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# Self-T-dual Brane Cosmology

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## Abstract

We show how T-duality can be implemented with brane cosmology. As a result, we obtain a smooth bouncing cosmology with features similar to the ones of the pre-Big Bang scenario. Also, by allowing T-duality transformations along the time-like direction, we find a static solution that displays an interesting self tuning property.

*Based on a talk given at the XI Marcel Grossmann Meeting, Berlin 23-29 July, 2006.*

# 1 Introduction

In the past years, various cosmological models were inspired by different aspects of string theory. In some cases, these rely upon the fundamental symmetries of string theory, the most notable example being the pre-Big Bang (PBB) scenario motivated by T-duality [1]. In other cases, models are based on extended objects such as branes [2]. These two approaches are often seen as competing. However, if our Universe is seen as a brane moving in a higher-dimensional bulk, obtained by compactification of string theory, it is likely that the effective cosmology inherits some of the symmetries of the uncompactified theory. A first application of this idea can be found in the context of type IIA/IIB supergravity. When compactified to five dimensions, these theories possess static black hole solutions with flat horizon [3], which are directly related by T-duality transformations. By studying a brane moving in these dual spaces, it was found that these transformations induce the inversion of the cosmological scale factor on the brane, along the lines of the PBB scenario [4]. The latter, however, is based on a self-T-dual action, with time-dependent background solutions. In the next section, we show that it is possible to construct a self-T-dual action, which, instead, has static background solutions. Also, an embedded moving brane displays an effective cosmological evolution, which smoothly connects a pre- and a post-big bang phase, through a non-singular bounce, in complete analogy with some of the PBB models. Finally, in the last section, we will also show how Self-T-dual brane models address the problem of fine-tuning between the brane vacuum energy density (tension) and the bulk cosmological constant.

## 2 Pre-Big Bang on the brane

To recreate a PBB scenario on the brane, we must find first a self-T-dual action, such that the related equations of motion have static solutions. Let us consider the dilaton-gravity action [5]

$$S_{\text{bulk}} = \int_{\mathcal{M}} d^5x \sqrt{g} e^{-2\phi} [\mathcal{R} + 4(\nabla\phi)^2 + V] , \quad (1)$$

where  $V$  is an exponential function of the so-called shifted dilaton  $\bar{\phi}$ . If we choose the line element

$$ds^2 = -A^2(r) dt^2 + B^2(r) dr^2 + R^2(r) \delta_{ij} dx^i dx^j , \quad (2)$$

then the shifted dilaton is defined as  $\bar{\phi}(r) = \phi(r) - \frac{3}{2} \ln R(r)$ . It can be shown that the action (1) is invariant under the T-duality transformation  $R(r) \xrightarrow{\text{T}} R(r)^{-1}$ , which leaves the shifted dilaton unchanged. Therefore, to any solution with metric (2), there exists another with  $R$  replaced by  $1/R$ . This property holds if we neglect the boundary terms springing from variation of the action with respect to the fields. However, if we want to preserve self-T-duality, these

terms must be kept when we introduce a  $Z_2$  symmetric 3-brane, which acts as a boundary. In this way, it turns out that the full action, obtained by adding Eq. (1) to the brane action

$$S_{\text{brane}} = - \int_{\Sigma} d^3x d\tau \sqrt{h} e^{-2\phi} [4K + \mathcal{L}] , \quad (3)$$

is still invariant under the transformation  $R(r) \xrightarrow{\text{T}} R(r)^{-1}$ , *provided*  $\mathcal{L} \rightarrow \mathcal{L}$ , i.e. provided the brane matter Lagrangian is itself T-duality invariant. In the expression above,  $h$  is the determinant of the induced FLRW metric on the brane,  $ds^2 = -d\tau^2 + R^2(r(\tau))\delta_{ij} dx^i dx^j$ ,  $K$  is the trace of the brane extrinsic curvature, and  $\tau$  is the proper cosmological time (and the parametric the position of the brane in the bulk). It is clear that the duality acting on the bulk metric leads to the inversion of the scale factor  $R$ . Now, let the matter on the brane be a perfect fluid, with equation of state  $p = \omega\rho$ . By carefully studying the Israel junction conditions, it can be shown that the self-T-duality of  $\mathcal{L}$  implies that  $\omega \xrightarrow{\text{T}} -\omega$ , exactly like in the PBB scenario <sup>1</sup>.

