An adaptive agent-based approach to traffic simulation

Barthelemy, Johan; Carletti, Timoteo

Published in:
Transportation Research Procedia

DOI:
10.1016/j.trpro.2017.05.142

Publication date:
2017

Document Version
Publisher's PDF, also known as Version of record

Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 25C (2017) 1238–1248

Transportation Research Procedia

www.elsevier.com/locate/procedia

World Conference on Transport Research - WCTR 2016 Shanghai. 10-15 July 2016

An adaptive agent-based approach to traffic simulation

Johan Barthélemy [a,b,*], Timoteo Carletti [b]

[a] *SMART Infrastructure Facility, University of Wollongong, Wollongong NSW 2522, Australia*
[b] *Namur Research Center for Complex Systems, University of Namur, 5000 Namur, Belgium*

Abstract

The aim of this work is to present the initial exploration of a behavioural Dynamic Traffic Assignment model, particularly suitable to be used and implemented in agent-based micro-simulations. The proposal relies on the assumption that travellers take routing policies rather than paths, leading us to introduce the possibility for each simulated agent to apply, in real time, a strategy allowing him to possibly re-route his path depending on the perceived local traffic conditions, jam and/or time spent.

The re-routing process allows the agents to directly react to any change in the road network. For the sake of simplicity, the agents' strategy is modelled with a simple neural network whose parameters are determined during a preliminary training stage. The inputs of such neural network read the local information about the route network and the output gives the action to undertake: stay on the same path or modify it. As the agents use only local information, the overall network topology does not really matter, thus the strategy is able to cope with large networks.

Numerical experiments are performed on various scenarios containing different proportions of trained strategic agents, agents with random strategies and non-strategic agents, to test the robustness and adaptability to new environments and varying network conditions. The methodology is also compared against MATSim and real world data. The outcome of the experiments suggest that this work-in-progress already produces encouraging results.

Peer-review under responsibility of WORLD CONFERENCE ON TRANSPORT RESEARCH SOCIETY.

Keywords: behavioural dynamic traffic assignment; agent-based model; strategic agents; neural networks; routing policy

* Corresponding author. Tel.:+61-242392329.
E-mail address: johan@uow.edu.au

2352-1465
Peer-review under responsibility of WORLD CONFERENCE ON TRANSPORT RESEARCH SOCIETY.
10.1016/j.trpro.2017.05.142

1. Introduction

Traffic flows simulation represents a central part of traffic micro-simulators such as MATSim Meister et al. (2010), DynaMIT Ben-Akiva et al. (1998) and AIMSUN Barceló and Casas (2005) as well as the traffic modelling part of UrbanSim Waddell (2002) and ILUTE Salvini and Miller (2005) integrated simulators. This module is in charge of executing the daily plans of simulated individuals in a physical environment, i.e. representing the traffic flows dynamics on a road network.

In recent decades, dynamics traffic assignment (for short DTA in the following) models emerged for solving this problem (see Chiu et al. (2011) for an extensive description of these techniques), which either aim at reaching a steady-state (user equilibrium) of the considered system or at simulating the agents route choice behaviours.

DTA techniques can also be distinguished by their analytical or simulation-based nature. Analytical methods formulate the traffic assignment as non-linear programming and optimisation problems or variational inequalities instead of focusing on the agents' behaviours. Examples of such works include Friesz et al. (1993), Merchant and Nemhauser (1978a) and Merchant and Nemhauser (1978b). Even though they have demonstrated their usefulness and are grounded on sound mathematical theories, their complexity and computational cost make their application to large-scale scenarios difficult Peeta and Ziliaskopoulos (2001).

Hence simulation-based methods, which explicitly model the individuals' mobility behaviours, have recently gained more attention in the literature (Nagel and Fltter ¨od (2009), Bazghandi (2012) and Ben-Akiva et al. (2012)). The underlying idea is to compute a user equilibrium by means of an iterative process. These successive steps generate traffic flows until the travel time of every agent becomes stationary, i.e. reaches a user equilibrium. This class of models is more suited to an agent-based approach than the analytical ones because of their very first assumption of focusing on agents' mobility behaviour rather than optimising a complex objective function. Nevertheless, due to their iterative nature they are also endowed with computational issues. Indeed if the road network and the number of agents involved are large, the DTA algorithms of this type may converge slowly to an equilibrium state Pan et al. (2012).

