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Sub-GeV-scale signatures of hidden braneworlds up to the Planck scale in a $SO(3,1)$-broken bulk

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Many-brane universes are at the heart of several cosmological scenarios related to physics beyond the Standard Model. It is then a major concern to constrain these approaches. Two-brane Universes involving $SO(3,1)$-broken 5D bulks are among the cosmological models of interest. They also allow considering matter exchange between branes, a possible way to test these scenarios. Neutron disappearance (reappearance) toward (from) the hidden brane is currently tested with high-precision experiments to constrain the coupling constant g between the visible and hidden neutron sectors. When dealing with the sub-GeV-scale quantum dynamics of fermions, any pair of braneworlds can be described by a non-commutative two-sheeted space-time $M_4 \times Z_2$ from which g emerges. Nevertheless, the calculation of the formal link between g for a neutron and $SO(3,1)$-broken 5D bulks remains an open problem until now although necessary to constrain these braneworld scenarios. Thanks to a phenomenological model, we derive g – for a neutron – between the two braneworlds endowed with their own copy of the standard model in a $SO(3,1)$-broken 5D bulk. Constraints on interbrane distance and brane energy scale (or brane thickness) are discussed. While brane energy scale below the GUT scale is excluded, energy scale up to the Planck limit allows neutron swapping detection in forthcoming experiments.

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I. INTRODUCTION

The desert – i.e. no new physics in colliders between the TeV and the GUT scales [1–4] – is a realistic but disappointing scenario feared in the context of the recent LHC results [5, 6]. However, some high-precision experiments at low energy could detect signatures of GUT scale and beyond. For instance, neutron electric dipole moment (nEDM) [7–9] or proton decay [10–12] are such signatures. As a consequence, prospecting for new low-energy tests of physics beyond the Standard Model is of crucial interest. It offers an alternative and additional route to colliders to probe new physics with experiments at quite low cost. Moreover, many works [13–48] describe our visible Universe – our visible world – as a domain wall [13, 18–21] (i.e. a 3-brane) inside a higher dimensional bulk, generally with five [22–26, 44–48] or sometimes six dimensions [28–33]. Some models also consider that many braneworlds could coexist within the bulk [17, 22, 25–27, 34–43, 47]. The interest for such many-world scenarios traces back to the Hořava-Witten [17] approach linking the $E_8 \times E_8$ heterotic super-string theory in ten dimensions to eleven-dimensional supergravity on the orbifold $R_{10} \times S_1/Z_2$. Assuming a six-dimensional compactification on a Calabi-Yau manifold, this model leads to a $M_4 \times S_1/Z_2$ Universe (i.e. a five-dimensional bulk with a compactified extra dimension) in which ordinary- and hidden-sector particles live on two different 3-branes (or braneworlds) located at each boundary of the orbifold. Along this line of thought, many models emerge such as the Randall and Sundrum's (RS) solution of the hierarchy problem [22], or the so-called ekpyrotic [25] universes as alternatives to cosmic inflation or to explain dark matter and dark energy [27, 34–37]. These frameworks have also been extended to a non-compact fifth dimension in $M_4 \times R_1$ or $M_4 \times R_1/Z_2$ bulks [22–26, 44–47] where the bulk metric can be warped or not [22, 44, 45]. In many scenarios, while our visible braneworld sustains particles of the Standard Model (the TeV brane in RS-like models), the hidden braneworld (or Planck brane) supports a Planck-scale physics. In some other approaches, the hidden world should possess its own copy of the Standard Model [34–37, 47, 49–53]. This is the case in the two-brane Universe scenarios where the bulk curvature violates the $SO(3,1)$ isometry thus leading to a violation of Lorentz invariance on branes

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[34–37, 47, 49, 50]. The typical metric is then the Chung-Freese metric such that [47, 49, 50]:

$$\begin{aligned} ds^2 &= g^{(5)}_{AB} dx^A dx^B, \text{ with } A, B = 0, 1, ..., 4 \\ &= g^{(4)}_{\mu\nu} dx^\mu dx^\nu - dz^2, \text{ with } \mu, \nu = 0, 1, ..., 3, \end{aligned} \tag{1}$$

where $g^{(4)}_{\mu\nu} = diag(1, -a^2(z), -a^2(z), -a^2(z))$. In the following, when we refer to x, it denotes the usual 4D space-time components while z is the fifth extra dimension along which the branes are located. At a cosmological scale this approach offers a non-inflationary solution to the cosmological horizon problem [34–37, 47, 49–53]. In addition, Lorentz non-invariant terms lead to an attractive phenomenology which can be tested [9, 54]. As a consequence, it is a major concern to further explore experimental expectations and constraints on these braneworlds scenarios with $SO(3,1)$-broken isometry, either at a cosmological scale or in particle physics. Since these scenarios contain a copy of the Standard Model in each brane, they should allow for matter exchange between both branes, even at low-energy physics. This is the phenomenology studied in the present paper.

High-energy phenomena are often considered when dealing with braneworld phenomenology. However, in the last decade, one of us (M.S.) and collaborators showed theoretically that matter exchange could occur at low energy between closest braneworlds in the bulk [38–43]. For instance, neutron swapping is allowed between our visible Universe and a parallel one hidden in the bulk [38–42]. Such an effect is probed in some experiments involving search for passing-through-walls neutrons in the vicinity of a nuclear reactor [40, 41, 55] or search for unusual leaks when dealing with ultra-cold neutron storage [42]. The ability for a neutron to leap from our visible world into a hidden braneworld (or reciprocally) is given by a swapping probability p [39], which can be experimentally probed [40–42, 55].

The study of this phenomenology is made possible by the fact that any Universe with two braneworlds – regardless of the underlying model – is equivalent to an effective non-commutative two-sheeted space-time $M_4 \times Z_2$ when following the dynamics of particles at low energies below the GeV scale [38]. Although many braneworlds can coexist within the bulk, in a first approximation, one can consider a two-brane Universe consisting of two copies of the Standard Model, localized in two adjacent 3-branes. While these two branes are mutually invisible to each other at the zeroth-order approximation, matter fields in separate branes mix at the first-order approximation mainly through $\mathcal{L}_c = ig\overline{\psi}_+\gamma^5\psi_- + ig\overline{\psi}_-\gamma^5\psi_+$, where $\psi_\pm$ are the Dirac fermionic fields in each braneworld – denoted $(+)$ and $(-)$ [38, 39]. g is the two-brane or interbrane coupling constant and can be related to the swapping probability such that $p \propto g^2$ [39–42].

