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# Detection of Laplace-resonant three-planet systems from transit timing variations

A.-S. Libert<sup>1,2,3★</sup> and S. Renner<sup>2,3</sup>

<sup>1</sup>*NaXys, Department of Mathematics, University of Namur, 8 Rempart de la Vierge, B-5000 Namur, Belgium*

<sup>2</sup>*Laboratoire d'Astronomie de Lille, Université Lille 1, 1 impasse de l'Observatoire, F-59000 Lille, France*

<sup>3</sup>*Institut de Mécanique Céleste et de Calcul des Ephémérides, UMR 8028 du CNRS, 77 avenue Denfert-Rochereau, F-75014 Paris, France*

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## ABSTRACT

Transit timing variations (TTVs) are useful to constrain the existence of perturbing planets, especially in resonant systems where the variations are strongly enhanced. Here we focus on Laplace-resonant three-planet systems, and assume that the inner planet transits the star. A dynamical study is performed for different masses of the three bodies, with special attention to terrestrial planets. We consider a maximal time-span of  $\sim 100$  yr and discuss the shape of the inner planet TTVs curve. Using frequency analysis, we highlight the three periods related to the evolution of the system: two periods associated with the Laplace-resonant angle and the third one with the precession of the pericentres. These three periods are clearly detected in the TTVs of an inner giant planet perturbed by two terrestrial companions. Only two periods are detected for a Jupiter–Jupiter–Earth configuration (the ones associated with the giant interactions) or for three terrestrial planets (the Laplace periods). However, the latter system can be constrained from the inner planet TTVs. We finally remark that the TTVs of resonant three or two Jupiter systems mix up, when the period of the Laplace-resonant angle matches the pericentre precession of the two-body configuration. This study highlights the importance of TTVs long-term observational programmes for the detection of multiple-planet resonant systems.

**Key words:** methods: observational – techniques: photometric – celestial mechanics – planetary systems.

## 1 INTRODUCTION

More than 100 multiplanetary systems have been detected.<sup>1</sup> At present, at least three extrasolar systems are believed to be in a multiply resonant configuration, similar to the Laplace resonance of the Jupiter's Galilean moons: HR 8799 (e.g. Reidemeister et al. 2009) and Gliese 876 (Rivera et al. 2010) in a Laplace 4:2:1 configuration, and the KOI-500 five-body exosystem that presents two three-body resonances (Lissauer et al. 2011).

In formation theories, it has been shown that, at least in the case of low-mass planets such as the ones in our Solar system, gas-driven migration can force the planets to enter into a multiple-planet resonance (Morbidelli et al. 2007). Recently, Libert & Tsiganis (2011a,b) have shown that a Laplace three-planet resonance can be achieved for Jupiter-mass planets and, provided that enough eccentricity damping is exerted on the planets, such a three-planet system could be stable after the dissipation of the gas disc.

Multiple-planet resonances can act as a phase-protection mechanism, especially for systems consisting of a lot of massive bodies in a particularly compact configuration. In their study of Gliese 876, Rivera et al. (2010) have shown that the Laplace resonance is essential for the long-term stability of the system. A lot of KOIs also show evidence of their proximity to multiple-planet commensurabilities (Lissauer et al. 2011), but it needs confirmation since the orbital parameters of the planets are still unknown.

The transit timing variations (TTVs) method is a powerful technique to infer the existence of additional non-transiting planets (Miralda-Escudé 2002; Agol et al. 2005; Holman & Murray 2005). Indeed, a companion (not necessarily transiting) planet interacts gravitationally with the transiting planet and generates deviations of the transit times with respect to the strictly periodic Keplerian case. This is especially the case for resonant systems where the variations are strongly enhanced. The TTVs technique has been applied using ground-based data (e.g. Steffen & Agol 2005; Collier et al. 2007), *MOST* data (e.g. Miller-Ricci et al. 2008), *Hubble Space Telescope* data (e.g. Agol & Steffen 2007) and recently the space missions *CoRoT* (e.g. Bean 2009; Csizmadia et al. 2010) and *Kepler* (e.g. Borucki et al. 2010, 2011). However, deriving the orbital

★E-mail: anne-sophie.libert@fundp.ac.be

<sup>1</sup> See the updated data base exoplanet.eu (Schneider et al. 2011).

elements and the mass of the perturber from TTVs is a difficult inverse problem (e.g. Nesvorný & Morbidelli 2008), and different configurations can generate similar TTVs with the same dominant period (e.g. Ford & Howell 2007; Boué et al. 2012). Despite these difficulties, Nesvorný et al. (2012) have recently achieved the first detection and characterization of a non-transiting planet by TTVs for the KOI-872 system. The TTVs of other *Kepler* confirmed/candidate systems are currently under investigation (Ford et al. 2011, 2012; Fabrycky et al. 2012; Steffen et al. 2012). The TTVs method is also promising for detecting Trojan planets (e.g. Ford & Howell 2007) and moons of transiting planets (e.g. Kipping 2009).