We can observe that both categories of DTA methods for steady-state solutions are not suited to temporal networks as the agents' lacks of real-time response to network modifications. For instance if an accident occurs at some point of an agent's trip, if the number of agents in the network changes, or if the network is modified by adding/removing streets, the whole optimisation/iterative stages must be repeated to compute a new equilibrium. Moreover these steady states approaches rely on strong assumptions and have several limitations now well identified. We refer the reader to Dehoux and Toint (1991) for a discussion of these limitations and why these models should be avoided in favour of purely behavioural models such as the ones proposed in PACSIM Cornélis and Toint (1998), FREESIM Rathi and Nemeth (1986) and CARSIM Benekohal and Treiterer (1988). The interested reader may find a recent review of these schemes in Pan et al. (2012).

The aim of this work is to present an original behavioural DTA model which is particularly appropriate in the context of agent-based micro-simulation. The proposal relies on the assumption that travellers take routing policies rather than paths Gao et al. (2010), leading us to introduce the possibility for each simulated agent to apply a strategy allowing it to possibly re-route his path depending on perceived local traffic conditions. This re-routing process allows the agents to directly react to any change in the road network, which removes the need of restarting the whole simulation process and consequently decreases the computational cost. For the sake of simplicity, we decide to model the agents strategy with a simple neural network whose parameters are determined during a preliminary learning stage. Of course more complex structures can be considered.

The paper is organised as follows. Section 2 formally details the design of the agents' strategies and their optimization process. The resulting mobility behaviour is then illustrated under various scenarios, testing the robustness of the strategies, in Section 3. Finally concluding remarks and perspectives are discussed in Section 4.

2. Methodology

2.1. A neural-network based strategy

Once each agent has received his own source and destination, he initially computes the shortest-path to perform is trip by assuming he can travel at free-flow speed, that is very diluted traffic conditions. This hypothesis can be relaxed by assuming each agent endowed with some baseline path, not necessarily the shortest - in time - one.

We represent each agent as a neural network whose inputs are the local information about the route network and whose output is the action to undertake: stay on the same path or modify it (keeping unchanged the destination). The take things simple, we assume the design of the neural network presented in Figure 1. The input nodes x_1 and x_2 respectively reads

- the *normalised*[1] time spent from the source up to the current position;
- and *normalised*[2] number of cars in the next link on the path.

The binary output node y_{out} gives 1 if the agent strategy is to change its path, or 0 otherwise. Thus the output is given by

$$y_{out} = \Theta(cos(\alpha)x_1 + sin(\alpha)x_2 - \theta) \quad (1)$$

where Θ is the Heaviside function, $\cos(\alpha)$ and $\sin(\alpha)$ are synapses weights and θ the threshold of the output node. If the agent chooses the re-routing, then he computes a new shortest-path avoiding the congested link between his current location and his destination.

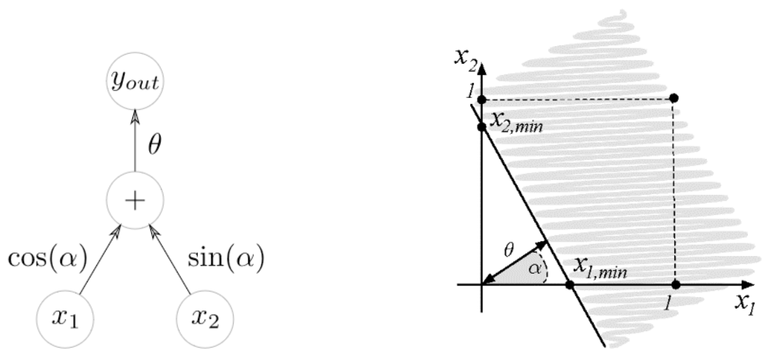

Fig. 1. The neural network design for strategic agents. Left panel: schematic representation of the neural network, the input layer consists of nodes x_1 and x_2 which are respectively weighted by $\cos(\alpha)$ and $\sin(\alpha)$. If their weighted combination exceeds a threshold θ then output node is activated and $y_{out} = 1$; otherwise $y_{out} = 0$. Right panel: the strategy, depending of the parameters α and θ, each agent, measuring x_1 and x_2, decides to re-route or not his path. The dashed area corresponds to values of number of cars and time already spent for which the agent will re-route his trip.

