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Dielectric effect on electric fields in the vicinity of the metal-vacuum-dielectric junction

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The dielectric effect was theoretically investigated for the electric field in the vicinity of a junction of metal, dielectric, and vacuum. The assumption of the two-dimensional symmetry of junctions led to a simple analytic form and to a systematic numerical calculation for the field. The electric field obtained for the triple junction was found to be enhanced and reduced according to the certain criteria determined by the contact angles along with the dielectric constant. Further numerical calculations of the dielectric effect exhibit that an electric field can experience a larger enhancement or reduction for a quadruple junction than for the triple one. It was also found that even though it changes slowly in comparison with the shape effect, the dielectric effect was noticeably large over the entire change of shape.

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Keywords: triple junction, field enhancement, dielectric effect, metal-dielectric contact

1. Introduction

Dielectrics are usually considered to reduce the electric field intensities. For a long time, however, strongly enhanced electric fields have often been observed in the phenomenon of electron emission avalanche or breakdown in the vicinity of the metal-dielectric contact [1-5]. Such an unexpected field enhancement, called the triple junction effect, must be due to

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dielectric in contact with metal (see Fig. 1). Two significant experiments to reveal the dielectric effect were made by the Geis group [4,6]. In 1977, they reported the enhanced field emission when the diamond portion of the triple junction was extended into the vacuum. Several years later, they observed the field emission which continued even after the bias was removed. Ma and Sudarshan [7,8] found that voids at the contact were crucial in vacuum insulation breakdown. The contact angle of the junction was considered to play a role in the enhancement mechanism [9-14]. Schächter [15] first used a two-dimensional model to obtain the theoretical result that the field emission current density was proportional to the dielectric constant for a specific triple junction.

The field enhancement at the triple junction is different from that produced by a protrusion or a sharpening of the metallic edge [16]. The electric field is enhanced or reduced according to the angular configuration of the triple junction [12-14]. The value of the dielectric constant is a factor to determine the magnitude of the enhancement. This implies that both the magnitude and direction of polarization are involved to determine the enhancement of the electric field. Thus it is interesting to find what configuration yields the largest dielectric enhancement through polarization with the triple junction as a basis. One of most probable candidates is a quadruple junction even though it may not appear as often as a triple junction (see Fig. 1). According to the previous work [17], we have found that a certain type of quadruple junction yields the larger dielectric effect than the triple junction. Even though it is not experimentally confirmed yet, the use of the triple or quadruple junction may lead to a new type of cold electron source. In addition, a deeper understanding of the dielectric effect may be used to avoid an avalanche of electron emission or breakdown. Thus we need to make a systematic approach to describe the field behavior in the region near the contact of metal and dielectric.

2. Metal-vacuum-dielectric triple junctions

As seen in Fig. 1, the metal-layer contact has several configurations of junction at which metal, vacuum, and dielectric meet [6]. Since it is a layer-by-layer contact, the junction structure has a relatively large longitudinal dimension in comparison with the facial size. Thus we assume that junctions have the two-dimensional symmetry. As a triple junction, we can consider the axis of a cylinder composed of three portions of metal, vacuum, and dielectric meet, as seen in Fig. 2. As far as the electric field in the vicinity of the junction is concerned, the cylinder plays the same role as the real triple junction configuration. It is supposed that the cylinder has a length of $\ell \approx 1$ mm and a radius of $R \approx 1$ μm ($R \ll \ell$).

Now we use the polar coordinates (r, θ) . The two-dimensional portions of metal, vacuum and dielectric are given by angles α , θ_1 and θ_2 , respectively. The electric potential Φ is given by the solution of the two-dimensional Laplace equation

$$r \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) + \frac{\partial^2 \Phi}{\partial \theta^2} = 0 \quad (1)$$

In general, the solution is written in the form $\Phi = \sum_n r^{\nu_n} (A_n \sin \nu_n \theta + B_n \cos \nu_n \theta)$, where r is the distance from the junction and A_n and B_n arbitrary constants. Since r is limited to the vicinity of the junction, i.e., $r \ll R$, only the lowest-power term is dominant. Then Φ is given by [18]

$$\Phi_1 = A_1 r^\nu \sin \nu \theta, \quad 0 < \theta < \theta_1 \text{ (vacuum)}, \quad (2)$$

$$\Phi_2 = A_2 r^\nu \sin \nu(\theta_1 - \theta), \quad \theta_1 < \theta < \theta_1 + \theta_2 = \theta_1 = 2\pi - \alpha \text{ (dielectric)}, \quad (3)$$

where subscripts 1 and 2 refer to the vacuum and dielectric regions, respectively and we put $\nu = \nu_1$, being positive. From Eqs. (2) and (3), the electric field magnitude $F = |\nabla\Phi|$ is given by

$$F_1 = A_1 \nu r^{\nu-1}, \quad 0 < \theta < \theta_1 \quad (\text{vacuum}), \quad (4)$$

$$F_2 = A_2 \nu r^{\nu-1}, \quad \theta_1 < \theta < \theta_t \quad (\text{dielectric}). \quad (5)$$

Both magnitudes F_1 and F_2 are independent of the angle variable θ and their behaviors are majorly characterized by ν only.

Now we use the boundary conditions which are the continuities of both Φ and $D_\theta = (\epsilon/r)(\partial\Phi/\partial\theta)$ at $\theta = \theta_1$. The two obtained relations are

$$\epsilon \tan \nu\theta_1 = -\tan \nu\theta_2. \quad (6)$$

$$A_2 / A_1 = \sin \nu\theta_1 / \sin \nu\theta_2 \equiv \eta \quad (7)$$

Equation (6) becomes the very equation for ν . Equation (7) is used to evaluate F_2 since η is equal to the field ratio F_2 / F_1 for $r_1 = r_2$.

The analytic solutions for ν are easily found in the two limiting cases of $\epsilon = 1$ and $\epsilon \rightarrow \infty$. For $\epsilon = 1$ (in the absence of dielectric), Eq. (6) is satisfied at

$$\nu = \nu_0 = \pi / \theta_1 = \pi / (2\pi - \alpha). \quad (8)$$

This is v just for the contact of metal and vacuum [18]. Next we rewrite Eq. (6) into the form $1/\varepsilon = \tan(\pi - v\theta_1)/\tan v\theta_2$. In the limit $\varepsilon \rightarrow \infty$, the right side is zero when the numerator is zero or when the denominator becomes infinity. Then we have two solutions of which the lower one is

$$v_\infty / v_0 = \begin{cases} 1/(\theta_1 / \theta_t) = 1/(1 - \theta_2 / \theta_t), & 0 < \theta_2 / \theta_t < 1/3 \\ 0.5/(\theta_2 / \theta_t), & 1/3 < \theta_2 / \theta_t < 1 \end{cases} \quad (9)$$

We plot the curve of v_∞ versus θ_2 in Fig. 3. Each coordinate is normalized with v_0 and θ_t . The reason is that even though both v_0 and v_∞ are a function of α , v_∞/v_0 is independent of α . We do not need to calculate v for all values of α but only one value, say $\alpha = \pi$. It is seen that v_∞/v_0 has the maximum of 1.5 at $\theta_2/\theta_t = 1/3$ and the minimum of 0.5 at $\theta_2/\theta_t = 1$. Then v_∞ has the minimum of 0.25 since v_0 has the minimum of 0.5, at $\alpha = 0$. It is interesting to see the difference metal and dielectric of $\varepsilon = \infty$. Metal gives one curve (dotted line) whereas dielectric of $\varepsilon = \infty$ gives two curves (solid line).