Here we propose an initial qualitative exploration of the theoretical TTVs of a three-planet multiply resonant system, assuming that only the inner body transits the star. We analyse the shape of its TTVs curve in a maximal time-span of  $\sim 100$  yr, in order to identify relevant features of the resonant evolution of the whole system. This allows us to define a simple criterion to distinguish three-body resonant systems from two-body ones. We only consider here the Laplace 4:2:1 resonance, varying the mass values of the three planets, but our results could be transposed to another three-planet resonance.

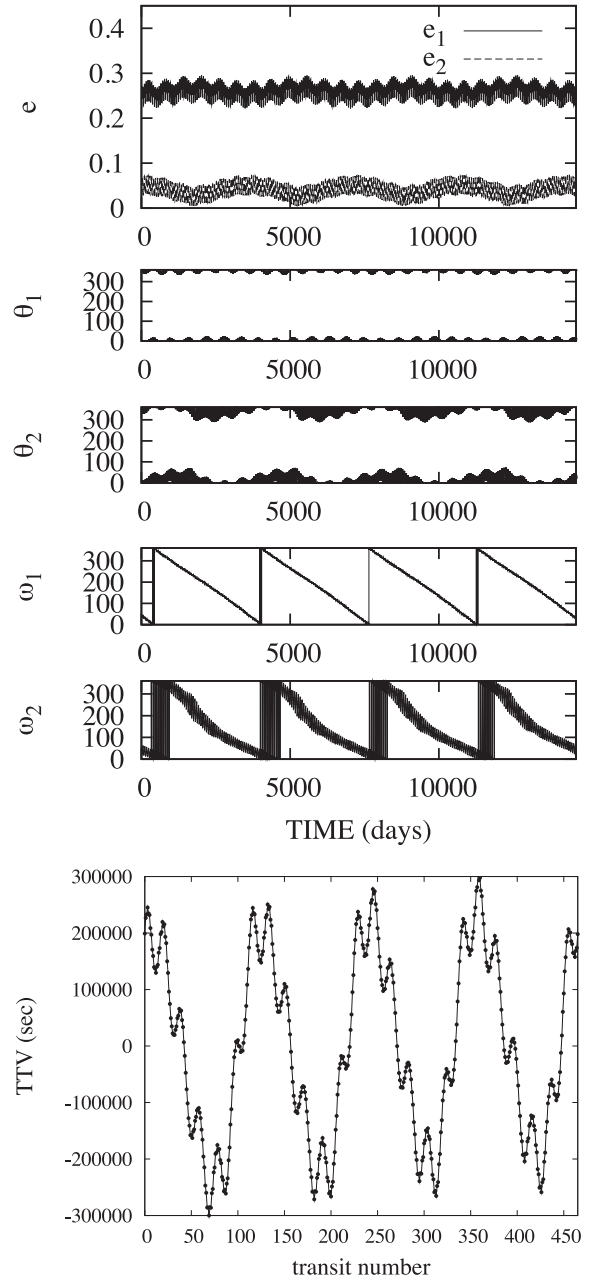
The paper is organized as follows. In Section 2, we analyse the TTVs signal of a Laplace-resonant system composed of three giant planets, identifying the periods with the time variation of the orbital elements. Section 3 focuses on the possible detection of terrestrial planets in multiply resonant systems. A discussion is given in Section 4, while our results are summarized in Section 5.

## 2 TTVs SIGNAL OF A LAPLACE-RESONANT SYSTEM

We assume in this work that the inner planet of the system transits the star, such that our analysis relies on the TTVs curve of this planet only. We first recall the features of the TTVs signal of a two-planet resonant system.

An example of a two-planet resonant dynamics is shown in Fig. 1, where the 2:1 mean-motion resonant evolution of the two-planet Gliese 876 system is given for 40 years [parameters of Rivera et al. (2010)]. The orbital periods of the two planets are 30.1 and 61.1 d, respectively. As stated by Holman & Murray (2005) and Agol et al. (2005), the TTVs curve of the inner planet in Fig. 1 (bottom panel) shows two periods: a short one associated with the resonant angles ( $\theta_i = \lambda_1 - 2\lambda_2 + \omega_i$ ,  $i = 1, 2$ ,  $\lambda$  being the mean longitude and  $\omega$  the argument of the pericentre), while the longer one corresponds to the precession rate of the arguments of the pericentres. Their values are given in Table 1 (top) where, by resorting to frequency analysis (Laskar 1993), we compute the main periods of the TTVs and the orbital elements. They are listed by decreasing amplitude of the trigonometric term: the coefficient  $c_1$  represents the highest amplitude, and bold type  $c_1$  indicates the precession rate of an angular variable in circulation. Let us note that only the period associated with the trigonometric term of highest amplitude is indicated for  $\theta_i$  and  $\omega$  (the other period and/or a period combination are also present, as can be seen in Fig. 1).