More sophisticated neural network could be considered by adding hidden layers or more inputs (see Bonsall, 1992). Nevertheless we focused on a simple strategy in the present work in order to reach a trade-off between simplicity and efficiency of the strategy. Moreover this strategy seems reasonably behaviourally consistent and previous research works (such asWachs, 1967), Ueberschaer, 1971 and Bonsall and May, 1986) highlighted the fact that if an agent

[1] Normalised means divided by the nominal time one should have spent, i.e. in free flow conditions.
[2] Normalised means divided by the link capacity.

already spent a larger amount of time en-route than it should have taken, and if he perceives congestion on the next road he intends to take, then the agent may reconsider a re-routing to avoid it.

2.2. Strategy learning with genetic algorithm

In our context we aim to minimise the time needed to perform a given source-destination trip. Stated differently, the goal is to allow the agent to choose the best links, i.e. the less congested ones on his path taking into account the dynamically varying traffic conditions in order to minimize his loss of time due to congestion.

The neural network has two strategy parameters to be determined, the weight α and the threshold θ. The resulting strategy, i.e. the networks with its parameters, is then associated with a fitness value $\in [0,1]$ that reflects how optimal it is: the higher the fitness, the lower the travel time. Assuming that $(l_1, \ldots, l_d)$ is the sequence of links covered by the agent as to reach his destination, then his fitness is given by

$$f(a_s) = \sum_{i=1}^{d} \frac{T(l_i)}{S(l_i)}, \tag{2}$$

where $T(l_i)$ is the free-flow travel time of the link l_i and $S(l_i)$ the time actually needed to cover this link with a speed that depends on the traffic condition and on the link type.

We decided to solve this optimisation problem using a Genetic Algorithm[3], because of their simplicity and robustness, but other choices could be possible.

In Figure 2 we illustrate a possible fitness function for a particular learning scenario involving a small congested network, computed for fine mesh of values for the parameters α and θ. We can observe that the objective function presents a large number of local maxima and it is highly oscillating, which further justifies the use of a genetic algorithm to explore the parameters space.

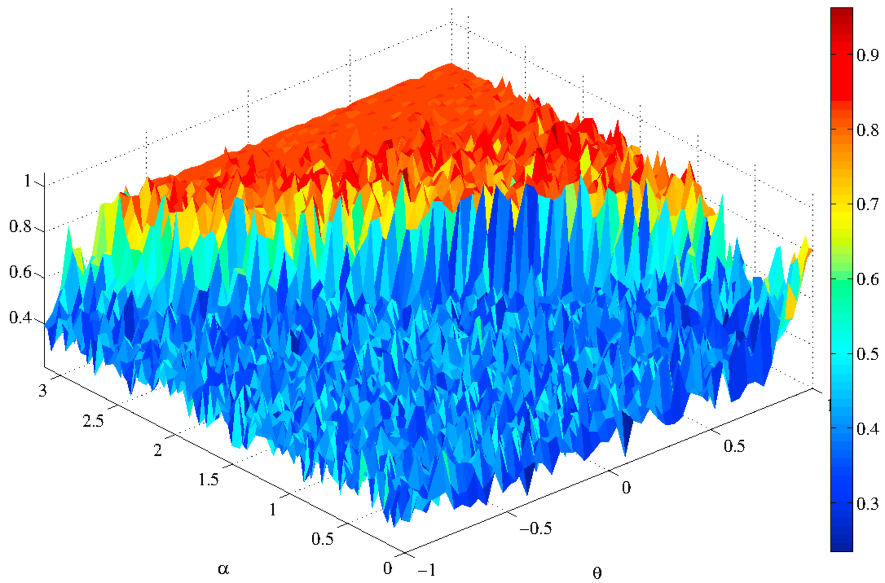

Fig. 2. Fitness value as a function of $\theta \in [-1,1]$ and $\alpha \in [0,\pi]$ for one learning scenario.

[3] We refer the reader to Eiben and Smith (2003) for a detailed overview of this methodology.

2.3. Traffic dynamic

The resulting traffic on the road network is simulated using a *queue* model where the links are modelled as simple *first-in-first-out* (FIFO) queues. This mesoscopic model, also retained by MATSim, is vehicle-based and has been known to offer a satisfactory representation of the travel times as well as excellent computational efficiency (Charypar et al., 2015). Consequently it offers a suitable compromise to more detailed and computationally costly car-following models.

