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Sarrazin, Michael; PETIT, Fabrice

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Artificially induced positronium oscillations in a two-sheeted spacetime: consequences on the observed decay processes

Michaël Sarrazin[1,*] and Fabrice Petit[2,†]

[1]*Laboratoire de Physique du Solide, Facultés Universitaires Notre-Dame de la Paix, 61 rue de Bruxelles, B-5000 Namur, Belgium*

[2]*Belgian Ceramic Research Centre, 4 avenue du gouverneur Cornez, B-7000 Mons, Belgium*

Following recent theoretical results, it is suggested that positronium (Ps) might undergo spontaneous oscillations between two 4D spacetime sheets whenever subjected to constant irrotational magnetic vector potentials. We show that these oscillations that would come together with o-Ps/p-Ps oscillations should have important consequences on Ps decay rates. Experimental setup and conditions are also suggested for demonstrating in non accelerator experiments this new invisible decay mode.

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I. INTRODUCTION

In recent years, there has been a renewed interest for Kaluza-Klein like scenarios suggesting that our usual spacetime could be just a slice of a larger dimensional manifold [1, 2, 3, 4, 5, 6, 7]. A huge number of papers have shown that by extending the number of dimensions, nice explanations of several physical phenomena can be found. The hierarchy between the electroweak and the Planck scales, the dark matter origin or the cosmic acceleration can be thus reasonably addressed in a multidimensional framework. Usually, those approaches perpetuate the tradition inherited from relativity by assuming that the whole universe behaves like a smooth continuum [1, 2, 3, 4, 5, 6, 7]. However, there have been some recent attempts to develop models where the continuous extra dimensions are substituted by discrete dimensions [8, 9, 10, 11, 12, 13, 14]. In those approaches, the extra-dimensions are replaced by a finite number of points and the whole universe can be seen as made up of a collection of 4D sheets. Besides keeping the physical richness of multidimensional spacetimes, such multi-sheeted approaches provide also a nice framework where the standard model may arise from pure geometry. For instance, it has been pointed out that ordinary gauge fields and Higgs field appear spontaneously in some non commutative models of spacetime [8, 9, 10]. All these new physical and mathematical developments are of great interest as they offer new ways for a better understanding of the building blocks of our universe.

To date, the physical forces have been accurately measured up to the weak scale distances. Since no divergence between experimental and theoretical results has been noted so far, any new physical effect arising from the existence of extra dimensions is actually severely constrained. As an illustration of this constraint, standard model particles are usually expected to be confined on a 4D space manifold without having any capability of moving throughout the bulk. Gravitation is the only interaction which is assumed to be able to connect adjacent spacetimes since otherwise causality and locality could be violated [4, 7]. Nevertheless, this question is still an open issue that needs to be further clarified. For instance, in an attempt to circumvent the classical restrictions against particle motion through the bulk, some works have tried to extend the hypothetical graviton capability to the case of massive particles as well [3, 5, 6]. Along these lines, it was suggested that highly massive and energetic particles may acquire a non zero five momentum and escape from a 4-dimensional submanifold to propagate into the dimensionally extended bulk.

Recently, at the crossroad of brane models and non commutative two-sheeted spacetimes, present authors have proposed to investigate the quantum behavior of massive fermions in a $M_4 \times Z_2$ two-sheeted universe. It was suggested that particles of usual matter might leave our 4D spacetime and reach another distant 4D spacetime sheet [15, 16]. In the proposed framework, the particle dynamics was studied through two-sheeted extensions of the classical Dirac, Pauli and Schrodinger equations. Although the usual particle behavior was recovered in the new framework, the existence of a tiny geometrical coupling between the two sheets was shown to provide new interesting physics [15, 16]. As the most striking result, oscillations of massive particles between the two spacetime sheets were predicted. Accordingly,

*Electronic address: michael.sarrazin@fundp.ac.be
†Electronic address: f.petit@bcrc.be

particles could disappear from one sheet to reappear in the second one. During oscillations, many parameters (such as, for instance, baryonic and leptonic numbers or electric charge) as measured by a M_4 observer are violated but they remain globally constant from a $M_4 \times Z_2$ point of view.