Let us perform a similar study for a system of three planets in a Laplace resonance. We choose to analyse one of the Laplace-resonant coplanar systems formed by migration in a gaseous disc in the work of Libert & Tsiganis (2011b). It consists of three planets with equal masses of  $1.5M_J$  ( $M_J$  = Jupiter's mass) and orbital periods of 20.1, 40.5 and 81.2 d, respectively (see Fig. 2). The TTVs of the inner planet are shown in Fig. 3 and exhibit three periods now.



**Figure 1.** Dynamical evolution of the two-planet resonant Gliese 876 system for 40 years. The parameters of planets b and c are taken from table 2 of Rivera et al. (2010). Masses of the planets are 0.72 and  $2.27 M_J$ , and the orbital periods are 30.1 and 61.1 d, respectively. From top to bottom: eccentricities, resonant angles ( $\theta_i = \lambda_1 - 2\lambda_2 + \omega_i$ ,  $i = 1, 2$ ), pericentre arguments and TTVs of the inner planet.

Results of the frequency analysis are given in Table 1 (bottom). As in the two-planet resonant case, the long period of the TTVs signal corresponds to the precession rate of the arguments of the pericentres. The other two periods are related to  $\theta_1$  and  $\theta_2$ , the libration angles of the 2:1 mean-motion resonance between the two inner planets, and  $\theta_3$  and  $\theta_4$ , the libration angles of the 2:1 mean-motion resonance between the two outer planets. These two periods are both easily detected in the time evolution of the Laplace-resonant argument  $\phi = \lambda_1 - 3\lambda_2 + 2\lambda_3$  (see Fig. 2, top right).

The three periods in the TTVs signal are a signature of the multiple resonance, it is not due to the fact that the system consists of

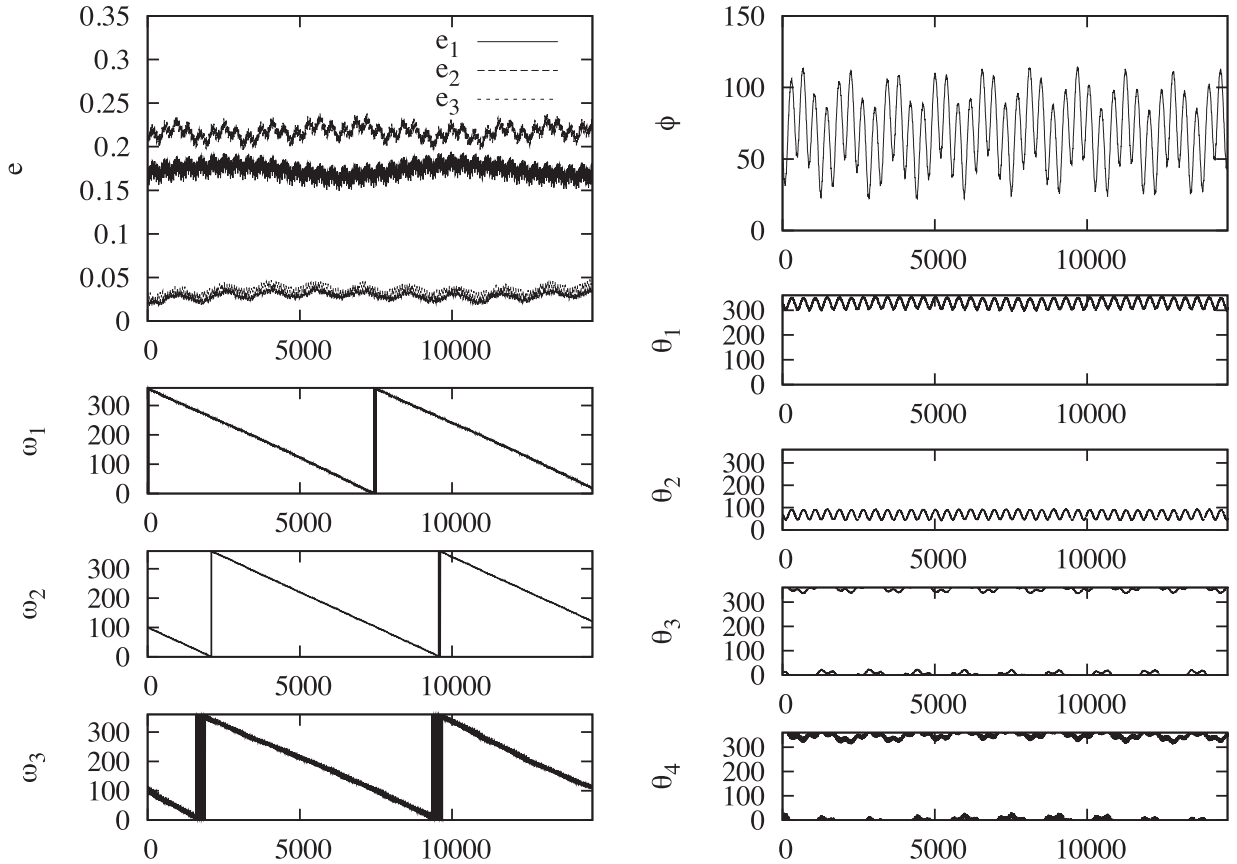
**Table 1.** Frequency analysis of the TTVs curve and the time variations of the resonant angles ( $\theta_i$ ), Laplace angle ( $\phi$ ) and arguments of the pericentre ( $\omega$ ). Top: the resonant two-planet Gliese 876 system; see Fig. 1. Bottom: the Laplace-resonant three-planet system; see Fig. 2. The main periods are listed by decreasing amplitude of the trigonometric term ( $c_1$  is the highest amplitude).