For real dielectrics of $1 < \varepsilon < \infty$, we make numerical calculations of Eq. (6) to obtain $v(\varepsilon)$, v in short, as a function of θ_2 , α and ε . We take $\varepsilon = 5.7$ (diamond), 10.4 (GaN), 100, and 1000. The obtained v are plotted in Fig. 4. We also insert v for metal (dashed line) for comparison. We have $v_0 < v < v_\infty$ or $v_\infty < v < v_0$ according to the value of θ_2 . That is, v can be smaller or larger than v_0 . The electric field becomes stronger or weaker in the presence of dielectric than that in the absence of dielectric.

Both the maximum and minimum points depend on ε . As ε increases from 1 to ∞ , the maximum position $(\theta_2/\theta_t)_{\max}$ moves from 1/4 to 1/3 whereas the minimum $(\theta_2/\theta_t)_{\min}$

moves from 3/4 to 1.0. On the other hand, the maximum value $(v/v_0)_{\max}$ moves from 1.0 to 3/2 whereas the minimum $(v/v_0)_{\min}$ moves from 1.0 to 1/2. It is worthwhile to note that the shape of the curve is asymmetry under reversal. We choose $\varepsilon=5.7$ (diamond) and 10.4 (GaN) for real materials, and $\varepsilon=100$ and 1000 for large values. The v is very close to v_∞ for $\varepsilon=100$ and v is almost the same as v_∞ for $\varepsilon=1000$. However, there is one thing to mention. The v/v_0 becomes 1 just at $\theta_2 = \theta_1$ for ε as large as possible in Fig 4, whereas v_∞/v_0 is 0.5 in Fig. 4. Two are not the same. We should be back to Eq. (6), which leads to the value $v = v_0$ at $\theta_2=0$, $\theta_1/2(=\theta_1)$, and θ_1 , regardless of ε . No change in F is made when the field region is occupied with vacuum or dielectric only, or is divided equally. It is then required that $v/v_0=1$ at $\theta_2 = \theta_1$, even for an extremely large value of ε .

At this point, we have to stress the two properties important for the field behavior even though they were mentioned in the previous work [12]. First, it depends on the ratio θ_2/θ_1 (or the ratio θ_2/θ_1) whether v is larger than v_0 or not. Simply, v is larger than v_0 for $\theta_2 < \theta_1/2$ (i.e., $\theta_2 < \theta_1$) and is smaller for $\theta_2 > \theta_1/2$ (i.e., $\theta_2 > \theta_1$). This implies that F is enhanced (or reduced) for θ_2 larger (or smaller) than θ_1 . At $\theta_2 = \theta_1$, v is equal to v_0 , implying no enhancement of F at all. Secondly, the value of ε is only the factor to determine the shape of v/v_0 versus θ_2/θ_1 . As ε increases, it has the higher peak and deeper valley, meaning a larger enhancement or reduction in the field [12-14].

3. Quadruple junctions

3.1 Metal-vacuum-dielectric-vacuum quadruple junctions

For a deeper understanding of the dielectric effect, we consider the metal-vacuum-dielectric-vacuum (MVDV) quadruple junction formed by introducing another vacuum portion between metal and dielectric in the triple junction (see Figs. 1, 2, and 5). Symbols α ,

θ_1 , θ_2 and θ_3 indicate the angles subtended by metal, vacuum, dielectric, and vacuum, respectively. In the same way as done for the triple junction, we make the two-dimensional treatments to find the solution, Φ , of the Laplace equation. In the region near the junction, Φ can be given in the forms [17,18]

$$\Phi_1 = A_1 r^\nu \sin \nu \theta, \quad 0 \leq \theta \leq \theta_1 \text{ (vacuum)}, \quad (10)$$

$$\Phi_2 = r^\nu (A_2 \sin \nu \theta + B_2 \cos \nu \theta), \quad \theta_1 \leq \theta \leq \theta_1 + \theta_2 \text{ (dielectric)}, \quad (11)$$

$$\Phi_3 = A_3 r^\nu \sin \nu (2\pi - \alpha - \theta), \quad \theta_1 + \theta_2 \leq \theta \leq \theta_1 + \theta_2 + \theta_3 = \theta_t \text{ (vacuum)}, \quad (12)$$

where the subscript refers to the region. In each region, the electric field is given in the form

$$F_1 = A_1 \nu r^{\nu-1}, \quad 0 \leq \theta \leq \theta_1 \text{ (vacuum)}, \quad (13)$$

$$F_2 = (A_2^2 + B_2^2)^{1/2} \nu r^{\nu-1}, \quad \theta_1 \leq \theta \leq \theta_1 + \theta_2 \text{ (dielectric)}, \quad (14)$$

$$F_3 = A_3 \nu r^{\nu-1}, \quad \theta_1 + \theta_2 \leq \theta \leq \theta_1 + \theta_2 + \theta_3 = \theta_t \text{ (vacuum)}. \quad (15)$$

Using the boundary conditions of both Φ and $\partial\Phi/\partial\theta$ at $\theta = \theta_1$ and $\theta_1 + \theta_2$, we have four relations for five unknowns, ν and four coefficients A_i ($i=1, 2, \text{ and } 3$) and B_2 . Three are the expressions of three coefficients in terms of the rest, say A_2 . The fourth is the transcendental equation for ν [17]:

$$\frac{(1 - \epsilon) \tan \nu \theta_1}{1 + \epsilon \tan^2 \nu \theta_1} = \frac{\tan \nu (\theta_1 + \theta_2) + \epsilon \tan \nu \theta_3}{1 - \epsilon \tan \nu (\theta_1 + \theta_2) \tan \nu \theta_3}. \quad (16)$$

It seems that θ_1 and θ_3 are not exchangeable in Eq. (16). However, the exchange of θ_1 and θ_3 does not give rise to any difference in v . As expected so, Eq. (16) reduces to Eq. (6) when $\theta_3=0$.

We make numerical calculations of Eq. (16) to obtain v as a function of θ_2 for several values of θ_3 . The obtained results are shown in Fig. 6 for $\theta_3 = 0$ (a triple junction), 30° , 60° , 90° , 120° , 150° (asymmetric), and θ_1 (symmetric). The two curves of $\theta_3=30^\circ$ and 120° , or 60° and 120° are complementary but are not symmetric. The reason is that θ_3 is fixed whereas $\theta_1 (=2\pi - \alpha - \theta_3 - \theta_2)$ varies with varying θ_2 . As seen in Fig. 6, we have two interesting properties for the MVDV junction. First, all v 's are lower than those for the triple junction. This implies that the MVDV quadruple junction can yield more enhancement of field than the triple junction. Second, v for the symmetric quadruple junction of $\theta_3 = \theta_1$ is the entire loci of valleys for asymmetric junctions. This means that the symmetric MVDV junction always yields the lowest v at given θ_2 and then the largest enhancement of F . It is also interesting that v for the symmetric MVDV junction has the minimum at $\theta_2 = \theta_1 / 2$, irrespective of values of ε . It is recalled that for the triple junction, it moves to the larger θ_2 as ε increases.