In Refs. [15] and [16], it was shown that the conditions for reaching such transfer between both spacetime sheets are far from trivial. Application of very intense curl-free (irrotational) magnetic vector potentials was shown to be necessary [27]. It must be noted that although the result establishing the oscillations was derived for point like particles, it was inferred that the equations could be also valid for more complex particles like protons or neutrons for instance, the essential point being that the particle exhibits a non zero magnetic moment [15, 16].

To address the issue of whether quantum motions between two spacetime sheets are indeed possible, the present paper discusses the possibility of a controlled production of these oscillations in the case of positronium (Ps). This choice was dictated for two reasons. Firstly, positronium is at the heart of many experimental researches [17, 18, 19, 20, 21] and secondly, there is already a wealth of papers suggesting that positronium could possibly exhibit an invisible decay [19, 20, 21]. Since each model of invisible positronium decay possesses its own specific signature, it is expected that the experimental confirmation of such decay could give insights on the form and size of the extra dimension(s) [19, 20, 21]. The purpose of this paper is then to clarify how such a Ps invisible decay would looks like in a $M_4 \times Z_2$ two-sheeted spacetime as described in Refs. [15] and [16].

II. MATHEMATICAL FRAMEWORK

In Refs. [15] and [16], a model describing the quantum dynamics of matter particles in a two-sheeted spacetime was introduced. It formally corresponds to the product of a four continuous manifold times a discrete two points space, i.e. $X = M_4 \times Z_2$. The model was built following two different approaches which are now briefly summarized.
The former approach was mainly based on the work of Connes [8, 9], Viet and Wali [11, 12] and relies on a non-commutative definition of the exterior derivative acting on the product manifold. Due to the specific geometrical structure on the discrete space, this operator is given by :

$$D_\mu = \begin{pmatrix} \partial_\mu & 0 \\ 0 & \partial_\mu \end{pmatrix}, \ \mu = 0,1,2,3 \text{ and } D_5 = \begin{pmatrix} 0 & g \\ -g & 0 \end{pmatrix} \tag{1}$$

Where the parameter g has the dimension of mass (in $\hbar = c = 1$ units) and acts as a finite difference operator along the discrete dimension. In the model discussed in Ref. [15] and contrary to previous works [8], g was considered as a constant geometrical field (not the Higgs field). As a consequence a mass term was introduced and a two-sheeted Dirac equation was derived :

$$(i\not{D} - M)\,\Psi = \left(i\Gamma^N D_N - M\right)\Psi = \begin{pmatrix} i\gamma^\mu\partial_\mu - m & ig\gamma^5 \\ ig\gamma^5 & i\gamma^\mu\partial_\mu - m \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = 0 \tag{2}$$

It can be noticed that by virtue of the two-sheeted structure of spacetime, the wave function ψ of the fermion comprises two components located on different 4D sheets. Therefore in this model, any massive particles is in fact a five dimensional entity. Although this $M_4 \times Z_2$ delocalization can be seen as a major drawback of the model, it was demonstrated that environmental interactions force the particle localization in one or the other sheet and act as a natural dimensional reduction mechanism [15, 16].

Obviously, the equation (2) can also be derived from the $M_4 \times Z_2$ lagrangian $\boldsymbol{L}$ defined as

$$\boldsymbol{L} = \bar{\Psi}\,(i\ \not{D} - M)\,\Psi \tag{3}$$

Where $\bar{\Psi} = \left(\bar{\psi}_+, \bar{\psi}_-\right)$ is the two-sheeted adjoint spinor of Ψ and with $\bar{\psi}_+$ and $\bar{\psi}_-$ the adjoint spinors respectively of ψ_+ and ψ_-.

Such a lagrangian can be also written into the following expended form

$$\boldsymbol{L} = \bar{\psi}_+\,(i\ \not{\partial} - m)\,\psi_+ + \bar{\psi}_-\,(i\ \not{\partial} - m)\,\psi_- + ig\bar{\psi}_+\gamma^5\psi_- + ig\bar{\psi}_-\gamma^5\psi_+ \tag{4}$$

At first sight, the doubling of the wave function can be seen as a reminiscence of the hidden-sector concept. While it is true that hidden sector models and present approach share several common points, it is equally true that they differ by the number of spacetime sheets they consider. For instance, the so-called Mirror matter approach, considers only one 4D manifold and justifies for the left/right parity by introducing new internal degrees of freedom to particles (see for instance Ref. [22]). In the present work, it can be noted that the number of particle families remains unchanged but the particles have now access granted to two distinct 4D spacetime sheets.