Periods (d)	TTVs	$\theta_1$	$\theta_2$	$\omega$			
3559	$c_1$		$c_1$	$\mathbf{c_1}$			
589	$c_2$	$c_1$					
Periods (d)	TTVs	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\phi$	$\omega$
7507	$c_1$						$\mathbf{c_1}$
391	$c_2$	$c_1$	$c_1$			$c_1$	
1522	$c_3$			$c_1$	$c_1$	$c_2$	

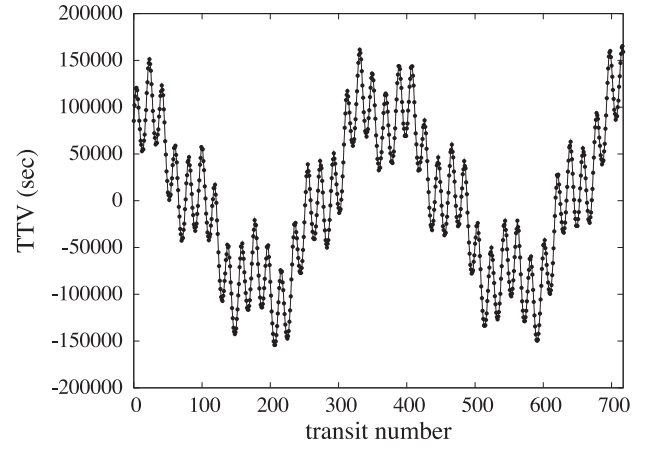
three planets. To illustrate this, we compare in Fig. 4 the TTVs of the same three-body system as before and another where the outer planet has been removed from the Laplace configuration.

### 3 ON THE POSSIBLE DETECTION OF TERRESTRIAL PLANETS

We now investigate if the three-period signature described in the previous section survives when considering one or more Earth-like planets in the Laplace resonance. Three different configurations are examined, depending on the number of terrestrial planets in the system.



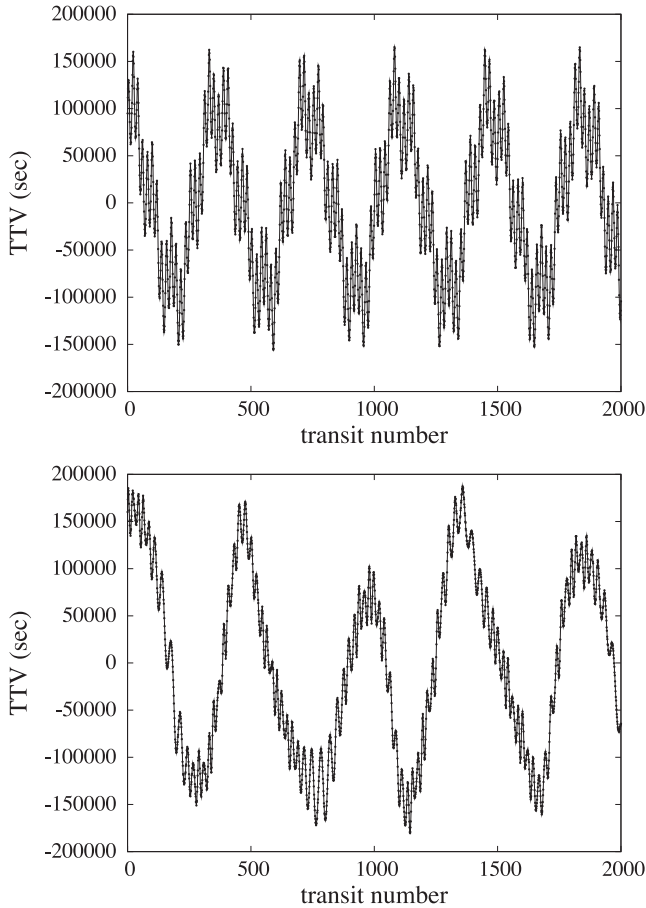
**Figure 2.** Dynamical evolution (for 40 years) of a Laplace-resonant three-planet system consisting of three giant planets of  $1.5M_J$  with orbital periods of 20.1, 40.5 and 81.2 d, respectively. The parameters are issued from the work of Libert & Tsiganis (2011b). Time is given in days.