3.2 Metal-dielectric-vacuum-dielectric quadruple junctions

Alternatively, we consider the metal-dielectric-vacuum-dielectric (MDVD) quadruple junction obtained by inserting an additional dielectric portion between metal and vacuum in the triple junction (see Figs. 1, 2, and 5). This structure may be the case that void exists on the side of dielectric at the triple junction. Symbols α , θ_3 , θ_1 , and θ_2 represent the angles subtended by metal, dielectric, vacuum, and dielectric, respectively. Numbering begins at 3. This choice is made so that the case of $\theta_3=0$ always represents the same triple junction as treated before. Since similar arguments is applicable in obtaining Φ , we write

$\Phi_3 = A_3 r^\nu \sin \nu \theta$ for $0 \leq \theta \leq \theta_3$, $\Phi_1 = r^\nu (A_1 \sin \nu \theta + B_1 \cos \nu \theta)$ for $\theta_3 \leq \theta \leq \theta_3 + \theta_1$, and $\Phi_2 = A_2 r^\nu \sin \nu (2\pi - \alpha - \theta)$ for $\theta_3 + \theta_1 \leq \theta \leq \theta_3 + \theta_1 + \theta_2$, where subscripts 3, 1, and 2 refer to the regions. The use of the boundary conditions at $\theta = \theta_3$ and $\theta_3 + \theta_1$ yields the associated transcendental equation

$$\frac{(1 - \varepsilon) \tan \nu \theta_3}{\varepsilon + \tan^2 \nu \theta_3} = \frac{\varepsilon \tan \nu (\theta_3 + \theta_1) + \tan \nu \theta_2}{-\varepsilon + \tan \nu (\theta_3 + \theta_1) \tan \nu \theta_2}. \quad (17)$$

,

As expected, Eqs. (16) and (17) are similar in form. The difference comes from the different composites and numbering.

Numerical calculations of Eq. (17) are carried out to obtain ν as a function of θ_1 for several values of θ_3 . As seen in Fig. 7, we have two interesting properties for the MDVD junction. First, all ν 's are higher than those for the triple junction. This implies that the field can be more largely reduced for the MDVD junction than for the triple junction. Second, ν for the quadruple junction of $\theta_3 = \theta_2$ is the entire loci of peaks for asymmetric junctions of given θ_3 . This means that the symmetric MDVD junction always yields the highest ν at given θ_2 . It is interesting that ν for the symmetric MDVD junction has the maximum at $\theta_2 = \theta_1 / 2$, irrespective of ε .

4. Electric fields and enhancements

4.1 Electric fields for the triple junction

It is supposed that a free negative charge Q is distributed on the surface of the metal (see Fig. 2). Since the surface charge density is given by the normal component of displacement,

we use Eqs. (4) and (5) to derive $Q = -\ell R^\nu (1 + \eta\varepsilon)A_1$. The field energy W is calculated to be $W = \nu R^{2\nu} \ell (\theta_1 + \eta^2 \varepsilon \theta_2)A_1^2 / 16\pi$. Then we have the potential [12,15]

$$V = 2W/Q = -\frac{\nu}{8\pi} R^\nu \frac{\theta_1 + \eta^2 \varepsilon \theta_2}{1 + \eta \varepsilon} A_1. \quad (18)$$

V is regarded as the electric potential imposed on the metal with free charge Q . Combining Eqs. (4), (5), (7), and (18), we have

$$F_1(r) = 8\pi \left(\frac{1 + \eta \varepsilon}{\theta_1 + \eta^2 \varepsilon \theta_2} \right) \left(\frac{r}{R} \right)^{\nu-1} \left(\frac{-V}{R} \right), \quad (19)$$

$$F_2 = \eta F_1(r). \quad (20)$$

Now we define the reference field F_0 as F_1 when dielectric is replaced with vacuum. At $\varepsilon=1$, where $\theta_2=0$, $\theta_1 = \theta_t$, $\eta=1$ and $\nu = \nu_0$, Eq. (19) becomes

$$F_0 = 16 \nu_0 \left(\frac{r}{R} \right)^{\nu_0-1} \left(\frac{-V}{R} \right). \quad (21)$$

The field enhancement β is defined as the ratio of the two fields, F_1 and F_0 :

$$\beta(r) = F_1(r)/F_0(r) = 0.5 \left(\frac{1 + \eta \varepsilon}{1 + (\eta^2 \varepsilon - 1) \theta_2 / \theta_t} \right) \left(\frac{R}{r} \right)^{\nu_0 - \nu}. \quad (22)$$

It is clear that Eq. (22) yields $\beta=1$ at $\varepsilon=1$. For $\varepsilon>1$, β is larger or smaller than unity according to the value of v/v_0 , or θ_2/θ_1 . That is, β is smaller (or larger) than unity for $\theta_2 < \theta_1$ (or $\theta_2 > \theta_1$). It is found that the prefactor in Eq. (22) varies approximately from unity to 5.5 over the entire range of θ_2 . We choose $R = 1\mu\text{m}$ and $r=0.1$ nm to make numerical calculations of β in the enhancement region (see Fig. 8). For $\varepsilon = 5.7$ (dotted line), we have the minimum enhancement $\beta_{\min} \approx 0.17$ at $\theta_2/\theta_1 \approx 0.29$ and the maximum enhancement $\beta_{\max} \approx 19.8$ at $\theta_2/\theta_1 \approx 0.91$. For $\varepsilon = 10.4$ (solid line), we have $\beta_{\min} \approx 0.12$ at $\theta_2/\theta_1 \approx 0.30$ and the maximum $\beta_{\max} \approx 47.1$ at $\theta_2/\theta_1 \approx 0.93$. This is the enhancement β of vacuum field F_1 (blue line). The dielectric field F_2 has the enhancement $\eta\beta$ (red line), which is smaller β . As seen in Figs. 4 and 8, both v_{\min} and β_{\max} do not take places at the same θ_2/θ_1 . The β has the maximum at a little larger θ_2 than v , due the prefactor of Eq. (22). It is worthwhile to mention that all the associated extrema v_{\min} , β_{\max} , $F_{1,\max}$ and $F_{2,\max}$ lie in the region $0.75 < \theta_2/\theta_1 < 1.0$.

The ε -dependence of β_{\max} is shown more clearly in Fig. 9. It seems that β_{\max} increases almost linearly with increasing ε . The slope depends on α and r . We take the values of $r=0.1$ and 1.0 nm with the fixed value of $R=1\mu\text{m}$. As ε increases from 1 to 100 (not shown here), β_{\max} increases from 1.0 to 122.5 for $\alpha=0$ and $r=1.0$ nm (red dashed line), from 1.0 to 153.2 for $\alpha=0$ and $r=0.1$ nm (red solid line), and from 1.0 to 669.4 for $\alpha=180^\circ$ and $r=0.1$ nm (blue solid line). The β_{\max} becomes very large for large ε , especially at small r and $\alpha=180^\circ$. Two plots of $\alpha=0$ and $r=0.1$ nm (red solid line), and $\alpha=90^\circ$ and $r=1.0$ nm (green dashed line) coincide accidentally.

Figure 10 shows the α -dependence of β_{\max} and F_0 . The α -dependence reflects the curvature of the metallic shape. From Eq. (22), we have the exponent $v_0 - v = v_0(1 - v/v_0)$.

The value v/v_0 depends on the dielectric property only. In contrast, v_0 depends on the curvature of the metallic shape. Thus the α -dependent enhancement comes majorly from the α -dependence of F_0 . Irrespective of dielectric, F_0 increases with decreasing α . In the presence of dielectric, β increases with increasing α . Since $F_1 = \beta F_0$, there is the competition between the two factors in determining F_1 (and F_2) as α varies. To figure out the curvature-dependence of both enhancements, we have made numerical calculations of F_0 , $F_{1,\max}$ and $F_{2,\max}$ as a function of α . The numerical values of F are calculated by setting $16\pi V/R = 4.0$ and then varies with the applied bias V . We use the arbitrary unit for F just to know the relative magnitudes.