To be consistent with the new framework the usual U(1) gauge field must be substituted by an extended U(1)$\bigotimes$U(1) gauge relevant for the discrete Z_2 structure of the universe. In addition, each sheet possesses its own current and charge density distribution as source of the electromagnetic field. The most general form for the new gauge (to be incorporated within the two-sheeted Dirac equation such that $\not{D} \rightarrow \not{D} + \not{A}$) is then defined by (see also Refs. [13] and [14] where such a gauge was also considered)

$$\not{A} = \begin{pmatrix} iq\gamma^{\mu}A_{+,\mu} & \gamma^5\chi \\ \gamma^5\chi^{\dagger} & iq\gamma^{\mu}A_{-,\mu} \end{pmatrix} \tag{5}$$

On the two sheets live then the distinct $\mathbf{A}_+$ and $\mathbf{A}_-$ fields. In the present model, the component χ cannot be associated to the usual Higgs field encountered in GSW model. An obvious consequence of this off-diagonal term χ is to couple the photons fields of the two sheets. Since this term introduces major complications unless to be weak enough compared to $\mathbf{A}_{\pm}$ (most notably it leads to unusual transformation laws of the electromagnetic field which are difficult to reconcile with observations) it is logical to set $\chi = 0$. The electromagnetic fields of both sheets are then completely decoupled and each sheet is endowed with its own electromagnetic structure. Note that the another consequence of having $\chi \neq 0$ is to couple each charged particle with the electromagnetic fields of both sheets, irrespective of the localization of this particle in the discrete space. For instance a particle of charge e localized in the "+" sheet would have been sensitive to the electromagnetic field of the "−" sheet with an effective charge εe ($\varepsilon < 1$). This kind of exotic interactions has been considered previously in literature within the framework of mirror matter paradigm and is not covered by the present paper [23, 24]. Another noticeable consequence of considering $\chi = 0$ is that the photons fields are now totally trapped in their original sheets : photons belonging to a given sheet are not able to go into the other sheet (and as a consequence, structures belonging to a given sheet are invisible from the perspective of an observer located in the other sheet). The classical gauge field transformation which reads

$$A'_{\mu} = A_{\mu} + \partial_{\mu}\Lambda \tag{6}$$

can now be easily extended in the Z_2 five dimensional framework using Eq. (5). To be consistent with the above hypothesis (notably $\chi = 0$), it can be shown that the gauge transformation is degenerated and reduced to a single $e^{iq\Lambda}$ which must be applied to both sheets simultaneously [16]. By setting $\chi = 0$ and by considering the same gauge transformation in the two sheets we can get photons fields $\mathbf{A}_{\pm}$ which behave independently from each other and in accordance with observations [15, 16].

After introducing the gauge field into the Z_2 Dirac equation and taking the non relativistic limit (following the standard procedure), a two-sheeted Pauli like equation can be derived

$$i\hbar\frac{\partial}{\partial t}\left|\Psi\right\rangle = \mathbf{H}\left|\Psi\right\rangle \tag{7}$$

with $\left|\Psi\right\rangle = \begin{pmatrix} \left|\psi_+\right\rangle \\ \left|\psi_-\right\rangle \end{pmatrix}$, where $\left|\psi_+\right\rangle$ and $\left|\psi_-\right\rangle$ correspond to the wave functions in the $(+)$ and $(-)$ sheets respectively. The Hamiltonian $\mathbf{H}$ involves different contributions [15, 16], listed below (using now natural units)

$$\mathbf{H}_k = -\frac{\hbar^2}{2m}\begin{bmatrix} \left(\nabla - i\frac{q}{\hbar}\mathbf{A}_+\right)^2 & 0 \\ 0 & \left(\nabla - i\frac{q}{\hbar}\mathbf{A}_-\right)^2 \end{bmatrix} \tag{8}$$