Fundamentally, branes in the bulk can be described by ξ-thick domain walls (with a brane energy scale $M_B \sim 1/\xi$), which are kink or anti-kink solitons of scalar fields allowing fermions – or gauge bosons – trapping on the 3D world [13, 18–21]. Surprisingly, the equivalence between two-brane models and the non-commutative two-sheeted space-time approach is general and neither relies on the domain walls features or on the bulk properties (dimensionality, number of compact dimensions, and metric) [38]. As a consequence, in the $M_4 \times Z_2$ model, the fundamental parameter g is a black box which implicitly contains the bulk properties, the distance d between branes in the bulk, but also their thickness ξ and some properties of the states localized on each brane [38].

In the first derivation of the $M_4 \times Z_2$ model of a two-brane Universe [38], it was shown that g can be explicitly computed against brane and bulk parameters [38] when dealing with a domain-wall description of branes. But for an illustrative purpose, and for the sake of simplicity, a simple scenario was considered [38], with a flat $M_4 \times R_1$ bulk with fermions trapped along two thick domain walls. In this naive scenario the mass of the fermions on each brane was only related to Kaluza-Klein states – the interbrane coupling could then be expressed by [38]:

$$g \sim (1/\xi) \exp(-d/\xi). \tag{2}$$

Here, it is worth noting that the brane energy scale could be as high as the Planck scale, i.e. well beyond the reach of direct searches at high energy with particle colliders. However, depending on the ratio d/ξ, g can still reach values permitting to explore the braneworld through matter swapping at low energy [39, 40]. This motivating result is also at the origin of the present paper.

To go further, considering the current experimental context [39–42] involving neutron experiments, it is crucial to explicitly derive g for a neutron, which is not a Kaluza-Klein state. In addition, regarding the models here under consideration, where the bulk possesses a $SO(3,1)$-broken isometry, one must also be able to consider warped metrics. Nevertheless, a full *ab initio* computation of g is out of reach such as and for now. Indeed, strictly speaking, a full description of the neutron in the domain wall approach would need for a valid description of the Standard Model on the domain-wall branes and to numerically solve the equations giving birth to the neutron. For instance, a lattice Quantum Chromodynamics (QCD) on a Universe with two domain-wall branes would be necessary, a huge work far beyond the scope of the present paper. As a consequence, before anything else, a phenomenological approach is first necessary to identify the main features of g for a neutron against the bulk metric and dimensionality, the brane energy scale, and interbrane distance. This is the purpose of the present paper. In section II, we recall the main results and technical difficulties of the framework which motivated the present work. In section III, we introduce the new phenomenological approach which allows us to address the problem of the derivation of g for the neutron for two thick branes in a $SO(3,1)$-broken bulk. At last, in section IV, constraints on interbrane distance and brane energy scale (or brane thickness) are considered according to existing experimental bounds on g, which are also discussed in the context of future experiments.

II. $M_4 \times Z_2$ LOW-ENERGY LIMIT OF A UNIVERSE WITH TWO DOMAIN-WALL 3-BRANES: MAIN RESULTS AND OPEN PROBLEMS

As introduced here above, whatever the high-energy theory of a two-brane Universe (i.e. whatever the number or properties of bulk scalar fields responsible for particle trapping on branes, the number of extra dimensions or the bulk metric, etc.), the fermion dynamics on both branes at low energy corresponds to the dynamics of fermions in a $M_4 \times Z_2$ space-time in the context of the non-commutative geometry [38]. Let us recall some fundamentals of this framework. For an illustrative purpose, one first considers two braneworlds described by two topological defects in the bulk. For instance, we can consider two domain walls corresponding to a kink − anti-kink pair of solitons in a $M_4 \times R_1$ flat 5D bulk. A simple Lagrangian for such a system is:

$$\mathcal{L}_{M_4 \times R_1} = \frac{1}{2}\left(\partial_A \Phi\right)\left(\partial^B \Phi\right) - V(\Phi) + \overline{\Psi}\left(i\Gamma^A\left(\partial_A + i\mathcal{A}_A\right) - \lambda\Phi\right)\Psi, \tag{3}$$

where $\mathcal{A}_A$ is a $U(1)$ bulk gauge field and Φ is the scalar field. The potential $V(\Phi)$ is assumed to ensure the existence of kink-like solutions, i.e. of domain walls, by following the Rubakov-Shaposhnikov concept [13]. Ψ is the massless fermionic matter field. Ψ is coupled to the scalar field Φ through a Yukawa coupling term $\lambda\overline{\Psi}\Phi\Psi$ with λ the coupling constant.

In our previous work [38], it was shown that $\mathcal{L}_{M_4 \times R_1}$ reduces to the effective Lagrangian $\mathcal{L}_{M_4 \times Z_2}$ for energies below the GeV scale. Then, the effective phenomenological discrete two-point space Z_2 replaces the continuous real extra dimension R_1. At each point along the discrete extra dimension Z_2, there is a four-dimensional space-time M_4 endowed with its own metric. Each M_4 sheet describes each braneworld considered as being separated by a phenomenological distance $\delta = 1/g$. This result is obtained from an approach inspired by the construction of molecular orbitals in quantum chemistry, here extended to fermionic bound states on branes. Then, g is proportional to an overlap integral of the fermionic wave functions of each 3-brane over the extra dimension R_1 [38].

The effective $M_4 \times Z_2$ Lagrangian for the fermion dynamics in a two-brane Universe is [38, 39]:

$$\mathcal{L}_{M_4 \times Z_2} \sim \overline{\Psi}\left(i\not{D}_A - M\right)\Psi. \tag{4}$$

Labelling (+) (respectively (−)) our brane (respectively the hidden brane), we write: $\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$ where $\psi_\pm$ are the wave functions in the branes (±) and:

$$i\not{D}_A - M = \begin{pmatrix} i\gamma^\mu(\partial_\mu + iqA_\mu^+) - m & ig\gamma^5 - im_r \\ ig\gamma^5 + im_r & i\gamma^\mu(\partial_\mu + iqA_\mu^-) - m \end{pmatrix}. \tag{5}$$

$A_\mu^\pm$ are the electromagnetic four-potentials on each brane (±). m is the mass of the bound fermion on a brane. The mass mixing term, due to the off-diagonal mass term m_r, results from the two-domain wall Universe [38]. The phenomenology related to m_r can be neglected when compared to the phenomenology related to g as shown elsewhere [39] and as briefly recalled in section III F. The derivative operators acting on M_4 and Z_2 are $D_\mu = \mathbf{1}_{8\times 8}\partial_\mu$ ($\mu = 0,1,2,3$) and $D_5 = ig\sigma_2 \otimes \mathbf{1}_{4\times 4}$, respectively, and the Dirac operator acting on $M_4 \times Z_2$ is defined as $\not{D} = \Gamma^N D_N = \Gamma^\mu D_\mu + \Gamma^5 D_5$ where: $\Gamma^\mu = \mathbf{1}_{2\times 2} \otimes \gamma^\mu$ and $\Gamma^5 = \sigma_3 \otimes \gamma^5$. γ^μ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ are the usual Dirac matrices and σ_k ($k = 1,2,3$) the Pauli matrices. Note that Eq. (5) is typical of non-commutative $M_4 \times Z_2$ two-sheeted space-times [38]. We refer to the terms proportional to g as geometrical mixing.