As α increases from 0 to 180° , $F_0(r)$ decreases from 63.7 to 1.27 (50 times smaller), irrespective of ϵ . Instead, β_{\max} increases from 7.77 to 19.8 (2.5 times larger) for $\epsilon = 5.7$ and from 15.0 to 47.1 (3.1 times larger) for $\epsilon = 10.4$. This implies that F_0 changes more rapidly with α than β or $\eta\beta$. As a result, both $F_{1,\max}$ and $F_{2,\max}$ are shown to increase with decreasing α . However, the dielectric enhancement is still significant or large over all α . The dielectric effect is not just apparently visible because it changes slowly in comparison with the shape effect.

4.2 Electric fields for the quadruple junction

As discussed in section 3, the MVDV junction is effective in enhancing the field whereas the MDVD in reducing the field. In the current work, we consider the MVDV junction. We apply the same calculation scheme as used for the triple junction to obtain

$$\beta(r) = F_1(r)/F_0(r) = 0.5 \left(\frac{\eta_1(\eta_1 + \eta_2)\theta_t}{\eta_1^2\theta_1 + \varepsilon(1 + \eta_2^2)\theta_t + \eta_3^2\theta_3} \right) \left(\frac{R}{r} \right)^{v_0-v}, \quad (23)$$

where $\eta_1 = A_1/A_2$, $\eta_2 = B_2/A_2$, $\eta_3 = A_3/A_2$. All η_i ($i=1, 2$ and 3) are obtained using the boundary condition [17]. Two $\beta(r)$ given by Eqs (22) and (23) are the same in form and are different in the configuration-dependence.

For the symmetric quadruple junction of $\theta_3 = \theta_1$, we make Eq. (23) to obtain $\beta(r)$ as a function of θ_2/θ_1 for $\varepsilon=5.7$. The results are shown in Fig. 11. We consider $r=0.1$ (solid line) and 1.0 nm (dashed line). We also plot $\beta(r)$ for the triple junction for comparison. We have the maximum enhancement $\beta_{\max}=21.8$ and 66.1 for $r=1.0$ and 0.1 nm, respectively. Those are 12.0 and 19.8 for the triple junction. The β_{\max} for the symmetric quadruple junction are 2 or 3 times larger than those for the triple junction. The dielectric effect is more effective for the quadruple junction.

The difference between enhancements for the triple and quadruple junctions supports the reason that the electric field is enhanced or reduced by the polarization of dielectric. In the absence of dielectric, the surface charge is distributed so that the potential is constant over the metallic surface, resulting in the α -dependence of F_0 . If dielectric is introduced to be in contact with metal, then the charge on the metal causes dielectric to polarize. In turn, the free surface charge on metal is also redistributed under the influence of polarization charges. The magnitude and direction of polarization depends on the contact angles of the constituents. If polarization charges attract free surface charges to the junction, then more free charges are accumulated near the junction. The field becomes strong. The strong field produces larger polarization. This process goes on until the perfect polarization of dielectric is reached. As a result, the very strong is formed due to dielectric for the triple and quadruple junctions in the

region near the junction of metal, vacuum, and dielectric.

5. Conclusions

We described the electric field in the vicinity of metal-dielectric contact with vacuum.. The assumption of the two-dimensional symmetry of junctions led to the formulation of the field enhancement due to dielectric. For the triple junction, the electric field was found to be enhanced and reduced according to the contact angle ratio as well as the dielectric constant. It was found that a certain type of quadruple junction yielded a larger enhancement than the triple junction. It was noted that the enhancement of the electric field at the metal-dielectric contact is the product of the two enhancements due to dielectric and the shape of the metallic emission portion. Such an field enhancement may be large enough for the strange triple junction phenomenon such as the vacuum or dielectric breakdown.

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Figure Captions

Fig. 1 Triple and quadruple junctions formed on an interface. There may exist a triple junction of metal-vacuum-dielectric (small blue circle) and quadruple junctions of metal-vacuum-dielectric-vacuum (red large circle) and metal-dielectric-vacuum-dielectric (red dotted circle) [6].

Fig. 2 Triple junctions made by Geis et al. [4](a) for experiment and suggested by authors (b) for calculation. Two are reduced to the two-dimensional junction (c), where the coordinate origin is taken at the triple junction (d). The contact angles of metal, vacuum, and dielectric are given by α , θ_1 , and θ_2 , respectively. The cylinder is considered to have the radius $R \approx 1\mu\text{m}$ and the length $\ell \approx 1\text{mm}$.

Fig. 3 Enhancement parameter v_∞ for the triple junction. The inset shows the plot of v_0 versus θ_1 for the metal-vacuum junction ($\theta_2=0$). The dotted line represents v when dielectric portion is metallic.

Fig. 4 Enhancement parameter v for $1 < \epsilon < \infty$. We take $\epsilon=5.7$ (diamond) and 10.4 (GaN), 100 , 1000 . As ϵ increases, v has higher peaks and lower valleys and both peaks and valleys move toward the larger θ_2 .

Fig. 5 Cylindrical quadruple junctions. Additional portion, vacuum (a) or dielectric (b), is added to the triple junction. Symbol θ_3 always indicates the angle subtended by the additional portion. .

Fig. 6 Enhancement parameter v for the metal-vacuum-dielectric-vacuum quadruple junction. We take $\theta_3=0$ (triple junction), 30° , 60° , 90° , 120° , 150° , and θ_1 (symmetric).

Fig. 7 Enhancement parameter v for the metal-dielectric-vacuum-dielectric quadruple junction. We take $\theta_3=0$ (triple junction), 30° , 60° , 90° , 120° , 150° , and θ_1 (symmetric).

Fig. 8 Field enhancements of vacuum and dielectric fields for the triple junction. We take $\epsilon=5.7$ (dashed line) and 10.4 (solid line). The enhancement, β , for the vacuum field F_1 (blue line) is larger than that for the dielectric field F_2 (red line).

Fig. 9 Plot for β_{\max} versus ϵ . The maximum enhancement β_{\max} is shown as a function of ϵ for $r=0.1$ (solid line) and 1.0 nm (dashed line) and $\alpha=0, 90^\circ$, and 180° . It is seen that β_{\max} increases almost linearly with increasing ϵ . The plot for $\alpha=0$ and $r=0.1$ nm is overlapped on that for $\alpha=90^\circ$ and $r=1.0$ nm.

Fig. 10 Shape-dependence of the field and the enhancement. As α varies from 0 to 180° , F_0 decreases rapidly while β increases slowly. Thus both F_1 (blue line) and F_2 (red line) are shown to follow the trend of F_0 . We take $\epsilon=5.7$ (dashed line) and 10.4 (solid line).

Fig. 11. Field enhancement for the symmetric quadruple junction. The symmetric metal-vacuum-dielectric-vacuum quadruple junction (blue line) yields the larger enhancement than the triple junction (red line). We take $r=0.1$ (solid line) and 1.0 nm (dashed line).