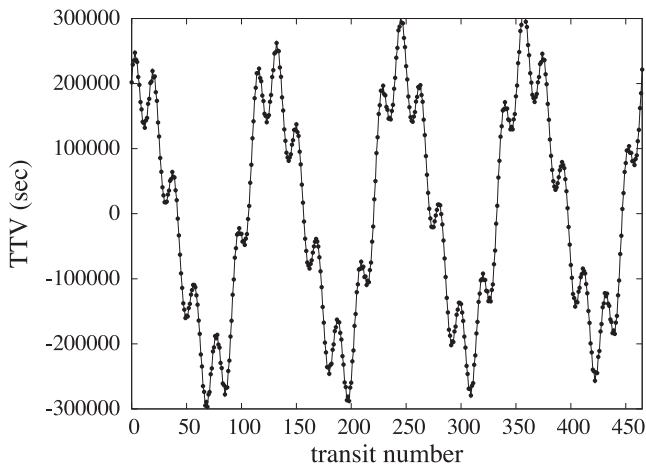
**Figure 3.** TTVs of the inner planet of the Laplace-resonant three-planet system of Fig. 2. The time interval is 40 yr.

#### 3.1 One terrestrial and two giant planets

This case can be understood by studying Gliese 876 (Rivera et al. 2010). It consists of two Jupiter-like planets ( $0.7M_J$  and  $2.3M_J$ ) and a Uranus-mass body ( $14.6M_E$ ,  $M_E$  = Earth's mass) in the Laplace resonance with orbital periods of 30.1, 60.1 and 124.3 d, respectively. The TTVs of the inner planet are given in Fig. 5. Only two periods are detected in the curve, despite the Laplace configuration. This is confirmed with the frequency analysis. The same result holds for a Jupiter–Jupiter–Earth system.



**Figure 4.** Comparison of the TTVs of the inner planet for the system given in Fig. 2 (top panel) and for a modified system where the outer planet has been removed from the Laplace resonance (bottom panel).



**Figure 5.** TTVs of the inner planet of the Laplace-resonant three-planet Gliese 876 system. The parameters are issued from table 3 of Rivera et al. (2010).

Therefore, we conclude that the three-period signature cannot be detected for a Jupiter–Jupiter–Earth Laplace-resonant configuration. This is a consequence of the weaker gravitational effect of the smallest planet on its giant companions. Comparing Figs 1 and 5, we note that the TTVs curves look very similar. However, the presence of the third planet slightly modifies the period values and

the amplitude of the signal. A deep analysis is needed to investigate if this difference can help to detect the third body. This is beyond the scope of this paper.

### 3.2 Two terrestrial and one giant planets

This case is particularly interesting. We analyse a giant planet of  $1M_J$  in the Laplace resonance with two super-Earths of  $10M_E$ , with orbital periods of 13.1, 26.2 and 52.5 d, respectively. As in Section 2, the giant TTVs show three periods; see the top-left panel of Fig. 6. The first one is the shortest ( $\sim 300$  d), arising from the 2:1 resonance between the two inner bodies. To identify the two other periods, the time variations of the Laplace-resonant angle  $\phi$  and the pericentre argument  $\omega_2$  are given. We remark that  $\phi$  and the TTVs curve reach extrema at the same time. The period associated with the argument  $\omega_2$  also appears in the TTVs; see the top-right panel of Fig. 6, where we consider, for comparison, the reduced system of the two inner planets only (in the 2:1 resonance).

As a result, the existence of two super-Earth companions of a giant planet in a Laplace-resonant configuration can be inferred from the TTVs of the giant. The amplitude of the TTVs signal is high ( $\sim 2.2$  h), and the time necessary to identify the three periods using frequency analysis (criterion: twice the longer period of the signal) is approximately 2000 transits, equivalent to  $\sim 70$  yr in that case.

Another example is given at the bottom-left panel of Fig. 6, with a system of a hot Jupiter of  $0.5M_J$  ( $P = 5.1$  d) and two terrestrial planets of  $2M_E$  ( $P = 10.2$  and  $20.5$  d, respectively). Since the terrestrial masses are smaller than in the previous case,  $\sim 3000$  transits ( $\sim 40$  yr) are needed to detect the three periods by frequency analysis.