Regarding the electromagnetic field, it was shown [38] that the five-dimensional $U(1)$ bulk gauge field must be substituted by an effective $U(1)_+ \otimes U(1)_-$ gauge field in the $M_4 \times Z_2$ space-time. The Dvali-Gabadadze-Shifman mechanism [44, 45] leads to the gauge field localization on the branes and $U(1)_\pm$ are the gauge groups of the photon fields on each brane: the bulk gauge field $\mathcal{A}_A$ splits into $A_\mu^\pm$. The electromagnetic field $\not{A} \sim diag(iq\gamma^\mu A_\mu^+, iq\gamma^\mu A_\mu^-)$ is then introduced in the Dirac equation through $\not{D}_A \to \not{D} + \not{A}$ [38].

In this approach [38], it must be noted that the mass m corresponds to a Kaluza-Klein state [38] and g is given by Eq. (2). So, in order to address the behaviour of the neutron, we should first consider massless fermionic Kaluza-Klein states. Next, many fermionic fields should be introduced in the model. Not only to get quark families, but also because two fermionic fields are required at least to get both chirality states on the brane when considering the fundamental Kaluza-Klein states [46]. Of course, gauge and Higgs fields should be also included in the domain-wall-brane model to dress massless states to retrieve the expected standard masses on branes. Indeed, the Higgs field contributes to quark masses while, considering the neutron, its mass mainly arises from the strong interaction between quarks. Of course, one should also ensure to retrieve the Standard Model symmetry as well as the efficiency of the mechanisms needed to confine each field on each brane [18–21] when dealing with the bulk metric. There is possibly many ways to reach these goals which meet the landscape problem encountered in superstring models [56]. Anyway, this issue is then far beyond the scope of the present paper. As a consequence, a more relevant and phenomenological approach must be considered for now to address the main features of the coupling constant g when dealing with neutron in a two-brane Universe with a given bulk metric. This is the mainspring of the approach introduced in the next section.

III. $M_4 \times Z_2$ LOW-ENERGY LIMIT OF A UNIVERSE WITH TWO D3-BRANES: AN APPROACH FOR THE NEUTRON PHENOMENOLOGY

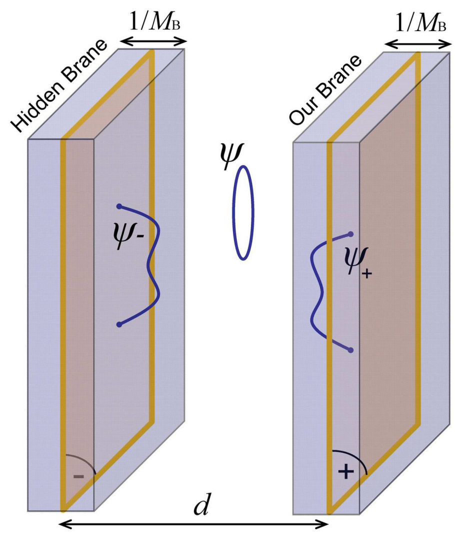

FIG. 1: (Color online). Sketch of the problem under study. A string-inspired approach allows treating the problem from a phenomenological point of view by introducing brane quark fields $\psi_\pm$ and a bulk quark-like field ψ. While D3-branes (yellow) correspond to strings' endpoints, they can be considered as thick 3-branes (bluish) when dressed with strings. Brane thickness is introduced through the brane energy scale M_B which is roughly the string energy scale. Interbrane distance d in the $M_4 \times R_1$ bulk is greater than M_B^{-1}.

In the present section, we derive the relevant phenomenological $M_4 \times Z_2$ low-energy limit for a universe with two D3-branes embedded in a Chung-Freese bulk with $SO(3,1)$-broken isometry. By doing so, we seek to estimate the interbrane coupling constant g for a neutron − i.e. a *udd* quark triplet − in such a universe. While the fermion - hidden fermion coupling is the core of our attention, bosons' coupling between both branes is a correction which can be here neglected [38, 39] without loss of generality. In the following, we will study the coupling between quarks' fields of each brane. We consider a simplified[1] string-inspired [56] phenomenological model (Fig. 1). We thus introduce two quark field families, each one located in each brane (like the open strings on a brane) and bulk fermionic fields free to propagate in the bulk (like the closed strings). The whole problem is described by the full action $S = S_{bulk} + S_{brane(+)} + S_{brane(-)} + S_{coupling}$ which contains the four terms described hereafter.

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[1] For instance, in the present approach, Kalb-Ramond and Ramond-Ramond fields are not under consideration [56].

A. Effective quark action on branes

We first consider that each brane is endowed with its own copy of the Standard Model. A quark field on a brane is then assumed as resulting from an open string initially in its zero-mass state and fixed on the brane. Fundamentally, a quark gets its mass from interaction with the Higgs field on branes only. In addition, the neutron will get its mass mainly from the strong force interaction between quarks in the *udd* quark triplet. As a consequence, one should introduce the Higgs and $SU(3)$ gauge fields on the brane in order to compute the whole neutron properties. Nevertheless, for instance, usual *ab initio* computations of masses or magnetic moments of hadrons use tedious numerical computation to describe gluons/quarks dynamics from lattice QCD. Such an approach is far beyond the scope of the present work. As a consequence, to identify the main behaviour of the neutron in our model, we use for now the Constituent Quark Model (CQM) [57–60]. In hadron, each idealized current quark (or naked quark) is dressed by the gluons/virtual quarks sea surrounding it. Dressed quarks are called constituent quarks. The effective masses of constituent quarks can be derived from experimental data, and of course, differ from current quarks' masses. Of course, *ab initio* computations in chromodynamics would be necessary to explain the values of the masses of the constituent quarks. But from a phenomenological point of view, the CQM works pretty well to calculate masses or magnetic moments [2] [57–60]. In section III F, the interbrane coupling constant of the neutron will be naturally derived from CQM [57–60]. As a consequence, in the following, each quark on brane will be endowed with its effective phenomenological mass m, which implicitly contains the effects of Higgs field and quark-to-quark interactions on branes. Then, the quark actions (whatever the flavour) on each brane located on $z = \pm d/2$ resume to:

$$S_{brane(\pm)} = \int d^4x \sqrt{\left|g_{\pm}^{(4)}\right|}\overline{\psi}_{\pm} \left(i\Gamma^{\mu}_{\pm}\left(\partial_{\mu} + C^{\pm}_{\mu} + iqA^{\pm}_{\mu}\right) - m\right)\psi_{\pm}, \tag{6}$$

where $\psi_{\pm}(x)$ (respectively $g_{\pm}^{(4)}$) are the usual 4D spinors of quark fields (respectively the induced metric tensors) on each brane. $C^{\pm}_{\mu}$ and $\Gamma^{\mu}_{\pm}$ are the spin connections and the Dirac matrices taken on the branes and deduced from the general 5D equations (8) to (10) introduced in the next section. m is the quark constituent mass induced on each brane. $A^{\pm}_{\mu}$ are the electromagnetic potentials on each brane. The expression of $S_{brane(\pm)}$ would correspond to infinitely thin branes, i.e. D3-branes. Considering string-dressed 3-branes, that means one expects that gauge fields and fermions of the Standard Model are contained in the thickness ξ of the 3-brane, i.e. they do not spread beyond $z > \xi = M_B^{-1}$. As a consequence, the brane thickness is assumed to be small compared to the interbrane distance d.