Every link $l \in L$ is a queue characterized by the following properties:

- a free flow travel time l_t;
- a length l_l and number of lanes l_n defining the queue storage capacity;
- a flow capacity l_c, i.e. the maximum number of vehicles per km per hour on the link;
- the number of agents currently in the queue l_v.

The traffic dynamic is then performed by updating the state of each queue at every time step: for each $l \in L$ such that $l_v > 0$, if the following conditions yield for the first agent of the queue

- it has spent at least a duration of l_t on the link;
- l_c has not been exceeded during the current time step;
- and the next link on its route has free storage capacity

then the agent is removed from l and put in the next link of its route. It must be noted that the time step retained in this work is one second.

This approach insure that the specifications of the network are accounted for as the flow capacity and the storage capacity constraints may cause congestion on the links (which can spill-back). Additionally it can easily be extended to capture congestion shock waves using for instance Newell's simplified model for kinematic waves (Newell, 1993; Zhou et al., 2014).

3. Results

The goal of this Section is to present some results concerning the robustness of the optimized strategies with respect to new environments and with respect to the impact of the proportion of strategic agents present in the population. A comparison with a classical user equilibrium approach is then conducted. Finally we examine the performance of the strategic agents against a well-known and validated agent-based traffic micro-simulator.

3.1. Impact of the strategic agents proportion

We firstly conducted experiments over three different scenarios, whose details are given in Table 1 and illustrated in Figure 3, with different proportions of strategic agents to test their adaptability to new conditions:

1. an artificial network designed to represent 2 urban centres linked by 3 roads possibly responsible of bottlenecks in transfers among centres. This network was also used during the learning stage.
2. a second artificial network consisting of 3 urban centres surrounded by main roads and joined by highways. This case somehow generalise the previous one by adding high, medium and low capacity routes.
3. the Chicago road network, available at http://www.bgu.ac.il/~bargera/tntp/. Note that the original capacities of the Chicago network have been downscaled to obtain congestion with less agents in order to keep reasonable computation times.

In order to compare the DTA resulting from our model we repeated each simulation twice on each scenario: a first time with agents trained on the 2 cities network and a second one with random strategic agents (i.e. with strategy whose weights are randomly drawn).

Table 1. Scenarios characteristics.

	Scenario		
Parameter	2 cities	3 cities	Chicago
nodes	59	99	933
slow links (50 km/h)	182	264	354
medium links (90 km/h)	0	54	2,532
fast links (120 km/h)	0	6	64
total length (km)	196	504	12,190
network total capacity (agent/hour)	390	5,820	46,718
mean capacity per km (agent/hour)	1.9	11.5	5.7
number of agents	100	750	4,000

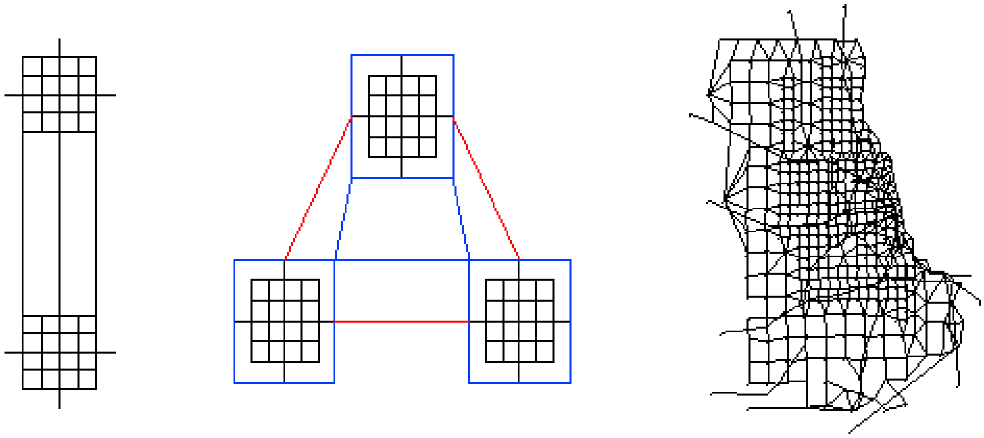

Fig. 3. Networks. Left panel: 2 cities. Centre panel: 3 cities (slows, medium and fast links are respectively black, blue and red). Right panel: Chicago.

