}^{N_{ph}} n_i}{N_{ph} N_F} \qquad QI = \frac{\sum_{i=1}^{N_{ph}} \frac{n_i}{n_{ci}}}{N_{ph} N_F}$$

n_i : number of flashes per phrase

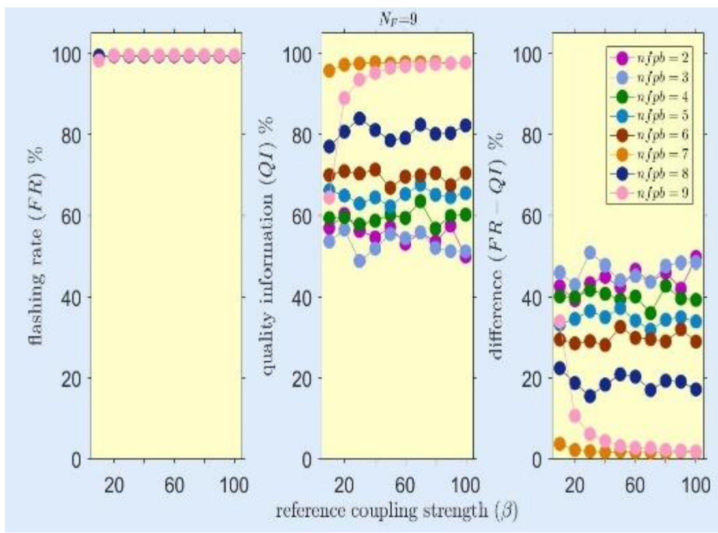
N_{ph} : total number of phrases

N_F : total number of NBOs

n_{ci} : number of clusters per phrase



FR = 0.6325, *QI* = 0.2727



Situation with only unidirectional coupling from BOs to NBOs, and where NBOs = 9. We observe that *FR* and *QI* almost coincide when the number of flashes per burst (nfpb) is 7. Results obtained with 100 simulations. It constitutes a **biological interesting result**.

Conclusions and perspectives

- Unravel the mechanism leading to response to synchronization
- Extension of the concept of response to synchronization
- Concepts on synchronization response (SR) and quality of information (QI)
- Multiple interesting phenomena related to biological aspects: controlling and enhancing synchronization and resonance
- Ideas for opinion formation models
- Following a previous work [3], implementation of some network configurations in a robotic platform of e-puck robots