$$\mathbf{H}_m = -g_s\mu\frac{\hbar}{2}\begin{bmatrix} \sigma\cdot\mathbf{B}_+ & 0 \\ 0 & \sigma\cdot\mathbf{B}_- \end{bmatrix} \tag{9}$$

$$\mathbf{H}_p = \begin{bmatrix} V_+ & 0 \\ 0 & V_- \end{bmatrix} \tag{10}$$

$$\mathbf{H}_{cm} = ig\gamma g_s\mu\frac{\hbar}{2}\begin{bmatrix} 0 & \sigma\cdot\left\{\mathbf{A}_+ - \mathbf{A}_-\right\} \\ -\sigma\cdot\left\{\mathbf{A}_+ - \mathbf{A}_-\right\} & 0 \end{bmatrix} \tag{11}$$

$$\mathbf{H}_c = \frac{g^2\hbar^2}{m}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{12}$$

where the first three terms, i.e. $\mathbf{H}_k$, $\mathbf{H}_m$ and $\mathbf{H}_p$, are reminiscent of the terms found in the classical Pauli equation in presence of an electromagnetic field. $\mathbf{A}_+$ and $\mathbf{A}_-$ denote the magnetic potential vectors in the sheet (+) and (-) respectively. The same convention is used for the magnetic fields. $\mathbf{H}_k$ relates to the kinetic part and includes the vector potential, $\mathbf{H}_m$ is the coupling term between the magnetic field and the magnetic moment of the particle $g_s\mu$ where g_s is the gyromagnetic factor and $\mathbf{H}_p$ the coulomb term. In addition to these "classical" terms, the hamiltonian comprises also a specific term involving an "electromagnetic coupling" between the two sheets. Note that this coupling $\mathbf{H}_{cm}$ arises through the magnetic vector potential and the magnetic moment only. For an electron, it can be shown that $\gamma = 1$ though it is clear that it could differ slightly from unity due to QED corrections in the case of composite particles like proton or neutron [16]. It can be noted that $\mathbf{H}_c$ is a simple constant term which can be obviously eliminated through an appropriate rescaling of the energy. The exact physical meaning of this term shall not be discussed here. In the model, g is the coupling constant between the two sheets. g^{-1} is simply the distance between the two sheets [15, 16].
The second approach developed in Ref. [16] starts from the usual covariant Dirac equation in 5D. Assuming a discrete structure of the discrete space with two four dimensional submanifolds requires to substitute the extra dimensional derivative by a finite difference counterpart defined as

$$(\partial_5\psi)_\pm = \pm g(\psi_+ - \psi_-) \tag{13}$$

Then as previously for the non-commutative approach, the Dirac equation breaks down into a set of two coupled differential equations similar to Eq. (2) [16]. After introducing electromagnetic fields into the model and taking the non relativistic limit, it was shown that the two-sheeted Pauli's equation takes the same form than Eq. (7) to (12) except for the $\mathbf{H}_c$ term

$$\mathbf{H}_c = \frac{g^2\hbar^2}{m}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{14}$$

By contrast to the previous $\mathbf{H}_c$ term, this new one cannot be made vanishing through a simple rescaling of the energy scale. The off diagonal part of $\mathbf{H}_c$ was demonstrated to be responsible of particle oscillations between the two spacetime sheets, even in the case of a free particle [16]. Nevertheless, as explained in Ref.[16], $\mathbf{H}_c$ can be neglected, in a first approximation, in comparison with $\mathbf{H}_{cm}$ since it is proportional to g^2, an obvious tiny value in order for the model to be consistent with known experimental results. As a consequence the classical finite difference [16] and the non commutative [15] approaches of the two-sheeted spacetime predict almost the same physics. The only difference arises from the presence of this tiny coupling term $\mathbf{H}_c$ in the finite difference approach. In this paper we will concentrate mainly on the coupling term $\mathbf{H}_{cm}$ proportional to g which is expected to impart most of the new physics contained in both models [15, 16].