Let us now examine how the proportion of strategic agents in the population impacts the average fitness of every agents. In the experiments we considered the following proportions $p \in \{0\%, 10\%, 25\%, 50\%, 75\%, 100\%\}$ of strategic agents. The evolution of the fitness, for each scenario, is reported in Figure 4. One can observe that the proportion of strategic agents (both optimised and random) does have an impact on the agents' average fitness, more precisely:

- for every tested scenario there is an increase of the average fitness for proportions of optimised agents up to 50% of the total number of agents, indicating that the strategy is efficient. Moreover a proportion from 75% up to 100% of such agents has a more relevant and strong impact in the scenario involving the most congested road networks, i.e. with lower mean capacities (*2 cities* and *Chicago*). For less congested network (3 cities), in case the number of optimised agents is high, the overall performance may not increase and even drop. Indeed when they perform re-routing, they may encounter links being more congested than the initial ones;
- agents performing random re-routing experience a decrease of the overall performance as their proportion increases with the exception of the *3 cities* scenario. This behaviour is certainly imputable to the uncongested nature of the network;
- additionally, the trained agents perform better than the random agents in term of average fitness for a majority of the conducted experiments, demonstrating that the learning process is necessary and produces efficient strategies.

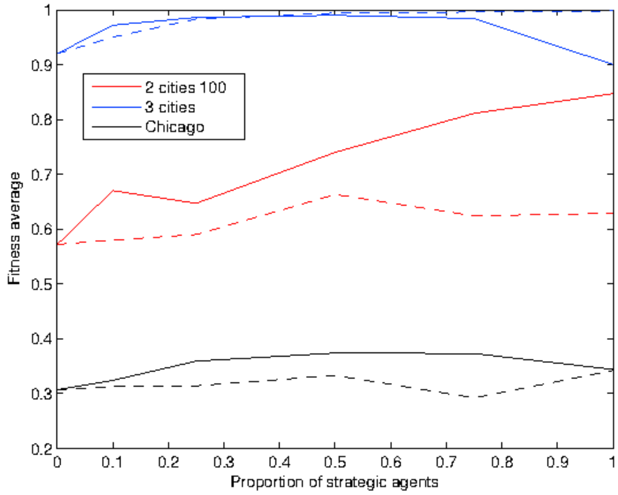

Fig. 4. Evolution of the agent average fitness with respect to the proportion of strategic agents in various scenarios. The average fitness is computed at the end of each simulation. The solid lines represent the agents provided with a strategy optimised while the dashed lines correspond to agents with random strategies.

These observations show that the provided strategy, optimised with a preliminary learning process, is effective compared to random behaviours and even with respect to agents without any strategy, i.e. performing the shortest path trip. Moreover these findings hold even if the learning phase has been performed on a small network compared to the simulated network.

3.2. Comparison with an user-equilibrium approach

In order to assess the validity of the proposed approach, we compare our results with the ones generated by the Origin-Based Assignment (OBA) algorithm developed by Bar-Gera (2002). This algorithm determines the classical deterministic user equilibrium as defined by Wardrop (1952). The retained network for these experiments is the well-known Sioux-Falls network[4] represented in Figure 5.

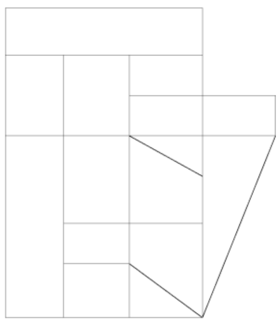

Fig. 5. Sioux-Falls road network: 24 nodes, 76 links (60 km/h) and 360600 trips.

[4] Available at http://www.bgu.ac.il/~bargera/tntp/

Figure 6 shows the impact of the proportion of strategic agents on various indicators on the Sioux-Falls network. We can observe that

- as the proportion of strategic agents increases, the average fitness, the ratio of used streets (i.e. the average load of the network) and the agents average speed slightly improve;
- the number of time steps required for every agents to reach his destination also decreases as the proportion of strategic agent increase up to 75%.

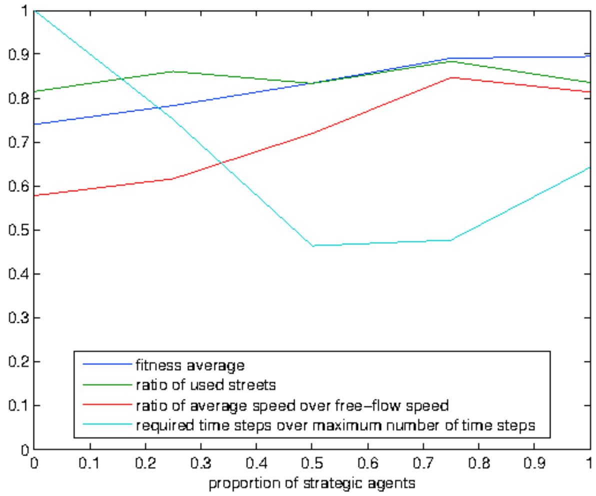

Fig. 6. Evolution of some performance indicators with respect to the proportion of strategic agents in the Sioux-Falls network.