B. Effective quark-like fermion action in the bulk

We assume that for each quark on branes (as an open string) there is a dual bulk fermion able to carry flavour and charges in the bulk (as a closed string). Since particles' interactions of the Standard Model are supposed to exist on branes only, we assume that bulk particles are sterile in relation to each other. The dual quark-like bulk state is assumed to be massless[3] and related to a closed string in its zero-mass state. Since we neglect quarks' interaction in the bulk, the bulk triplet is also massless. The bulk quark-like field $\psi(x, z)$ follows then the action:

$$S_{bulk} = \int d^4x dz \sqrt{\left|g^{(5)}\right|}\overline{\psi}\left(i\Gamma^{A}\left(\partial_A + C_A + iq\mathcal{A}_A\right)\right)\psi \tag{7}$$

where the Dirac matrices Γ^A follow the relationship $\left\{\Gamma^A, \Gamma^B\right\} = 2g^{(5)AB}\mathbf{1}$. By contrast, we define the "flat" Dirac matrices γ^A through: $\left\{\gamma^A, \gamma^B\right\} = 2\eta^{(5)AB}\mathbf{1}$ with $\eta^{(5)AB}$ the Minkowski metric with a $(+,-,-,-,-)$ signature. $\mathcal{A}_A$ is the effective five dimensional electromagnetic vector potential such that $\mathcal{A}_4 = 0$, and which is supposed to be zero everywhere except on braneworlds. $q\overline{\psi}\Gamma^A\mathcal{A}_A\psi$ ensures charge conservation through the bulk. The five dimensional Dirac matrices in curved space-time take the form:

$$\Gamma^A(x, z) = e^A_B(x, z)\gamma^B, \tag{8}$$

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[2] Effective magnetic moments μ_q of constituent quarks can be easily derived from their effective masses m and from quarks' charges q since $\mu_q = q\hbar/2m$. Effective magnetic moments can be simply combined to retrieve the hadron magnetic moment. The hadron mass can also simply be retrieved by summing effective masses of each constituent quark plus a spin-spin corrective term [57–60].

[3] Strings can have excitation states leading to effective masses, which here are model-dependent multiple of M_B [56]. Then, we neglect the small corrections of these states as we are mainly concerned by the order of magnitude of g in the current experimental context.

where e_a^A defines the vielbein according to:

$$g^{(5)AB} = e_C^A(x,z)e_D^B(x,z)\eta^{(5)CD}. \tag{9}$$

The spin connection must satisfy the expression:

$$C_A(x,z) = \frac{1}{4}\Gamma_B\left[\partial_A\Gamma^B + \Gamma_{CA}^B\Gamma^C\right], \tag{10}$$

where Γ_{CA}^B are the Christoffel symbols for the metric field under consideration.

C. Effective bulk – brane fields coupling action

The fermion fields on each brane can be considered as source/well terms for the fermionic bulk field. The coupling between the brane fields and the bulk fields occurs naturally as a mass coupling on branes, and we get:

$$\begin{aligned} S_{coupling} &= -\int d^4xdz\sqrt{\left|g^{(4)}\right|} \\ &\quad\times\left\{\frac{m}{M_B^{1/2}}\left(\overline{\psi}_+\psi + \overline{\psi}\psi_+\right)\delta(z-d/2)\right. \\ &\quad\left.+\frac{m}{M_B^{1/2}}\left(\overline{\psi}_-\psi + \overline{\psi}\psi_-\right)\delta(z+d/2)\right\}. \end{aligned} \tag{11}$$

From the point of view of the domain-wall approach, ψ and $\psi_\pm$ are fundamentally the same fields [38], this is why we use the simple mass mixing implying m, as the coupling occurs only on each brane. Now, though the expression of $S_{brane(\pm)}$ corresponds to infinitely thin branes, $S_{coupling}$ introduces the finite thickness $\xi \sim 1/M_B$ of the branes along which the coupling occurs (due to the spatial extent of strings). In $S_{coupling}$, the power 1/2 of M_B ensures the correct dimensionality of the problem.

D. Validity domain of the model

• If both 3-branes are too close to each other, typically $d \leq 1/M_B$, direct interactions between fermion fields of each 3-brane could occur and exotic fields related to open strings stretched between both 3-branes could appear [56]. In such a case, both 3-branes should be then considered as a single 3D world, with visible and dark sectors. The last is a component which can be added to the Standard Model to restore some symmetries [61–63]. This has been studied previously in the literature for instance in the context of mirror particle paradigm [61–63]. This topic is out of the scope of the present work. To ensure two independent 3-branes and so the validity of our model, we roughly assume that $d > 2/M_B$.

• In the bulk, the neutron becomes a triplet of free but entangled quark-like bulk states. In the bulk, due to the relativistic free motion of bulk quarks, the triplet can stretch along usual spatial dimensions when propagating along the extra dimension. This could prevent neutron reappearance in a brane. Indeed, due to the finite range of the strong force and in order to restore the neutron, quarks reappearing on a brane must not be too distant from each other. Then, the interbrane distance must not be too large to avoid bulk quarks to separate from each other beyond the strong force range in a brane. As a consequence, one can claim that neutron exchange between braneworlds is not possible if the interbrane distance d exceeds roughly 0.5 fm, i.e. the coupling constant g then collapses to zero.