III. INVISIBLE POSITRONIUM DECAY IN A TWO-SHEETED SPACETIME

To illustrate how the two-sheeted geometrical structure of spacetime governs the quantum behavior of particles, let us examine its effects on the Ps decay processes. At the fundamental state $1S$ (about -6.8 eV), Ps exists in two forms. The first form 1^3S_1 called ortho-positronium (o-Ps), corresponds to parallel orientation of electron and positon spins. The second form 1^1S_0 or para-positronium (p-Ps), corresponds to antiparallel spins states. O-Ps decay time is about $1,4 \cdot 10^{-7}$ s with an emission of three photons whereas p-Ps decay time is about $1,25 \cdot 10^{-10}$ s with an emission of two photons. Besides, the hyperfine structure interval of positronium ($1^3S_1 \to 1^1S_0$) is about $8.41 \cdot 10^{-4}$ eV (i.e. ~ 203.39 GHz) [17, 18]. Following the standard description of Ps, the o-Ps wave function can be expressed as [17, 25]

$$|\psi_1\rangle = |\varphi_e(\uparrow)\rangle \otimes |\varphi_p(\uparrow)\rangle \tag{15}$$

$$|\psi_{-1}\rangle = |\varphi_e(\downarrow)\rangle \otimes |\varphi_p(\downarrow)\rangle \tag{16}$$

$$|\psi_0\rangle = (1/\sqrt{2})\left\{|\varphi_e(\uparrow)\rangle \otimes |\varphi_p(\downarrow)\rangle + |\varphi_e(\downarrow)\rangle \otimes |\varphi_p(\uparrow)\rangle\right\} \tag{17}$$

and similarly for the p-Ps wave function, we have

$$|\psi\rangle = (1/\sqrt{2})\left\{|\varphi_e(\uparrow)\rangle \otimes |\varphi_p(\downarrow)\rangle - |\varphi_e(\downarrow)\rangle \otimes |\varphi_p(\uparrow)\rangle\right\} \tag{18}$$

where $|\varphi_e(\updownarrow)\rangle$ and $|\varphi_p(\updownarrow)\rangle$ relate to the wave functions of the electron and positon respectively both taking into account the spin state.

We now consider the influence of a constant irrotational magnetic vector potential $\mathbf{A}$ located in our own spacetime sheet (arbitrarily taken to be the (+) sheet). For reasons explained before, let us also neglect all other terms in the Hamiltonian except the coupling

$$\mathbf{W} = ig\gamma g_s\mu\frac{\hbar}{2}\begin{bmatrix} 0 & [\sigma_e - \sigma_p]\cdot\mathbf{A} \\ -[\sigma_e - \sigma_p]\cdot\mathbf{A} & 0 \end{bmatrix} \tag{19}$$

Note that this term is simply the sum of two $\mathbf{H}_{cm}$ hamiltonian operators corresponding to the contributions of the electron and the positon. The minus sign arises from the opposite magnetic moment of these particles.

Choosing $\mathbf{A} = A\mathbf{e}_z$ for instance and noticing that

$$[\sigma_{z,e} - \sigma_{z,p}]\,|\psi_0\rangle = 2\,|\psi\rangle \tag{20}$$

$$[\sigma_{z,e} - \sigma_{z,p}]\,|\psi\rangle = 2\,|\psi_0\rangle \tag{21}$$

and

$$[\sigma_{z,e} - \sigma_{z,p}]\,|\psi_{\pm 1}\rangle = 0 \tag{22}$$

one can see that the only states connected through the term (19) are of the form

$$|\Psi_0\rangle = \begin{bmatrix} |\psi_0\rangle \\ 0 \end{bmatrix} \tag{23}$$

and

$$|\Psi\rangle = \begin{bmatrix} 0 \\ |\psi\rangle \end{bmatrix} \tag{24}$$

such that the corresponding matrix terms of $\mathbf{W}$ take the form

$$\langle\Psi|\,\mathbf{W}\,|\Psi_0\rangle = -ig\gamma g_s\mu\hbar A \tag{25}$$

$$\langle\Psi_0|\,\mathbf{W}\,|\Psi\rangle = ig\gamma g_s\mu\hbar A \tag{26}$$

We now look for a solution of the form

$$|\Phi(t)\rangle = a_0(t)e^{-iE_0t/\hbar}\,|\Psi_0\rangle + a_p(t)e^{-iE_pt/\hbar}\,|\Psi\rangle \tag{27}$$