### 3.3 Three terrestrial planets

Finally, let us consider a Laplace-resonant system of three terrestrial planets ( $1.5M_E$ ) with periods of 31.9, 63.8 and 127.5 d, respectively. The TTVs curve of the inner planet is shown in Fig. 7 (left-hand panel). As expected, the amplitude of the TTVs signal is extremely high ( $\sim 3.5$  d), but only two periods are detectable. The long period associated with the argument  $\omega$  is longer than the time-span considered, and thus visible by increasing the mass values up to  $0.5M_J$  (middle and right-hand panels). Indeed, the smaller the masses of the planets, the longer the precession rate of the arguments of the pericentres.

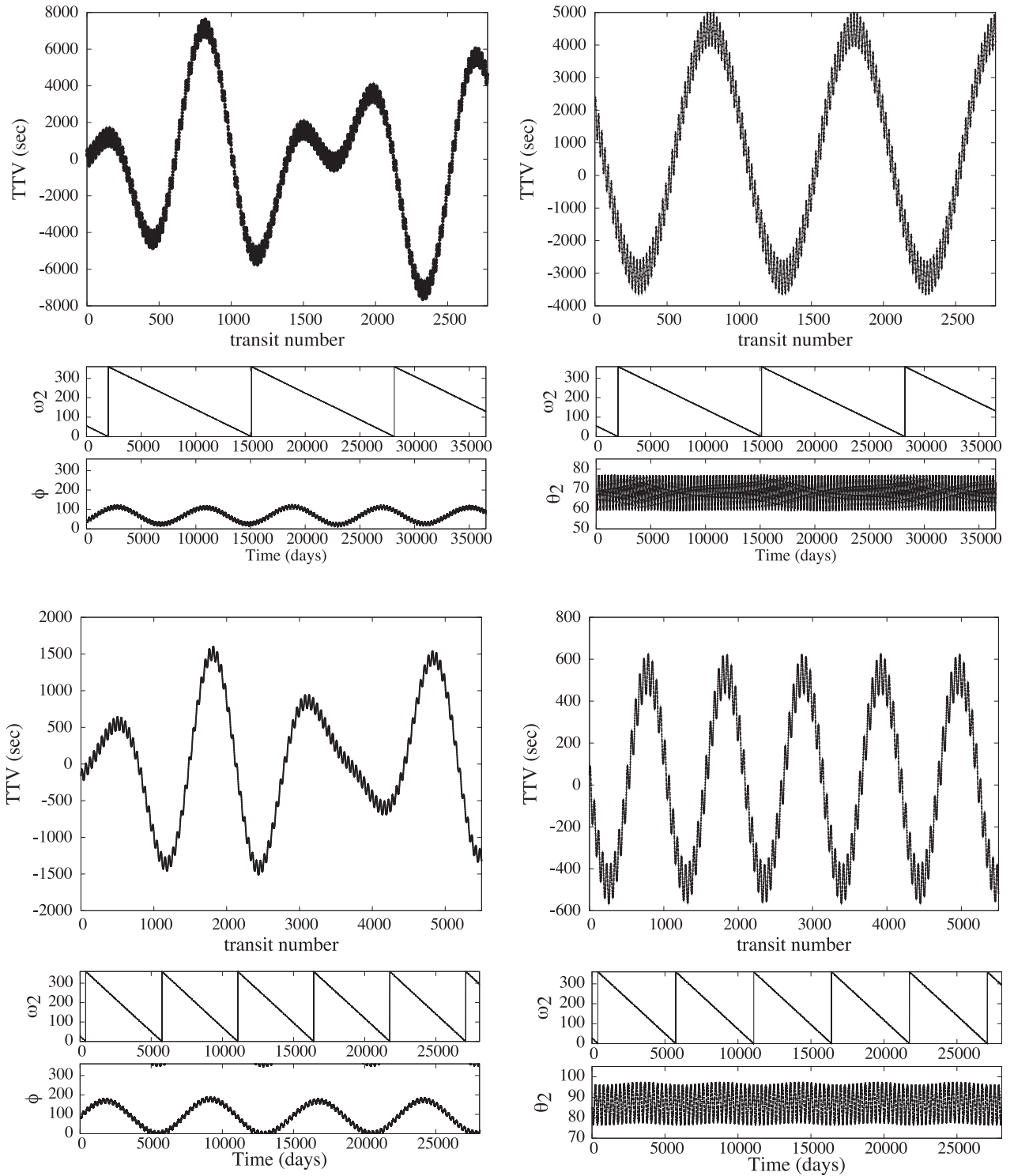
Therefore, for a system of three Earth-like planets in a multiple resonance, the secular period cannot be detected in the TTVs in a time-span of  $\sim 100$  yr. Only the two periods associated with the Laplace angle can be deduced, the longer being, from our example,  $\sim 1500$  transits ( $\sim 120$  yr) for  $1.5M_T$  planets. Nevertheless, the double-frequency TTVs signal is here the signature of a double 2:1 resonance between the three Earth-like bodies, with amplitudes of  $\sim 42$  and  $83$  h. This can be retrieved from two-body resonance calculations (e.g. section 6 of Agol et al. 2005).