E. $M_4 \times Z_2$ limit of the two-brane universe by propagating equations of motion over the extra dimension

Let us first consider the equation of motion for the bulk field from S. The bulk field simply follows the relationship:

$$\begin{aligned} &\sqrt{\left|g^{(5)}\right|}\left(i\Gamma^A\left(\partial_A + C_A + iq\mathcal{A}_A\right)\right)\psi \\ &= \sqrt{\left|g^{(4)}\right|}\frac{m}{M_B^{1/2}}\psi_+\delta(z-d/2) + \sqrt{\left|g^{(4)}\right|}\frac{m}{M_B^{1/2}}\psi_-\delta(z+d/2), \end{aligned} \tag{12}$$

which is the expected 5-dimensional Dirac equation supplemented by the source/well terms induced by the boundary conditions on the branes. Equation (12) can be then easily propagated over the extra dimension. Due to mass shell constraint on the branes, one imposes the condition:

$$\left(i\Gamma^{\mu}\left(\partial_{\mu}+C_{\mu}+iqA_{\mu}\right)-m\right)\psi=0 \tag{13}$$

and Eq. (12) becomes:

$$\begin{aligned}&\left(\gamma^5\partial_z+m\right)\psi \qquad (14)\\ &=\sqrt{\frac{\left|g_+^{(4)}\right|}{\left|g_+^{(5)}\right|}}\frac{m}{M_B^{1/2}}\psi_+\delta(z-d/2)+\sqrt{\frac{\left|g_-^{(4)}\right|}{\left|g_-^{(5)}\right|}}\frac{m}{M_B^{1/2}}\psi_-\delta(z+d/2),\end{aligned}$$

since $C_4=0$ and $\Gamma^4=-i\gamma^5$ due to the metric choice in Eq. (1). Now, we can introduce the Green function G – of the free field ψ – which obeys to:

$$\left(\gamma^5\partial_z+m\right)G(z-z')=\delta(z-z'), \tag{15}$$

leading to:

$$\begin{aligned}G(z) &= \frac{1}{2\pi}\int\frac{i\gamma^5\kappa+m}{\kappa^2+m^2}e^{-i\kappa z}d\kappa\\ &= (1/2)e^{-m|z|}\left(\mathbf{1}+sign(z)\gamma^5\right),\end{aligned} \tag{16}$$

with $\gamma^0G^{\dagger}(z)\gamma^0=G(-z)$. Solving Eq. (14), ψ can be simply expressed as:

$$\psi(x,z)=\sqrt{\frac{\left|g_+^{(4)}\right|}{\left|g_+^{(5)}\right|}}\frac{m}{M_B^{1/2}}G(z-d/2)\psi_+(x)+\sqrt{\frac{\left|g_-^{(4)}\right|}{\left|g_-^{(5)}\right|}}\frac{m}{M_B^{1/2}}G(z+d/2)\psi_-(x). \tag{17}$$

From Eqs. (17) and (16), one deduces that bulk field originating from a brane, does not propagate along the extra dimension, but has an evanescent component with a decay constant equal to m along the extra dimension[4]. As a consequence, despite the coupling between brane and bulk fields, quarks remain localized on their respective brane and cannot propagate through the bulk. Then, as shown hereafter, the only way for quarks to escape from a brane, is to jump towards another brane thanks to a quantum-tunnelling-like effect.

Injecting the expression of ψ given by Eq. (17) in $S_{coupling}$ (see Eq. (11)), we retrieve the mass mixing and the geometrical mixing terms found in the non-commutative $M_4\times Z_2$ approach (see section II):

$$S_{coupling}=\int d^4x\left(im_r\overline{\psi}_+\psi_--im_r\overline{\psi}_-\psi_++\overline{\psi}_+ig\gamma^5\psi_-+\overline{\psi}_-ig\gamma^5\psi_+\right), \tag{18}$$

where:

$$g=(1/2)\frac{m^2}{M_B}\left(R^{-3/2}+R^{3/2}\right)e^{-md}, \tag{19}$$

and $m_r=g$, with $R=a_-/a_+$ ($a_{\pm}=a(z=\pm d/2)$). Note that Eq. (19) is invariant through $R\to R^{-1}$. In Eq. (18) we successively applied a convenient SU(2) rotation:

$$\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}\to\begin{pmatrix}e^{-i\pi/4} & 0\\ 0 & e^{i\pi/4}\end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}, \tag{20}$$

[4] If massive bulk modes $M\propto M_B$ were considered, one needs to replace m by $|m-kM_B|$ (where k is a number greater than 1) in the argument of the exponential term of the bulk field propagator. Then, for an interbrane distance greater than $1/M_B$, the contributions from massive bulk modes quickly drop with a decay constant about kM_B and can be neglected as expected since $kM_B\gg m$.

and a convenient rescaling:

$$\left(\begin{array}{c}\psi_+\\ \psi_-\end{array}\right)\rightarrow\left(\begin{array}{cc}a_+^{3/2} & 0\\ 0 & a_-^{3/2}\end{array}\right)\left(\begin{array}{c}\psi_+\\ \psi_-\end{array}\right),\tag{21}$$

to ensure a more usual wave-function normalization. In Eq. (18), self-coupling terms of the form $\overline{\psi}_\pm\Gamma\psi_\pm$ are neglected. Indeed, $\overline{\psi}_\pm\Gamma\psi_\pm = m^2/M_B$ thus introducing a mass correction such that $m\rightarrow m(1+m/M_B)$. For $m = 340$ MeV [57–60] and M_B ranging between GUT and Planck scales, such a correction is far beyond any current experimental accuracy [5, 6].

Now, following the same procedure, Eq. (6) describing quarks on each brane becomes (after rotation (20) and rescaling (21)):

$$\begin{aligned}S_{brane(\pm)} = & \int d^4x\overline{\psi}_\pm\left(i\gamma^0\left(\partial_0+iqA_0^\pm\right)\right.\\ & +ia_\pm^{-1}\gamma^\eta\left(\partial_\eta+iqA_\eta^\pm\right)\\ & \left.+(3/2)\left(\partial_z a\right)_\pm a_\pm^{-1}\gamma^5-m\right)\psi_\pm.\end{aligned}\tag{22}$$

In the following, the terms $(3/2)\left(\partial_z a\right)_\pm a_\pm^{-1}\gamma^5$ will be neglected without any loss of generality since the Lorentz symmetry breaking introduced by these terms [54] can be neglected in our present problem thanks to current experimental data [9].