By studying the bulk equations of motion, one can find black hole solutions with one regular horizon. A brane moving in such a background encounters a turning point outside the horizon. By assuming that the T-duality transition occurs at the bounce, one can construct a non-singular transition between a pre-big bang phase (with, say,  $-\omega$  and scale factor  $1/R(\tau)$ ) and post-big bang phase (with  $\omega$  and scale factor  $R(\tau)$ ). Finally, it also turns out that the cosmic evolution far away from the bounce, both in the past and in the future, is always of de Sitter type.

### 3 Time-like T-duality and the fine tuning problem

We now turn our attention to the fine tuning problem of Randall-Sundrum-like models. We consider again the action (1), but now we assume that the bulk metric has the Poincaré-invariant form

$$ds^2 = e^{2\sigma(z)} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 , \quad (4)$$

while the shifted dilaton reads  $\bar{\phi} = \phi - 2\sigma$ . Along the lines sketched in the previous section, one can show [6] that the action is invariant under T-duality transformations along both the time and space coordinates  $x^\mu$ , i.e. under  $\sigma(z) \xrightarrow{\text{T}} -\sigma(z)$ . Of course, the same holds for the equations of motion, for which the only non-singular solutions are given by a constant shifted dilaton  $\bar{\phi}_0$  and  $\sigma = \pm\lambda(z - z_0)$ , where  $\lambda$  and  $z_0$  are integration constants. The positive and negative sign solutions are related by T-duality, and they simply correspond to two slices of anti-de Sitter space <sup>2</sup>. By inserting a  $Z_2$ -symmetric brane on this background, with a perfect

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<sup>1</sup>It is important to remark that in this model,  $\omega$  is a function of  $\tau$ .

<sup>2</sup>This is consistent with time-like T-duality, which requires the time-like direction to be compact.

fluid as matter, we can preserve the self-T-duality of the total action provided we impose  $(\omega + 1) \xrightarrow{T} -(\omega + 1)$ . Interestingly, this duality transformation is identical to the one found in the context of phantom cosmology <sup>3</sup>. But the most important result is that, in the case of a static brane, the (constant) energy density of the brane matter is given by

$$\rho^2 = 4V_0 e^{\beta\bar{\phi}_0} , \tag{5}$$

where  $\beta$  and  $V_0$  are arbitrary constants. Therefore, any value of  $\rho$  can be reached given any vacuum expectation value of the shifted dilaton.

These encouraging results call for further investigations into self-T-dual brane cosmology, and the main target is to find some signatures (such as particular CMB fluctuations or relic gravitons) of this model, which might be tested by observations.

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## References

- [1] M. Gasperini and G. Veneziano, *Phys. Rept.* **373** 1 (2003).
- [2] P. Horava and E. Witten, *Nucl. Phys. B* **460** 506 (1996); L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** 3370 (1999); L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** 4690 (1999).
- [3] M. Rinaldi, *Phys. Lett. B* **547** 95 (2002).
- [4] M. Rinaldi, *Phys. Lett. B* **582** 249 (2004).
- [5] M. Rinaldi and P. Watts, *JCAP* **0503** 006 (2005).
- [6] O. Corradini and M. Rinaldi, *JCAP* **0601** 020 (2006).
- [7] M. P. Dabrowski, T. Stachowiak and M. Szydlowski, *Phys. Rev. D* **68** 103519 (2003).

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<sup>3</sup>Thanks to Prof. M. P. Dabrowski for pointing out this similarity [7].