with $|\Phi(t=0)\rangle = |\Psi_0\rangle$. Putting Eq. (27) into the Schrodinger equation leads to the following system of coupled differential equations

$$\frac{d}{dt}a_0 = \kappa a_p e^{i\omega_0 t} - \Gamma_o a_0 \tag{28}$$

$$\frac{d}{dt}a_p = -\kappa a_0 e^{-i\omega_0 t} - \Gamma_p a_p \tag{29}$$

with $\kappa = g\gamma g_s\mu A$.

Note that the decay constants Γ_o and Γ_p have been conveniently introduced in the equations in agreement with the known lifetime of the Ps states, i.e. $\Gamma_o \sim 3.57\cdot 10^6$ s^{-1}and $\Gamma_p \sim 4\cdot 10^9$ s^{-1}. In addition we set $\omega_0 = (E_0 - E_p)/\hbar \sim 1.28\cdot 10^{12}$ s^{-1} corresponding to the hyperfine structure interval of Ps. The solution of the above system is straightforward

$$a_0 = \frac{1}{4\sigma}e^{-(1/2)(\Gamma_o+\Gamma_p)t}\left[\left((\Gamma_o - \Gamma_p + i\omega_0) + 2\sigma\right)e^{-\sigma t} - \left((\Gamma_o - \Gamma_p + i\omega_0) - 2\sigma\right)e^{\sigma t}\right]e^{(i/2)\omega_0 t} \tag{30}$$

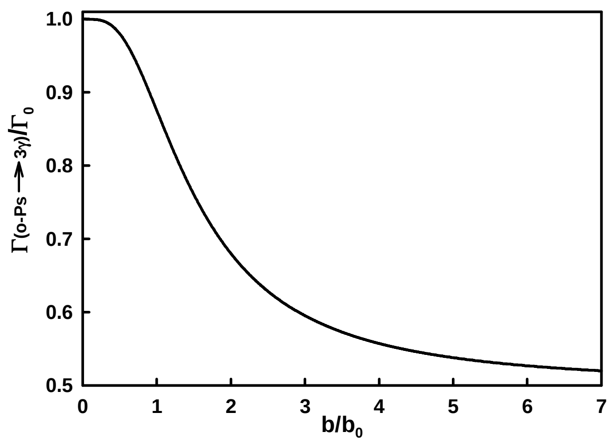

FIG. 1: Theoretical decay rate of o-Ps under the influence of an irrotational constant magnetic vector potential.

and

$$a_p = \frac{\kappa}{2\sigma} e^{-(1/2)(\Gamma_o+\Gamma_p)t} \left[e^{-\sigma t} - e^{\sigma t}\right] e^{-(i/2)\omega_0 t} \tag{31}$$

with

$$\sigma = \frac{1}{2}\sqrt{(\Gamma_o - \Gamma_p + i\omega_0)^2 - 4\kappa^2} \tag{32}$$

A first interesting case to be considered is the one where Γ_o and Γ_p are both equal to zero (the natural decay processes are not taken into account). Then, it can be shown that

$$|a_p|^2 = \frac{\kappa^2}{\rho^2} \sin^2 gt \tag{33}$$

and $|a_0|^2 = 1 - |a_p|^2$, with $\rho = (1/2)\sqrt{\omega_0{}^2 + 4\kappa^2}$.