Finally, the relevant action for the dynamics of the quark fields $\psi_\pm$ on each brane is the effective $M_4\times Z_2$ action $S_{M_4\times Z_2}$ which is the restriction of $S = S_{bulk}+S_{brane(+)}+S_{brane(-)}+S_{coupling}$. Indeed, from Eq. (18) and Eq. (22), one gets $S_{M_4\times Z_2} = S_{brane(+)}+S_{brane(-)}+S_{coupling} = \int\mathcal{L}_{M_4\times Z_2}d^4x$, with:

$$\mathcal{L}_{M_4\times Z_2}\sim\overline{\Psi}\left(i\not{D}_A-M\right)\Psi,\tag{23}$$

where $\Psi = \left(\begin{array}{c}\psi_+\\ \psi_-\end{array}\right)$, and:

$$i\not{D}_A-M = \left(\begin{array}{cc}i\widehat{\gamma}_\pm^\mu(\partial_\mu+iqA_\mu^+)-m & ig\gamma^5-im_r\\ ig\gamma^5+im_r & i\widehat{\gamma}_\pm^\mu(\partial_\mu+iqA_\mu^-)-m\end{array}\right),\tag{24}$$

with $\left(\widehat{\gamma}_\pm^0,\widehat{\gamma}_\pm^{1,2,3}\right) = \left(\gamma^0,a_\pm^{-1}\gamma^{1,2,3}\right)$. When $a_\pm^{-1}\longrightarrow 1$, Eq. (24) fully matches Eq. (5) above. It is noticeable that the present approach still leads to a non-commutative $M_4\times Z_2$ space-time description of the two-brane Universe [38, 64] (see section II). In fact, Eq. (23) is simply the generalization of the $M_4\times Z_2$ action for the Chung-Freese metric as shown in a previous work [43]. Strictly speaking, the choice of such a metric does not fundamentally change the physics of the two-brane Universe at low energy as described in our previous papers [39–42]. All the more, the value of g changes against the ratio $R = a_-/a_+$ of the warp factors $a_\pm$ as shown by Eq. (19). The low-energy phenomenology induced by $a_\pm^{-1}\neq 1$ is fully detailed in a previous work [43]. We just note that the factors $a_\pm^{-1}$, occurring in the "usual" Dirac operator in each brane, will affect the values of the momentum and of the kinetic energy which now differ in each brane [43]. Anyway, the new contributions occurring from the non-null differences between the kinetic energies (and the momenta) of each brane are negligible for neutrons with a kinetic energy lower or equal to that of thermal neutrons (i.e. about 25 meV or less) [43]. At last, regarding the values of g (see Eq. (19)), it is not possible to experimentally discriminate the contribution of M_B from the warp contribution R. One can then conveniently substitute the brane thickness M_B^{-1} by an effective one including the warped metric effect such that $M_B^{-1}\rightarrow M_B^{-1}(1/2)\left(R^{-3/2}+R^{3/2}\right)$. Anyway, M_B remains lower or equal than M_{Planck} whatever R.

F. Phenomenology and neutron - hidden neutron coupling constant

Assuming the non-relativistic character of the CQM [57–60], it is instructive to consider the non-relativistic limit of the two-brane Dirac equation (see Eq. (24)). One gets a two-brane Pauli equation: $i\hbar\partial_t\Psi = \left\{\mathbf{H}_0+\mathbf{H}_{cm}+\mathbf{H}_c\right\}\Psi$, with $\mathbf{H}_0 = diag(\mathbf{H}_+,\mathbf{H}_-)$ where $\mathbf{H}_\pm$ are the usual four-dimensional Pauli Hamiltonian expressed in each braneworld, and $\Psi = \left(\begin{array}{c}\psi_+\\ \psi_-\end{array}\right)$ where $\psi_\pm$ are now the Pauli spinors. Moreover, coupling terms appear (in natural units) [38, 39]:

$$\mathbf{H}_c = \left(\begin{array}{cc}0 & im_rc^2\\ -im_rc^2 & 0\end{array}\right),\tag{25}$$

which is obviously the mass mixing term, and [38, 39]

$$\mathbf{H}_{cm} = \begin{pmatrix} 0 & -ig\widehat{\mu}\cdot(\mathbf{A}_+ - \mathbf{A}_-) \\ ig\widehat{\mu}\cdot(\mathbf{A}_+ - \mathbf{A}_-) & 0 \end{pmatrix}, \tag{26}$$

where $\mathbf{A}_\pm$ are the local magnetic vector potentials in each brane and $\widehat{\mu} = \mu\sigma$ is the magnetic moment operator of the fermion. There is no pure geometrical mixing, instead $\mathbf{H}_{cm}$ relates to a mixed geometrical/electromagnetic coupling involving the magnetic moment. This is allowed by the pseudo-scalar-coupling character of the geometrical mixing. Obviously $\mathbf{H}_{cm}$ also exists at the relativistic energy scale, like the magnetic moment, although it does not explicitly appear in the Dirac equation. Here, the coupling strength between fermion spinors of the visible and hidden worlds becomes clearly dependent on the magnetic potentials which enhance the magnitude of the geometrical mixing. Since here $m_r c^2 = g\hbar c$, from Eqs. (25) and (26), one can easily check that $\mathbf{H}_{cm}$ dominates $\mathbf{H}_c$ when $|\mathbf{A}_+ - \mathbf{A}_-| > A_c$. The critical field A_c is given by $A_c = 2mc/q$, where m is here the mass of the constituent quark, and q its charge. $A_c \approx 7$ Tm for the constituent quark down. This value must be compared with these of expected astrophysical magnetic potentials about 10^9 Tm [65, 66]. Then, here the geometrical/electromagnetic coupling $\mathbf{H}_{cm}$ would be larger than the mass mixing $\mathbf{H}_c$ by 8 orders of magnitude. As a consequence, we usually neglect $\mathbf{H}_c$ [39].

From the previous equations, one can show that a neutron should oscillate between two states, one localized in our brane, the other localized in the hidden world [38] following a similar two-brane Pauli equation[5]. Then, assuming that g (respectively μ) refers here to the coupling constant (respectively the magnetic moment) of the neutron, using the quark constituent model [57–60], one must verify:

$$g\widehat{\mu} = \sum_q g_q \widehat{\mu}_q \tag{27}$$

where g_q (respectively $\widehat{\mu}_q$) refers to the coupling constant (respectively the magnetic moment operator) of each quark constituting the neutron with $\widehat{\mu} = \sum_q \widehat{\mu}_q$. Since $m_{up} \approx m_{down} \approx m = 340$ MeV [57–60], one simply gets $g \approx g_{up} \approx g_{down}$. This approach could be generalized to any chargeless baryon[6] endowed with a magnetic moment. Obviously, the large neutron lifetime and the huge number of neutrons produced in a nuclear reactor [40, 41, 55] make neutron highly competitive to probe two-brane physics by contrast to more exotic hadrons [67].