The comparison is performed by computing the average and maximum absolute deviations between the traffic flows produced by our approach and the ones determined by the OBA algorithm. These deviations, respectively denoted by D_a and D_m, are formally defined by

$$D_a = \sum_{k \in L} \frac{|v_l - v_l^*|}{l_{tot}} \quad \text{and} \quad D_m = \max_{k \in L} |v_l - v_l^*| \tag{2}$$

where L is the set containing the l_{tot} network links, v_l the number of agents going through $l \in L$ during the simulation and v_l^* the traffic flow computed by the OBA algorithm for the same link.

Figure 7 illustrates the behaviour of D_a and D_m as a function of the proportion of strategic agents. Indeed for a proportion of 75%, D_a is less than 0.5% while D_m is about 1.6%, showing that the solution of our approach produces results close to the theoretical ones. Hence the strategy computed with the genetic algorithm is close to the optimal one.

3.3. Comparison with MATSim

MATSim is an agent-based traffic micro-simulator which has been used in several applications all around the world (see http://www.matsim.org/scenarios for a listing of different scenarios). The simulator initially assigns to every agent a plan for each of his trip, i.e. a pre-defined path, a time to leave and a desired arrival time. Then MATSim tries to reach a user equilibrium by repeating K times the following iterative process:

1. each agent carries out his plans (simulation step);
2. after the simulation step, MATSim assesses the performance of every agent;
3. MATSim then modify the plans of the agents with the worst performance.

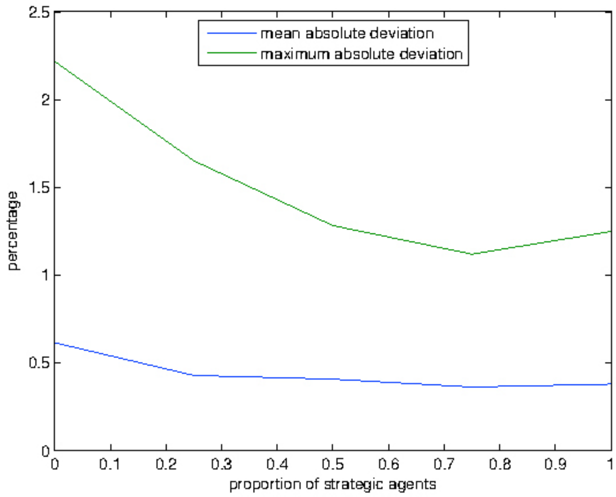

Fig. 7. Average and maximum absolute deviations (in percent) between theoretical flows computed by the Origin-based assignment algorithm and the proposed approach as a function of the proportion of strategic agents.

In this Section we compares the results of our strategic agents-based simulator (using 25% of strategic agents) against the ones generated by MATSim for Namur, a city in the Walloon Region of Belgium (after 10 iteration, only allowing the modification of the pre-defined paths for 10% of the agents). The simulation involved 100,000 agents performing a total of 439,500 trips over a typical weekday in a road network made of 23,000 nodes and 36,700 links. The travel demand generation is fully detailed in Barthelemy (2014) and Barthelemy and Toint (2015). A snapshot of the simulated traffic at 8:00 am is illustrated in Figure 9.

A comparison of the simulated daily counts on each link by the two approach is shown in Figure 9. One can easily observe the strong positive correlation (0.905) between the counts, indicating that our methodology and MATSim produce similar traffic patterns.

Fig. 8. Saturation of the Namur road network at 8:00 am - 100,000 agents, 360,000 trips, road network made of 25,000 nodes and 17,000 links.