This term simply relates to the probability of finding the Ps in the second spacetime sheet at time t. Clearly, Ps oscillates between the two sheets. The average probability of finding the Ps in the second sheet is simply given by $\kappa^2/2\rho^2$. Since Ps was assumed to be in a o-Ps state in the first sheet, it is clear that the larger $\kappa^2/2\rho^2$ is, the larger is the probability for an invisible decay with a p-Ps state in the second sheet. To get an experimental confirmation of this result, it will be interesting to achieve experimental conditions such that $\kappa \gg \omega_0/2$. Let us now return to the more general case including natural decay modes also. We are now going to calculate the whole decay rate Γ. The probabilities for each Ps state, $P_o = |a_0|^2$ and $P_p = |a_p|^2$ allow to define the Ps decay probability $P = 1 - P_o - P_p$. The related distribution function for the decay probability is $f(t) = (d/dt)P$ such that the Ps lifetime is then given by $\tau = \int_0^\infty t f(t) dt$ and the decay rate by $\Gamma = (1/2)\tau^{-1}$. Using Eq. (30) and Eq. (31), Γ can be easily derived

$$\Gamma = \frac{(\Gamma_o + \Gamma_p)^2(\kappa^2 + \Gamma_o\Gamma_p) + \Gamma_o\Gamma_p\omega_0^2}{2\kappa^2(\Gamma_o + \Gamma_p) + \Gamma_p((\Gamma_o + \Gamma_p)^2 + \omega_0^2)} \tag{34}$$

Reminding that $\omega_0 \gg \Gamma_p > \Gamma_o$, the above solution can be approximated with a very good accuracy by

$$\Gamma = \Gamma_o(1 - (1/2)(1 + \eta^2)^{-2}) + (1/2)\Gamma_p(1 + \eta^2)^{-1} \tag{35}$$

where $\eta = \omega_0/(\kappa\sqrt{2})$. The following substitution $\eta = b_0/b$ with $b = gA$ and $b_0 = \omega_0 m_e/(e\sqrt{2})$, appears to be convenient for displaying the solution since the actual value of g is presently unknown. Note that b has the dimension of a magnetic field like b_0. In the present case $b_0 \sim 5.14$ T. From Eq. (35), the interpretation of the global decay rate can be made quite easily. The first term corresponds to the decay rate of the o-Ps in our spacetime sheet, i.e.

$$\Gamma(\text{o-Ps} \to 3\gamma) = \Gamma_o(1 - (1/2)(1 + \eta^2)^{-2}) \tag{36}$$

The second term relates to the decay rate of the p-Ps after its conversion from o-Ps to p-Ps right after its "jump" in the second sheet (and seen from the perspective of an observer living in the second spacetime sheet)

$$\Gamma(\text{o-Ps} \to \text{p-Ps}' \to 2\gamma') = (1/2)\Gamma_p(1 + \eta^2)^{-1} \tag{37}$$

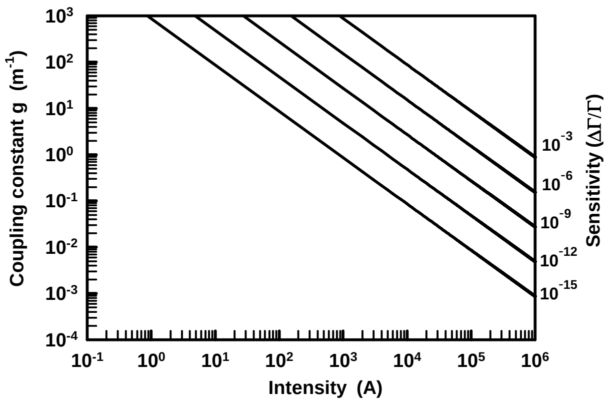

FIG. 2: Measurable coupling constant vs current intensity for various apparatus sensitivities.

From an experimental point of view, our results suggest that an o-Ps population in a constant irrotational vector potential should present an abnormal decay rate according to Eq. (36) (see Fig.1). A noticeable point is that one should observe a decreasing photon production rate as the magnetic potential increases, such a behavior being related to the p-Ps invisible decay in the second sheet.

It must be pointed out that these results remain unchanged if the initial o-Ps state is substituted by a p-Ps state (except for a permutation of the Γ_o and Γ_p terms in expressions (34) and (35) respectively).
In previous works [15, 16], it was mentioned that a constant irrotational potential $\mathbf{A}$ could be achieved in a hollow conducting cylinder with a constant current flow I inside. For such a configuration, we get typically $A \sim \mu_0 I$ [28]. So, for a specific decay rate Γ, we can expect to measure the variation $\Delta\Gamma$ of the decay as A, i.e. I varies. For the proposed experimental setup, the sensitivity of the apparatus $\Delta\Gamma/\Gamma$ would thus be simply related to the current value circulating in the cylinder shell. The figure 2 shows the value of the coupling constant we can thus expect to measure for different achievable intensities [29]. Reciprocally, since the value of the coupling constant is presently unknown, the current required to produce vector potentials allowing particle oscillations can be hardly estimated. However, it is very likely that the required intensities are above the present technology capabilities.