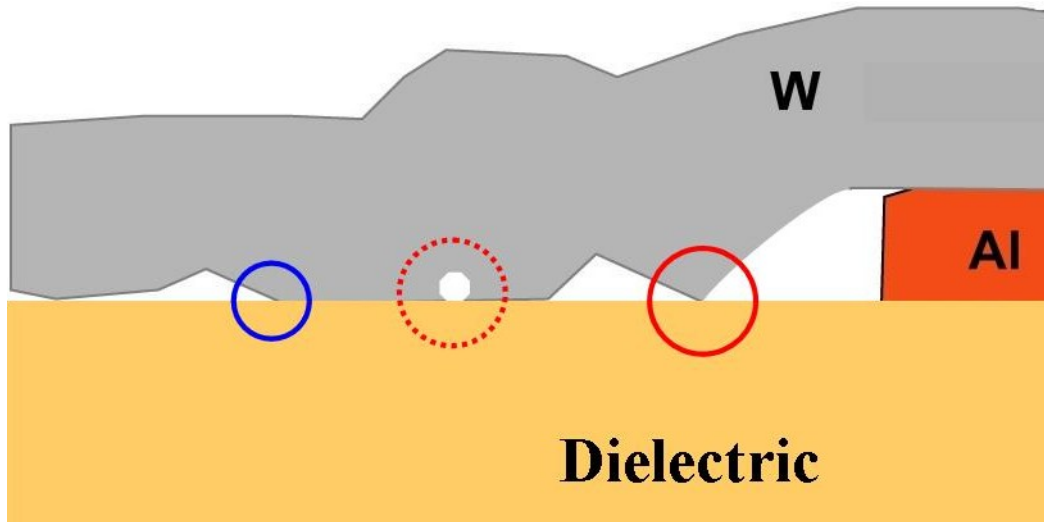


Fig. 1

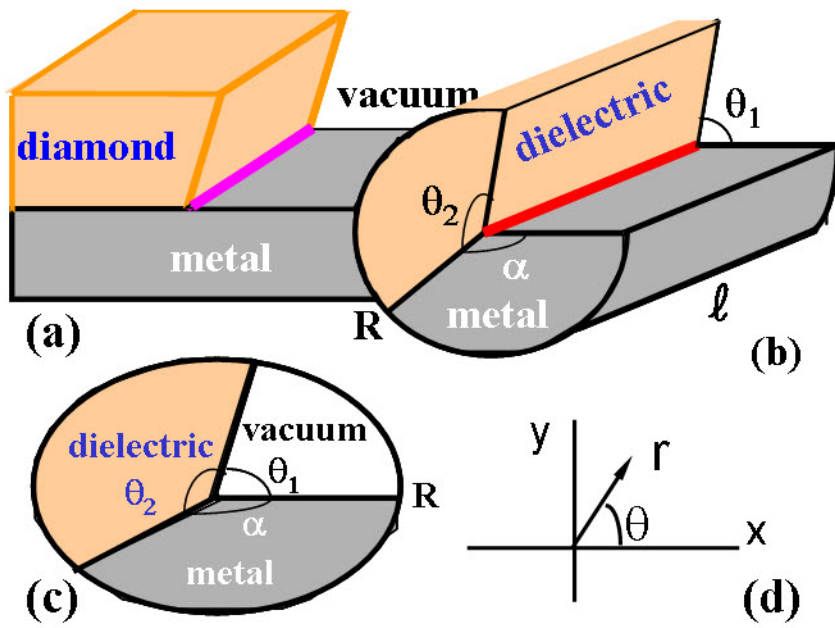


Fig. 2

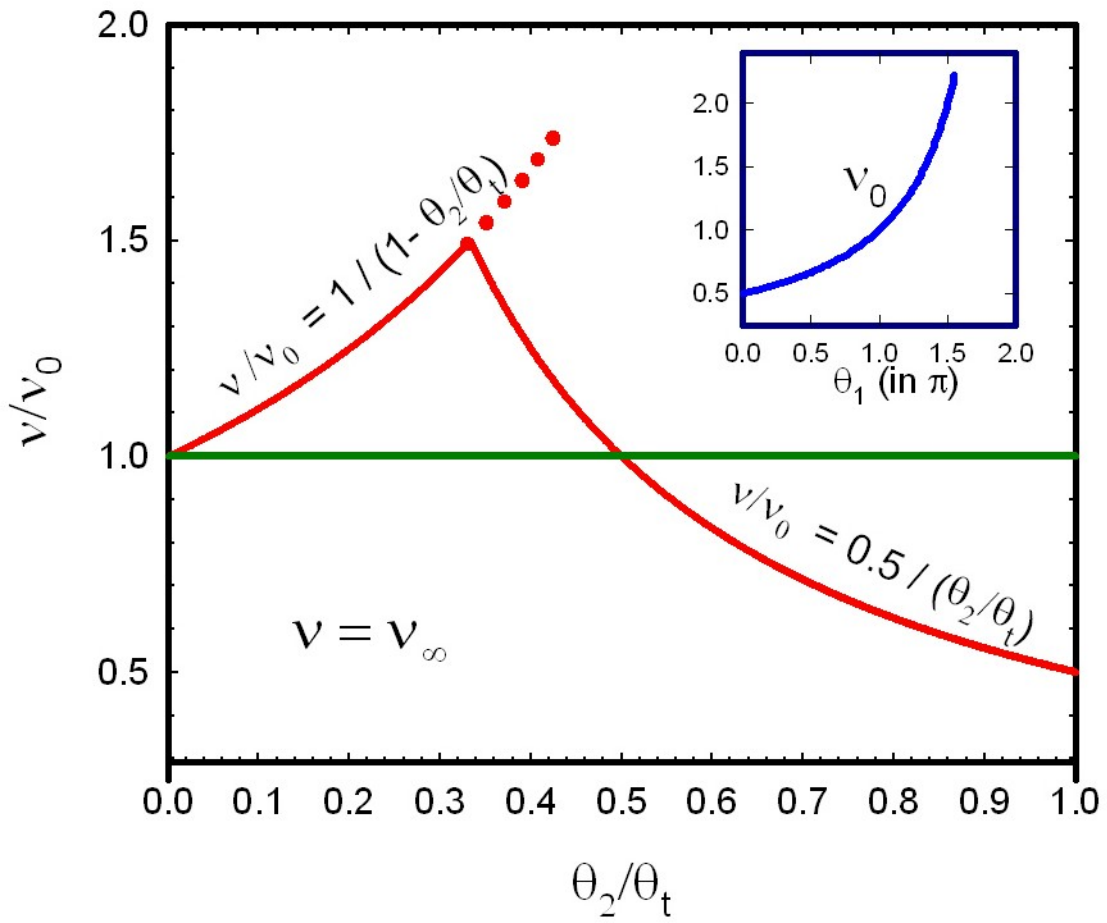