IV. BOUNDS ON INTERBRANE DISTANCE AND BRANE ENERGY SCALE AGAINST NEUTRON-HIDDEN NEUTRON COUPLING

Figure 2 shows bounds on the interbrane distance d in a 5D bulk, and on the brane energy scale M_B, against the coupling constant g. The M_B scale includes GUT and Planck energies. Values greater than 200 peV (i.e. 10^{-3} m^{-1}) are excluded with confidence from experimental data [40]. For interbrane distances greater than 0.5 fm, neutron exchange is supposed to be precluded ($g = 0$) by the model (see section III), in agreement with the exponential decay of g (see Eq. (19)) as $m^{-1} \approx 0.58$ fm. As a significant result, Fig. 2 shows that neutron disappearance/reappearance observation would imply a brane energy scale greater than the GUT scale. From Eq. (19), assuming a Chung-Freese metric, the ratio R of the warp factors should not exceed 20 to stay in the correct energy scale domain. Regarding now the green domains in Fig. 2, it is noteworthy that the coupling constant g gets values ranging between 2 and 200 peV – i.e. $10^{-3} < g < 10^{-5}$ m^{-1} – compatible with the observation of neutron disappearance/reappearance in the next coming experiments [40, 41]. If one doubts of a fine tuning of the interbrane distance around $1/m$, and if one assumes that neutron swapping can occur (i.e. d is lower than 0.5 fm), then short interbrane distances should prevail leading to: $g \sim m^2/M_B \approx 2.4 \times 10^{-4}$ m $^{-1} \approx 50$ peV at the (reduced) Planck scale. Such a value of g is very promising for next coming experiments [55].

V. CONCLUSION

In the context of the $M_4 \times Z_2$ low-energy description of a two-brane Universe, we have derived the explicit expression of the coupling constant g between the visible state of the neutron in our visible braneworld and the hidden state of the

[5] Reader will find details of the related phenomenology in our previous works [39].
[6] For charged particles, the swapping between braneworlds must be dramatically frozen [41, 42].

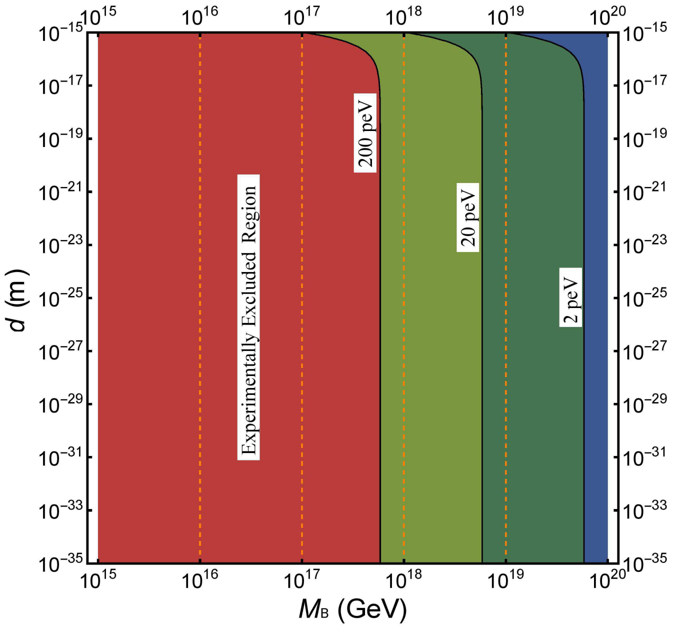

FIG. 2: (Color online). Bounds on the interbrane distance d in a 5D bulk and on the brane energy scale M_B against the coupling constant g. The M_B scale includes GUT and Planck energies (10^{16} GeV and 10^{19} GeV respectively). The red region for values greater than $g = 200$ peV (or 10^{-3} m^{-1} in natural units) is excluded with confidence from experimental data [40]. The light green region with $20 < g < 200$ peV (or $10^{-4} < g < 10^{-3}$ m^{-1}) is expected to be likely reachable in future experiments. The dark green region with $2 < g < 20$ peV (or $10^{-5} < g < 10^{-4}$ m^{-1}) will be partially reachable in future experiments. The blue region is out of range for now. For interbrane distances greater than 0.5 fm, neutron exchange is supposed to be precluded ($g = 0$ m^{-1}) by the model.

neutron in the hidden braneworld. This phenomenological approach allows studying g against the interbrane distance and the brane thickness (or brane energy scale), here in the framework of the Chung-Freese two-brane Universes involving $SO(3,1)$-broken 5D bulks. According to current experimental bounds on g, we have shown that a successful detection of the neutron swapping would lead to reject a braneworld energy scale below the GUT energy scale. While colliders tend to reject new physics at TeV and beyond, it is shown that even if the brane energy scale corresponds to the Planck scale, g can reach values motivating new passing-through-walls-neutron experiments purposed to detect neutron swapping between braneworlds [40, 41, 55]. While we have focused here on a 5D bulk with a Chung-Freese metric, our approach could be extended to a 6D bulk and beyond, compact or non-compact, warped or not. This will be considered in further work to test the robustness of the present results.

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[1] D. Hooper, S. Profumo, *Phys. Rep.* **453**, 29 (2007), arXiv:hep-ph/0701197.
[2] P. Langacker, M. Luo, *Phys. Rev. D* **44**, 817 (1991).
[3] J. Ellis, S. Kelley, D.V. Nanopoulos, *Phys. Lett. B* **260**, 131 (1991).
[4] S. Dimopoulos, *Phys. Lett. B* **246**, 347 (1990).
[5] Particle Data Group, *Chin. Phys. C* **40**, 100001 (2016).
[6] ATLAS Collaboration, *JHEP* **10**, 134 (2015), arXiv:1508.06608 [hep-ex].
[7] P. Schmidt-Wellenburg, *AIP Conf. Proc.* **1753**, 060002 (2016), arXiv:1602.01997 [nucl-ex].
[8] T. Bhattacharya, V. Cirigliano, R. Gupta, *PoS LATTICE2013*, 299 (2014), arXiv:1403.2445 [hep-lat].
[9] I. Altarev, *et al.*, *Europhys. Lett.* **92**, 51001 (2010), arXiv:1006.4967 [nucl-ex].