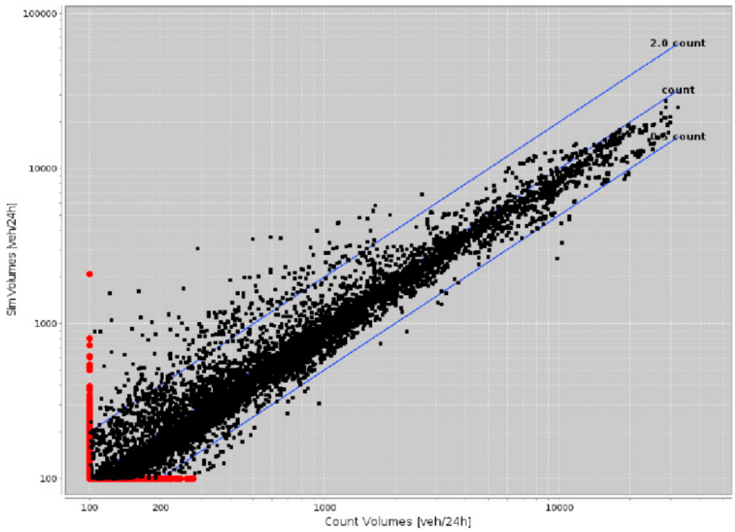

Fig. 9. Strategic agents traffic counts (x axis) against MATSim traffic counts (y axis). Each dot represents a link and the daily traffic volume predicted by the proposed model and MATSim. For a perfect match the dots should be aligned on a diagonal line. Pearson correlation coefficient = 0.905.

4. Conclusions

In this work we presented the premise of a novel alternative model to existing simulation-based dynamic traffic assignments models by endowing the travelling agents with a strategy. The proposed strategy is coded using a neural network whose inputs rely on the current trip duration and the perceived traffic conditions. Instead of fine tuning the neural network parameters, we decided to use a training phase using a standard genetic algorithm allowing us to derive the optimal neural network parameters. The trained agents can then use their own strategy to face to changing conditions and new environments.

In the conducted experiments, the strategic agents have demonstrated an efficient behaviour. The robustness and adaptability to new environments have also been demonstrated and thus indicate promising results. A key advantage of this approach is that it does not require several computationally expensive iterations to take account of network modifications or changing traffic conditions. As a result this model seems well-suited to large-scale applications. This last property will be investigated in further works, as well as testing different neural networks formulations.

Acknowledgements

The authors would like to thank Eric Cornelis, Guillaume Deffuant and Philippe Toint for their insightful comments and their suggestions. Helpful corrections from Nam Huynh, Laurie Hollaert, Jonathan Dehaye and MarieMoriamé are also gratefully acknowledged. Computational resources have been provided by the Consortium des équipements de Calcul Intensif (CCI), funded by the Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under Grant No. 2.5020.11. Finally we gratefully acknowledge the support of NVIDIA Corporation with the donation of the TITAN X used for this research.

References

Bar-Gera, H., 2002. Origin-based algorithm for the traffic assignment problem. Transportation Science 36, 398–417.

Barcel ´o, J., Casas, J., 2005. Dynamic network simulation with AIMSUN, in: Simulation Approaches in Transportation Analysis. Springer, pp. 57–98.