Fig. 3

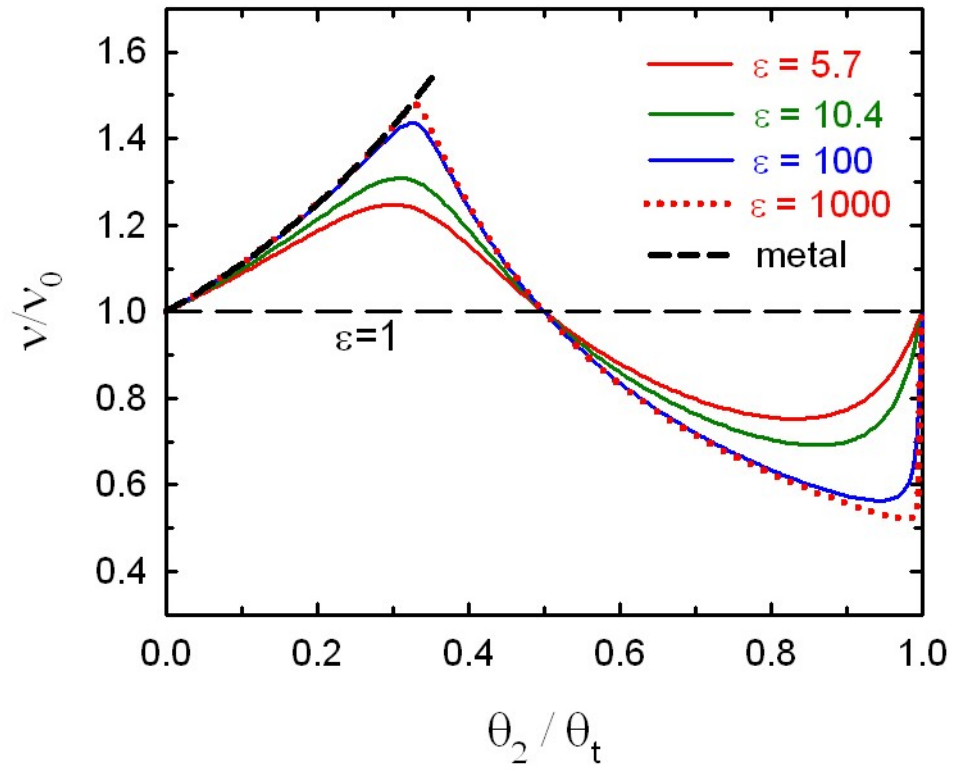


Fig. 4

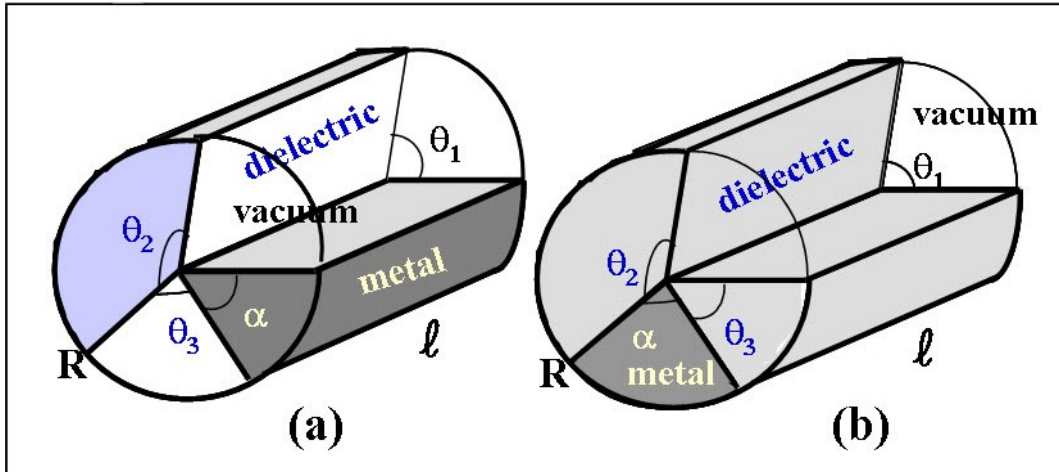


Fig. 5

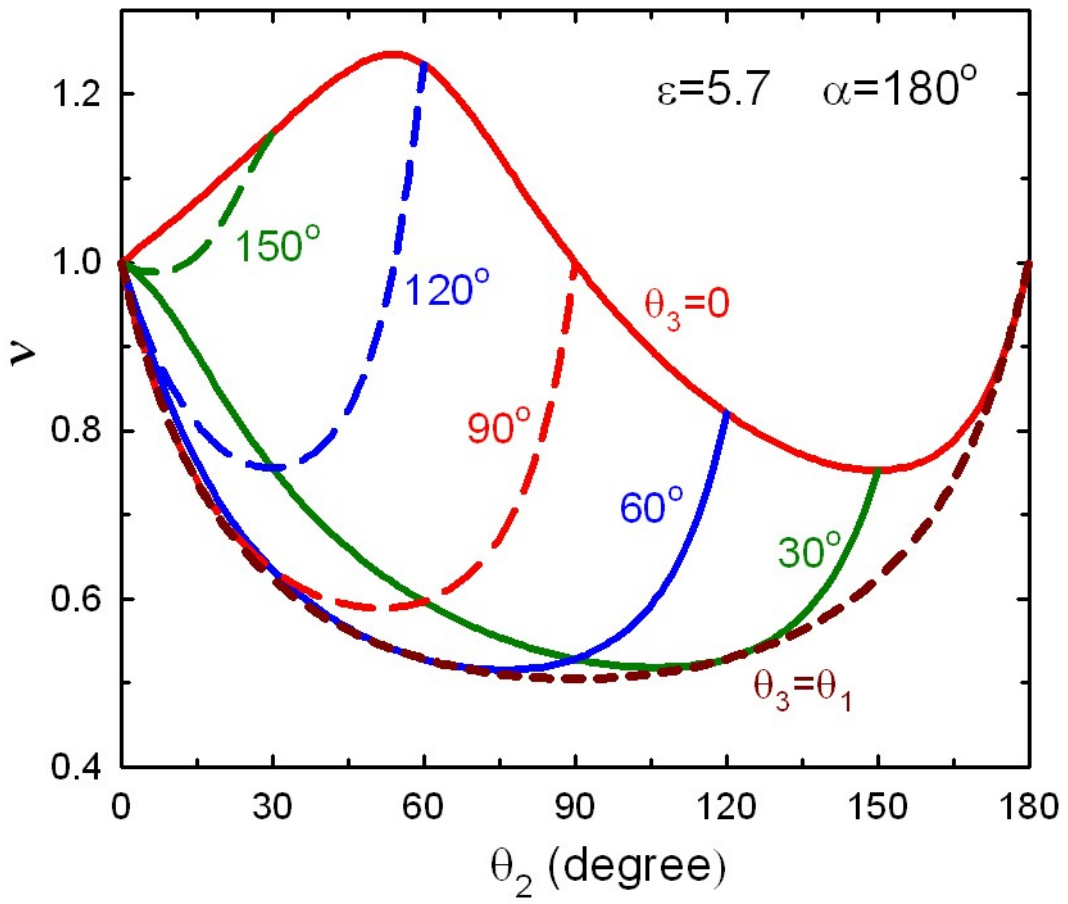