## 4 DISCUSSION OF THE RESULTS

### 4.1 Limitations due to observation time-span

We have shown in the previous section that at least several decades are necessary to practically identify Laplace-resonant three-planet systems containing terrestrial bodies. In a given short observation time-span, only two periods can usually be detected in the TTVs of a Laplace-resonant system, thus leading to a curve very similar to

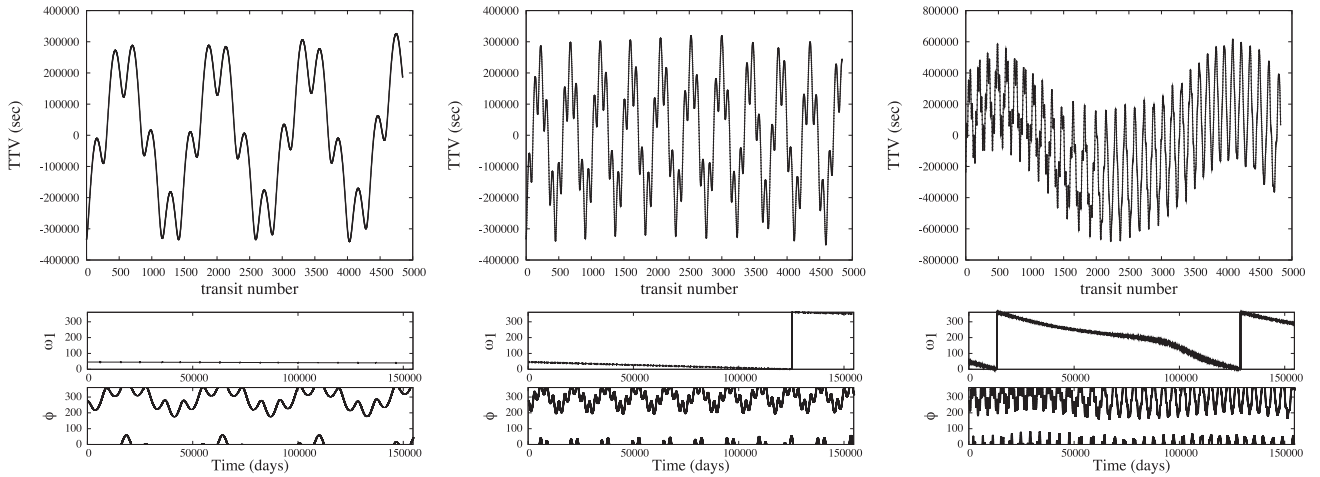




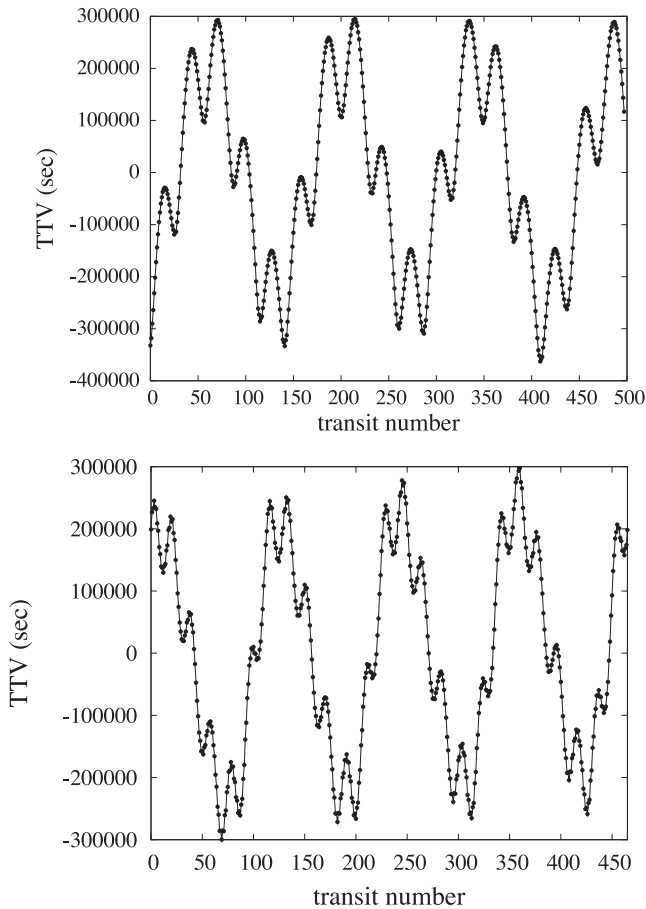
**Figure 6.** Left: TTVs and dynamical evolution of Laplace-resonant systems consisting of a giant planet and two terrestrial planets, in a time interval of  $\sim 100$  yr: a giant of  $1M_J$  with  $P = 13.1$  d + two super-Earths of  $10M_E$  on the top, a hot Jupiter of  $0.5M_J$  with  $P = 5.1$  d + two Earth-like planets of  $2M_E$  at the bottom. Right: comparison with the systems formed by the two inner resonant planets only.

the one of two planets in resonance. This is illustrated in Fig. 8: the long-period resonant argument of a three-Jupiter system matches the pericentre precession of a two-body configuration. This example suggests possible degeneracy between two- and three-planet resonant configurations in short observation time-spans, and this question should be investigated in more detail in the future.