Barthelemy, J., Toint, P., 2015. A stochastic and flexible activity based model for large population. Application to Belgium. Journal of Artificial Societies and Social Simulation 18, 15. URL: http://jasss.soc.surrey.ac.uk/18/3/15.html.
Barthélemy, J., 2014. A parallelized micro-simulation platform for population and mobility behaviour-Application to Belgium. Ph.D. thesis. University of Namur.
Bazghandi, A., 2012. Techniques, advantages and problems of agent based modeling for traffic simulation. International Journal of Computer Science 9.
Ben-Akiva, M., Bierlaire, M., Koutsopoulos, H., Mishalani, R., 1998. Dynamit: a simulation-based system for traffic prediction, in: DACCORS Short Term Forecasting Workshop, The Netherlands, Citeseer.
Ben-Akiva, M.E., Gao, S., Wei, Z., Weng, Y., 2012. A dynamic traffic assignment model for highly congested urban networks. Transportation Research Part C: Emerging Technologies 24, 62–82.
Benekohal, R.F., Treiterer, J., 1988. Carsim: Car-following model for simulation of traffic in normal and stop-and-go conditions. Transportation Research Record 1194.
Bonsall, P., 1992. The influence of route guidance advice on route choice in urban networks. Transportation 19, 1–23.
Bonsall, P., May, A.D., 1986. Route choice in congested urban networks, in: Research for Tomorrow's Transport Requirements. Proceedings of the Fourth World Conference on Transport Research.
Charypar, D., Axhausen, K., Nagel, K., 2015. Event-driven queue-based traffic flow microsimulation. Transportation Research Record: Journal of the Transportation Research Board .
Chiu, Y., Bottom, J., Mahut, M., Paz, A., Balakrishna, R., Waller, T., Hicks, J., 2011. Dynamic traffic assignment: A primer. Transportation Research E-Circular E-C153.
Cornélis, E., Toint, P.L., 1998. Pacsim: a new dynamic behavioural model for multimodal traffic assignment, in: Labbé, M., Laporte, G., Tanczos, K., Toint, P.L. (Eds.), Operations Research and Decision Aid Methodologies in Traffic and Transportation Management, Springer Verlag, Heidelberg, Berlin, New York. pp. 28–45.
Dehoux, P., Toint, P., 1991. Some comments on dynamic modelling, in the presence of advanced driver information systems, in: Argyrakos, G., Carrara, M., Cartsen, O., Davies, P., Mohlenbrink, W., Papageorgiou, M., Rothengatter, T., Toint, P.L. (Eds.), Advanced Telematics in Road Transport, Commission of the European Communities - DG XIII. pp. 964–981.
Eiben, A., Smith, J.E., 2003. Introduction to Evolutionary Computing. Springer.
Friesz, T.L., Bernstein, D., Smith, T.E., Tobin, R.L.,W.Wie, B., 1993. A variational inequality formulation of the dynamic network user equilibrium problem. Operations Research 41, 179–191.
Gao, S., Frejinger, E., Ben-Akiva, M., 2010. Adaptive route choices in risky traffic networks: A prospect theory approach. Transportation Research part C: Emerging Technologies 18, 727–740.
Meister, K., Balmer, M., Ciari, F., Horni, A., Rieser, M.,Waraich, R.A., Axhausen, K.W., 2010. Large-scale agent-based travel demand optimization applied to switzerland, including mode choice. paper presented at the 12th World Conference on Transportation Research.
Merchant, D.K., Nemhauser, G.L., 1978a. A model and an algorithm for the dynamic traffic assignment problems. Transportation Science 12, 183–199.
Merchant, D.K., Nemhauser, G.L., 1978b. Optimality conditions for a dynamic traffic assignment model. Transportation Science 12, 200–207.
Nagel, K., Fltteröd, G., 2009. Agent-based traffic assignment: going from trips to behavioral travelers, in: 12th International Conference on Travel Behaviour Research (IATBR), Jaipur.
Newell, G.F., 1993. A simplified theory of kinematic waves in highway traffic, part i: General theory. Transportation Research Part B: Methodological 27, 281–287.
Pan, J., Khan, M.A., Popa, I.S., Zeitouni, K., Borcea, C., 2012. Proactive vehicle re-routing strategies for congestion avoidance, in: Distributed Computing in Sensor Systems (DCOSS), 2012 IEEE 8th International Conference on, IEEE. pp. 265–272.
Peeta, S., Ziliaskopoulos, A.K., 2001. Foundations of dynamic traffic assignment: The past, the present and the future. Networks and Spatial Economics 1, 233–265.
Rathi, A.K., Nemeth, Z.A., 1986. Freesim: A microscopic simulation model of freeway lane closures (abridgment). Transportation Research Record 1091.
Salvini, P., Miller, E.J., 2005. Ilute: An operational prototype of a comprehensive microsimulation model of urban systems. Networks and Spatial Economics 5, 217–234. URL: http://dx.doi.org/10.1007/s11067-005-2630-5, doi:10.1007/s11067-005-2630-5.
Ueberschaer, M.H., 1971. Choice of routes on urban networks for the journey to work. Highway Research Record 369.
Wachs, M., 1967. Relationships between drivers'attitudes toward alternate routes and driver and route characteristics. Highway Research Record 197, 70–87.
Waddell, P., 2002. Urbansim: Modeling Urban Development for Land Use, Transportation and Environmental Planning. Journal of the American Planning Association 3, 297–314.
Wardrop, J., 1952. Some theoretical aspects of road traffic research. Proceedings of the Institute of Civil Engineers, part II 1, 325–378.
Zhou, X., Taylor, J., Pratico, F., 2014. Dtalite: A queue-based mesoscopic traffic simulator for fast model evaluation and calibration. Cogent Engineering 1, 961345.