Fig. 6

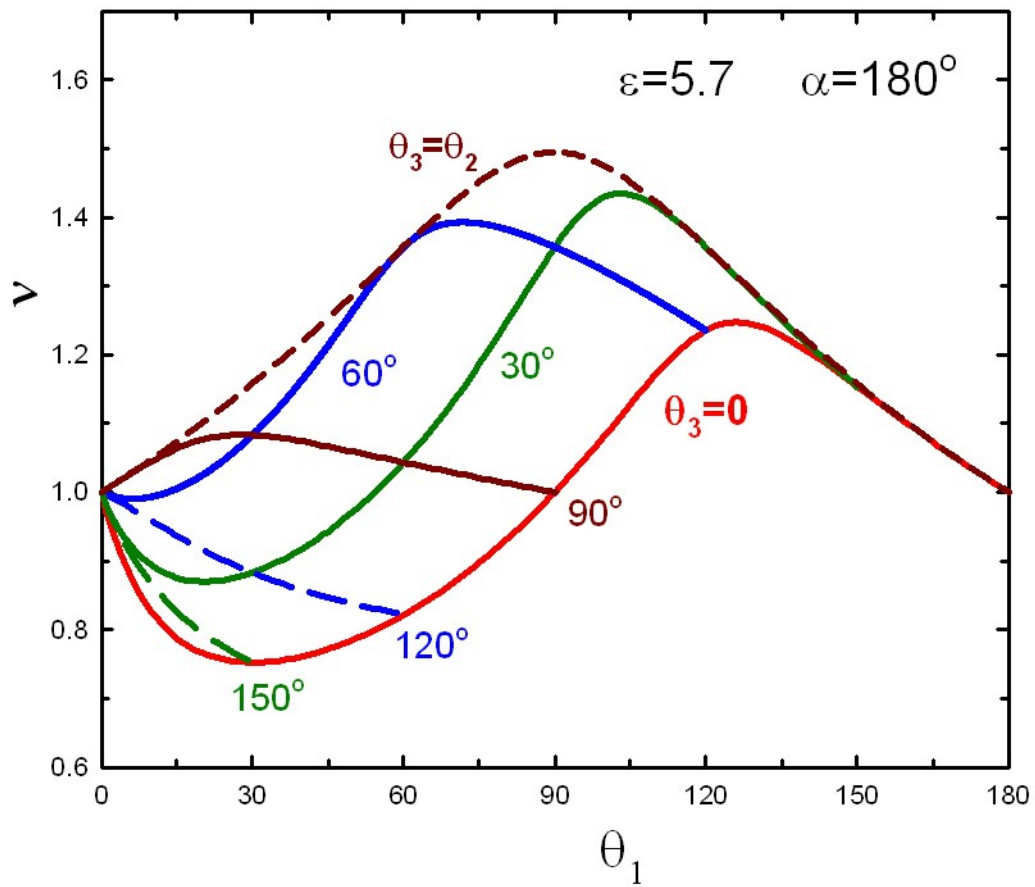


Fig. 7

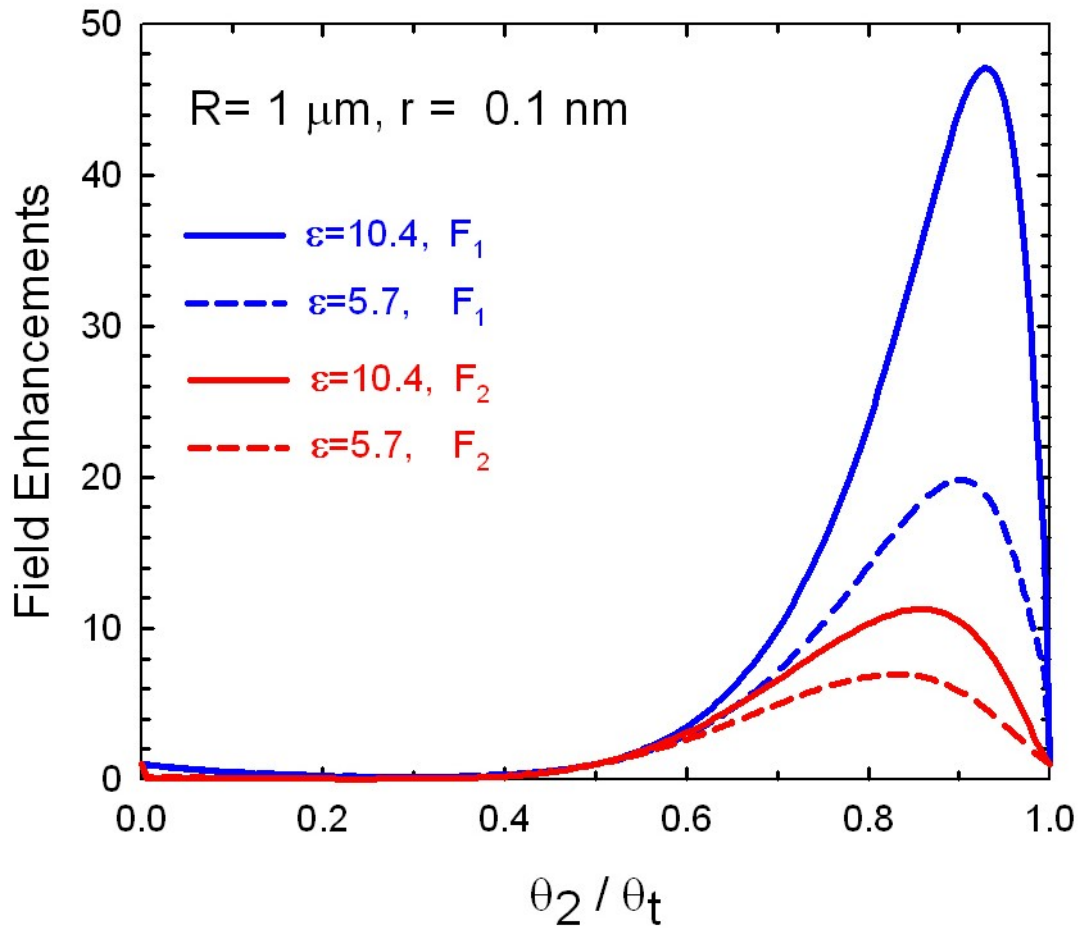


Fig. 8