In Ref. [23] dealing with mirror matter, a mixing between o-Ps and mirror o-Ps was also suggested. However the model treated in this paper [23] is different from that considered in this work in several aspects. First, this paper [23] does not localize mirror matter on a distinct spacetime sheet. Secondly, it considers that the mixing between both worlds occurs through a photon/mirror photon kinetic mixing (gravitation also is assumed to mediate interactions between matter and mirror matter but it is not relevant in the present discussion). Typically, this mixing is then given by the Lagrangian [23]

$$L = -\varepsilon F_{\mu\nu} F^{'\mu\nu} \tag{38}$$

with ε the coupling strength. Considering matter/mirror matter quantum electrodynamics with this coupling term shows that oscillations of o-Ps into mirror o-Ps effectively occur in a vacuum experiment. The result is an apparent invisible decay similar to that proposed in the present paper. Nevertheless, it must be stressed that an obvious advantage of the two-sheeted structure discussed presently is to demonstrate the existence of oscillations without recourse to a photon/mirror photon kinetic mixing (thus keeping safe the usual electromagnetic laws). Most important, our model suggests a way to artificially modify the decay rate of positronium. At present time, the possible experimental signals for the existence of extra-dimensions are limited. They rely mainly on the observation of deviations from the inverse square law of gravity or spectrum determination of Kaluza-Klein tower states. The two-sheeted spacetime paradigm suggests a new possibility involving usual matter particles oscillations between adjacent 4D sheets. An artificial control of the positronium decay rate by using a device similar to that proposed in this paper should clearly be investigated. If an abnormal decay rate could indeed be noticed by applying an irrotational magnetic potential onto Ps, then it could reveal the existence of another spacetime sheet and confirms that our spacetime is just a sheet embedded in a more complex manifold.

IV. CONCLUSION

In this paper, we have sketched a possible experimental consequence of the two-sheeted spacetime introduced in Refs. [15] and [16]. Ps decay was just considered here as an illustration of the model but more complex situations can be easily envisaged. It has been demonstrated that invisible decay of Ps should occur in presence of a constant irrotational magnetic vector potential. As described, the invisible decay occurs through particle oscillations between two distinct spacetime sheets. A simple experimental setup designed to produce such oscillations has been suggested. The predictions of the model could be relevant for research teams aiming at demonstrating abnormal Ps behavior not accommodated by the standard model.

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[27] It is important to recall that if a magnetic field results from a magnetic vector potential, by contrast, a magnetic vector potential not necessary induces a magnetic field. In this case, the magnetic vector potential is called irrotational or curl-free. The particle disappearance can be obtained with an irrotational magnetic vector potential only since any magnetic field

will tend to inhibit the effect [15, 16].

[28] Note that an equivalent although perhaps more convenient geometry than an hollow cylinder can be used. It consists of a complex toroidal coil and was described in Ref. [26].

[29] Note that the reintroduction of the offdiagonal terms of $\mathbf{H}_c$ also leads to an invisible decay of Ps in the second spacetime sheet. Nevertheless a careful analysis shows that Γ weakly decreases according to $\Delta\Gamma/\Gamma \sim (1/2)(\chi^2/\Gamma^2)$ where $\chi = 2\hbar g^2/m_e$. Since this term is proportional to the square of the coupling constant, it can be neglected. For instance, even with $g \sim 10^3$ m^{-1} (i.e. a distance about 1 mm between sheets) one obtains relative variations of the decay rate of nearly $2 \cdot 10^{-9}$ and $2 \cdot 10^{-15}$ for o-Ps and p-Ps respectively. In that case, Ps freely oscillates between sheets without p-Ps/o-Ps conversion. It is a matter of evidence that such a low contribution to the decay rate is still far from the present experimental accuracies [17, 18, 21].