Therefore, long-term TTVs observations are necessary to constrain dynamically multiple-planet resonant systems, especially if no additional data such as radial velocity data are available. Let us note that, even if this work identifies the three periods in the TTVs signal as a signature of a resonant three-body configuration, the same three periods could correspond to different parametrizations



**Figure 7.** TTVs and dynamical evolution for  $\sim 400$  years for a system of three Earth-like planets ( $1.5M_E$ , left), three super-Earth planets ( $15M_E$ , middle) and three Jupiter-like planets ( $1.5M_J$ , right). Periods of the planets are 31.9, 63.8 and 127.5 d, respectively. The same initial orbital elements have been used for the three systems.



**Figure 8.** Double-period TTVs for different system configurations for  $\sim 40$  years. Top panel: the three Jupiter-like planets given in Fig. 7. Bottom panel: the Gliese 876 two-planet system shown in Fig. 1. In each case, the transiting giant planet is  $\sim 0.5M_J$  with an orbital period of  $\sim 30$  d.

of the system, as the degeneracy in the characterization of non-transiting planets pointed out for a two-body resonance (see e.g. Boué et al. 2012). Furthermore, only coplanar systems have been investigated here. The detection of inclined Laplace-resonant sys-

tems should also be addressed, as the possible formation of (highly) inclined two- and three-body resonant systems has recently been demonstrated (e.g. Thommes & Lissauer 2003; Libert & Tsiganis 2009, 2011b).

## 4.2 Kepler systems

At the time of writing, three confirmed *Kepler* systems consist of three planets: Kepler-9 (Holman et al. 2010), Kepler-18 (Cochran et al. 2011) and Kepler-30 (Fabrycky et al. 2012). The TTVs of Kepler-9b and Kepler-9c present the characteristic signature of a 2:1 mean-motion resonance. These two giant planets also have an inner non-resonant super-Earth companion. Kepler-18 exhibits a similar dynamics, with two Neptune-mass planets in a 2:1 mean-motion resonance confirmed by TTVs and an inner non-resonant super-Earth planet. As argued in Section 3.1, such systems could host a terrestrial Laplace-resonant planet, but this would require further investigations. Kepler-30 is an interesting three-planet system with significant TTVs. While the Neptune-size Kepler-30b lies interior to a 2:1 mean-motion resonance with the Jupiter-size Kepler-30c, an additional Jupiter-size planet orbits the star with a period that roughly corresponds to 5 times the one of Kepler-30b and 2.5 times the one of Kepler-30c. The possible proximity to such a three-planet resonance could explain the difficult interpretation of the TTVs on the current short observation time. A longer time-span will help to identify the possible signature of three periods in the signal – if they exist. Let us note that most recent *Kepler* systems (e.g. Kepler-51, Kepler-53 and Kepler-60) show potential multibody resonances (see Steffen et al. 2012). Furthermore, a lot of KOIs are close to multiple-period-ratio commensurabilities. For instance, KOI-500 (Lissauer et al. 2011) is particularly interesting with possibly two three-body resonances. However, a study of the inner TTVs curve only is not sufficient for the dynamics of a five-body system.

## 5 CONCLUSION

We studied the possible detection of Laplace-resonant three-planet systems in a maximal time-span of  $\sim 100$  yr, assuming that only the inner planet transits the star. We have shown that a three-body resonant configuration has a specific TTVs shape: three periods are

present, the long one associated with the precession of the arguments of the pericentres, and the other two with the time evolution of the Laplace-resonant angle. Our criterion for detecting a signal is that the frequency is sampled completely, not that the amplitude of the Fourier component is large.

The three periods are detectable for a giant planet perturbed by two terrestrial bodies. A system of three Earth-like planets can be constrained from the inner planet TTVs, even if only two periods are visible. We discussed the number of transits necessary to secure a detection. We pointed out that two- and three-body resonant configurations could be mistaken on a short observation time, as the long-period resonant argument of a three-Jupiter system can be confused with the pericentre precession of a two-body configuration.

Many *Kepler* extrasolar systems (and KOIs) reveal to be in a closely packed configuration (Lissauer et al. 2011), and multiple-planet resonances can act as a phase-protection mechanism which ensures long-term stability. Formation scenarios (during the gas disc phase) can also favour multiple resonant systems. We emphasize the importance of TTVs long-term observing programmes for the detection of such systems in the future and the constraints they will provide on the formation history of planetary systems.

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