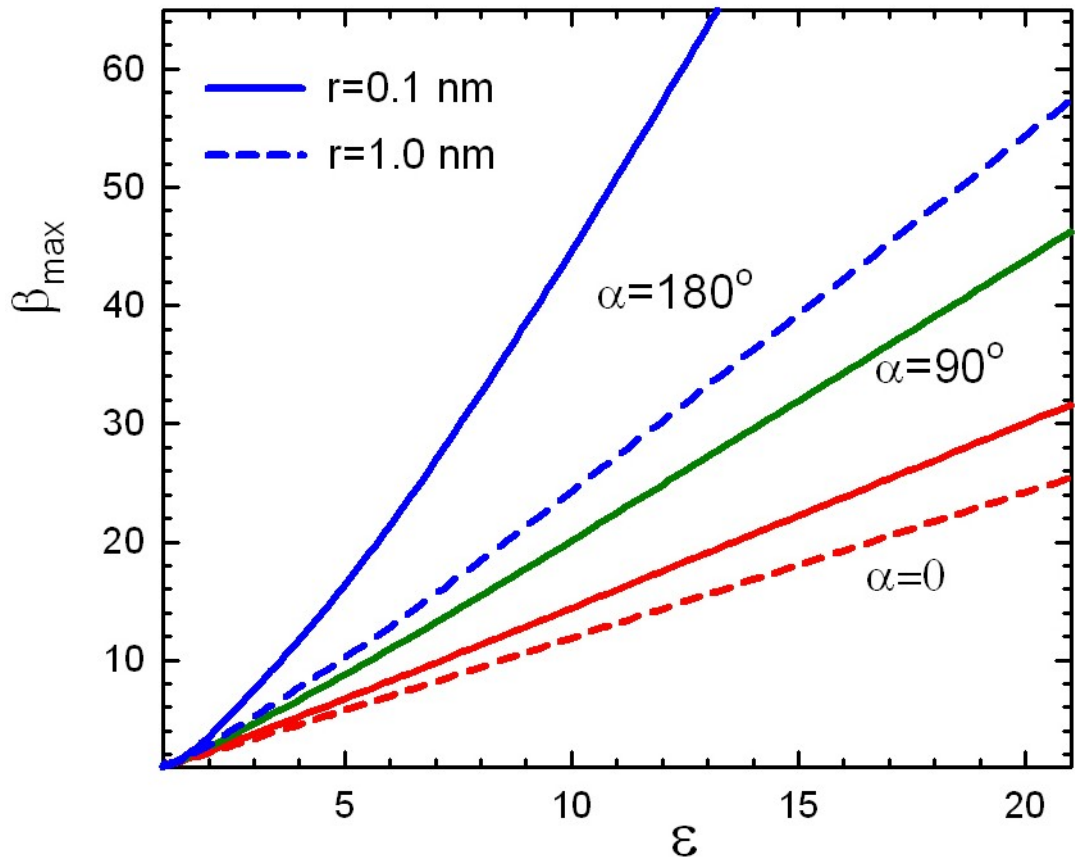


Fig. 9

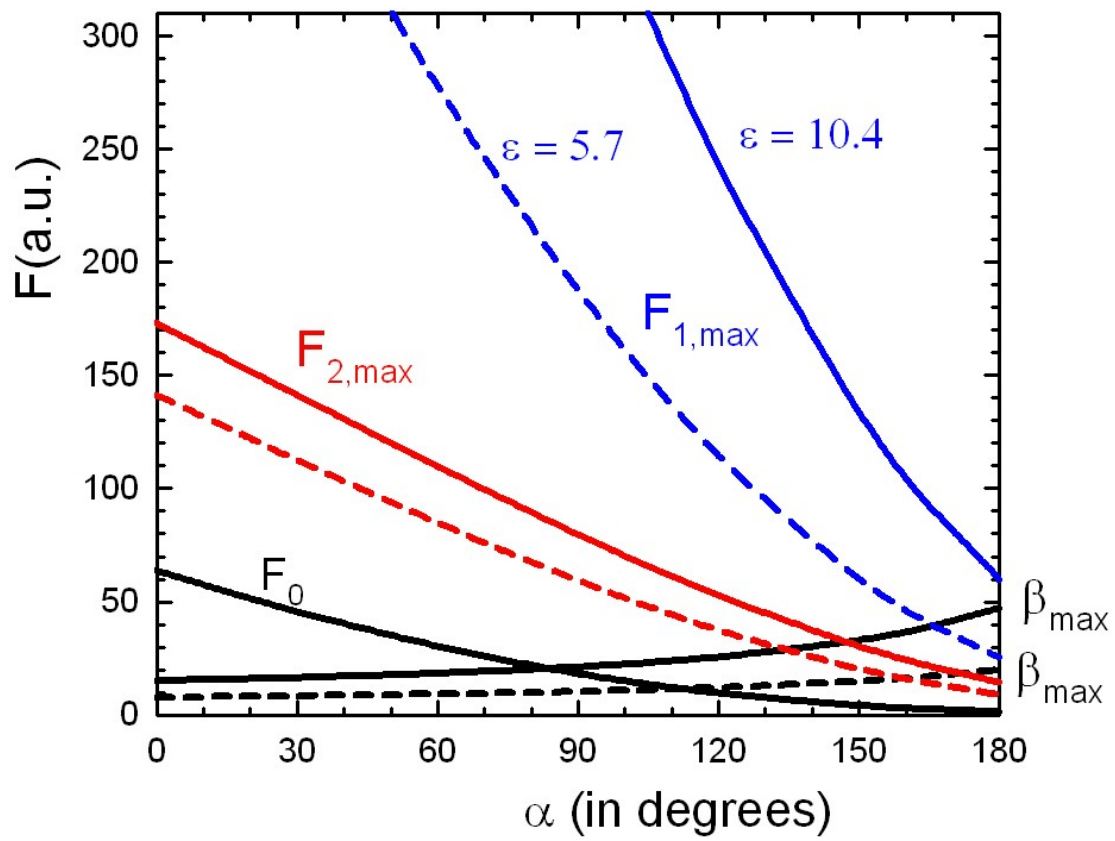


Fig. 10

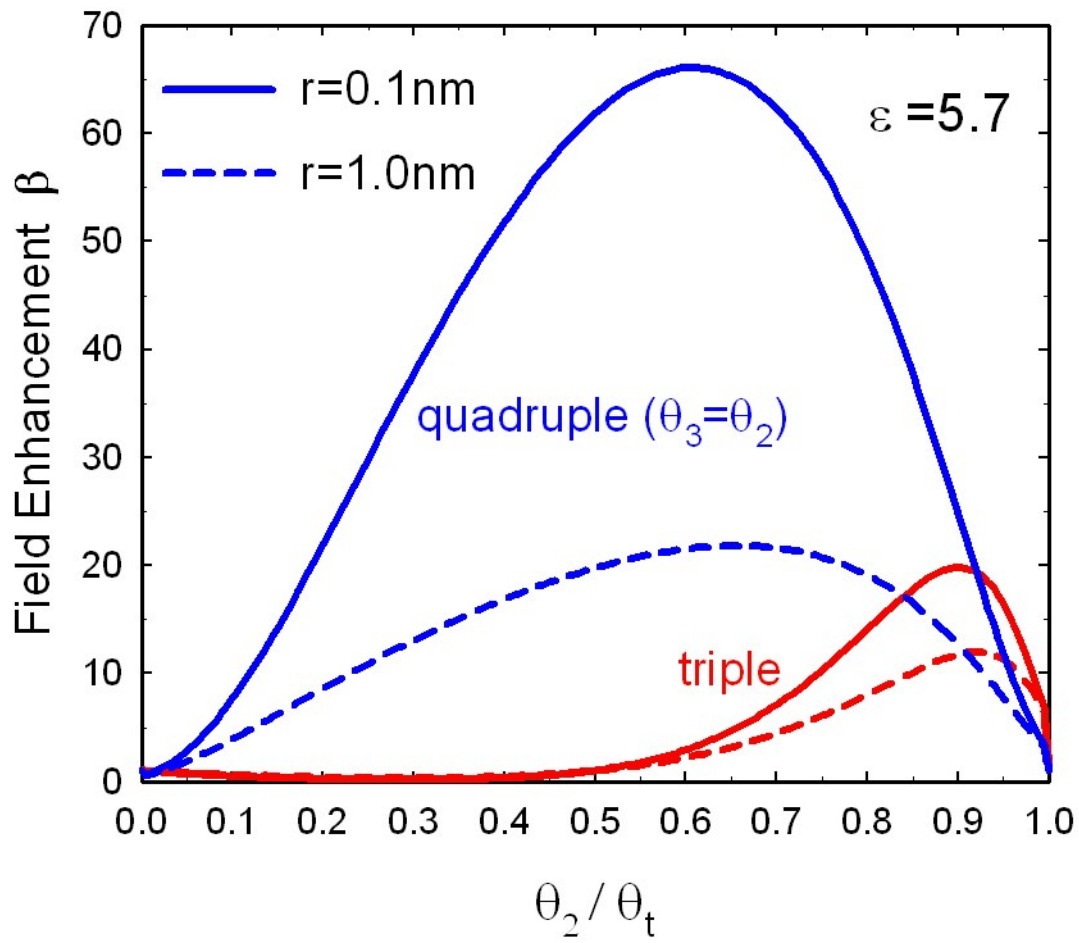


Fig. 11