[10] The Super-Kamiokande Collaboration, *Phys. Rev. D* **90**, 072005 (2014), arXiv:1408.1195 [hep-ex].
[11] C. McGrew, *et al.*, *Phys. Rev. D* **59**, 052004 (1999).
[12] W. J. Marciano, G. Senjanović, *Phys. Rev. D* **25**, 3092 (1982).
[13] V.A. Rubakov, M.E. Shaposhnikov, *Phys. Lett.* **125B**, 136 (1983).
[14] M. Pavsic, *Phys. Lett. A* **116**, 1 (1986), arXiv:gr-qc/0101075.
[15] K. Akama, *Lect. Notes Phys.* **176** (1983) 267, arXiv:hep-th/0001113.
[16] J. Hughes, J. Liu, J. Polchinski, *Phys. Lett. B* **180**, 370 (1986).
[17] P. Horava, E. Witten, *Nucl. Phys. B* **460**, 506 (1996), arXiv:hep-th/9510209;
[18] Y.-Y. Li, Y.-P. Zhang, W.-D. Guo, Y.-X. Liu, *Phys. Rev. D* **95**,115003 (2017), arXiv:1701.02429 [hep-th].
[19] R. Davies, D.P. George, R.R. Volkas, *Phys. Rev. D* **77**, 124038 (2008), arXiv:0705.1584 [hep-ph].
[20] G.A. Palma, *Phys. Rev. D* **73**, 045023 (2006), arXiv:hep-th/0505170.
[21] A. Lukas, B.A. Ovrut, K.S. Stelle, D. Waldram, *Phys. Rev. D* **59**, 086001 (1999), arXiv:hep-th/9803235.
[22] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999), arXiv:hep-ph/9905221.
[23] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999), arXiv:hep-th/9906064.
[24] V.A. Rubakov, *Physics-Uspekhi* **44**, 871 (2001), arXiv:hep-ph/0104152.
[25] J. Khoury, B.A. Ovrut, P.J. Steinhardt, N. Turok, *Phys. Rev. D* **64**, 123522 (2001), arXiv:hep-th/0103239.
[26] R. Kallosh, L. Kofman, A. Linde, *Phys. Rev. D* **64**, 123523 (2001), arXiv:hep-th/0104073.
[27] T. Koivisto, D. Wills, I. Zavala, *JCAP* **06**, 036 (2014), arXiv:1312.2597 [hep-th].
[28] D.M. Dantas, D.F.S. Veras, J.E.G. Silva, C.A.S. Almeida, *Phys. Rev. D* **92**, 104007 (2015), arXiv:1506.07228 [hep-th].
[29] Y.-X. Liu, L. Zhao, Y.-S. Duan, *JHEP* **0704**, 097 (2007), arXiv:hep-th/0701010.
[30] C. Ringeval, J.-P. Uzan, *Phys. Rev. D* **71** (2005) 104018, arXiv:hep-th/0301172.
[31] E.I. Guendelman, E. Spallucci, *Phys. Rev. D* **70**, 026003 (2004), arXiv:hep-th/0311102.
[32] P. Kanti, R. Madden, K. A. Olive, *Phys. Rev. D* **64**,044021 (2001), arXiv:hep-th/0104177.
[33] T. Gherghetta, M. Shaposhnikov, *Phys. Rev. Lett.* **85**,240 (2000), arXiv:hep-th/0004014.
[34] D. Battefeld, P. Peter, *Phys. Rept.* **571**, 1 (2015), arXiv:1406.2790 [astro-ph.CO].
[35] J.-L. Lehners, *Phys. Rept.* **465**, 223 (2008), arXiv:0806.1245 [astro-ph].
[36] R. Maartens, K. Koyama, *Living Rev. Relativity* **13**, 5 (2010), arXiv:1004.3962 [hep-th].
[37] P. Brax, C. van de Bruck, A.-C. Davis, *Rep. Prog. Phys.* **67**, 2183 (2004), arXiv:hep-th/0404011.
[38] M. Sarrazin, F. Petit, *Phys. Rev. D* **81**, 035014 (2010), arXiv:0903.2498 [hep-th].
[39] M. Sarrazin, F. Petit, *Eur. Phys. J. C* **72**, 2230 (2012), arXiv:1208.2014 [hep-ph].
[40] M. Sarrazin, *et al.*, *Phys. Lett. B* **758**, 14 (2016), arXiv:1604.07861 [hep-ex].
[41] M. Sarrazin, *et al.*, *Phys. Rev. D* **91**, 075013 (2015), arXiv:1501.06468 [hep-ph].
[42] M. Sarrazin, G. Pignol, F. Petit, V.V. Nesvizhevsky, *Phys. Lett. B* **712**, 213 (2012), arXiv:1201.3949 [hep-ph].
[43] F. Petit, M. Sarrazin, *Phys. Rev. D* **76**, 085005 (2007), arXiv:0706.4025 [hep-th].
[44] G. Dvali, G. Gabadadze, M. Shifman, *Phys. Lett. B* **497**, 271 (2001), arXiv:hep-th/0010071.
[45] S.L. Dubovsky, V.A. Rubakov, *Int. J. Mod. Phys. A* **16**, 4331 (2001), arXiv:hep-th/0105243.
[46] S.L. Dubovsky, V.A. Rubakov, P.G. Tinyakov, *Phys. Rev. D* **62**, 105011 (2000), arXiv:hep-th/0006046.
[47] D.J.H. Chung, K. Freese, *Phys. Rev. D* **62**, 063513 (2000), arXiv:hep-ph/9910235.
[48] L. Visinelli, N. Bolis, S. Vagnozzi, *Phys. Rev. D* **97**, 064039 (2018), arXiv:1711.06628 [gr-qc].
[49] D.J.H. Chung, E.W. Kolb, A. Riotto, *Phys. Rev. D* **65**, 083516 (2002), arXiv:hep-ph/0008126.
[50] D.J.H. Chung, K. Freese, *Phys. Rev. D* **67**, 103505 (2003), arXiv:astro-ph/0202066.
[51] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, N. Kaloper, *JHEP* **0012**, 010 (2000), arXiv:hep-ph/9911386.
[52] R.W. Rasmussen, *et al.*, *Phys. Rev. D* **96**, 083018 (2017), arXiv:1707.07684 [hep-ph].
[53] R.R. Caldwell, D. Langlois, *Phys. Lett. B* **511**, 129 (2001), arXiv:gr-qc/0103070.
[54] D. Colladay, A. Kostelecky, *Phys. Rev. D* **55**, 6760 (1997), arXiv:hep-ph/9703464.
[55] C. Stasser, M. Sarrazin, G. Terwagne, to appear in *EPJ Web of Conferences* (2019), arXiv:1810.12800 [nucl-ex].
[56] R.J. Szabo, *An Introduction to String Theory and D-brane Dynamics*, Imperial College Press (2007).
[57] D. Griffiths, *Introduction to Elementary Particles*, Wiley-VCH Verlag GmbH & Co. KGaA (2008).
[58] W. Lucha, F.F. Schöberl, D. Gromes, *Phys. Rep.* **200**, 127 (1991).
[59] H.J. Lipkin, *Phys. Lett. B* **233**, 446 (1989).
[60] I. Cohen, H.J. Lipkin, *Phys. Lett.* **93B**, 56 (1980).
[61] L.B. Okun, *Physics-Uspekhi* **50**, 380 (2007), arXiv:hep-ph/0606202.
[62] R. Foot, R.R. Volkas, *Phys. Rev. D* **52**, 6595 (1995), arXiv:hep-ph/9505359.
[63] E.W. Kolb, D. Seckel, M.S. Turner, *Nature* **314**, 415 (1985).
[64] M. Sarrazin, F. Petit, *Eur. Phys. J. B* **87**, 26 (2014), arXiv:1209.1712 [hep-ph].
[65] R. Lakes, *Phys. Rev. Lett.* **80**, 1826 (1998).
[66] J. Luo, C.-G. Shao, Z.-Z. Liu, Z.-K. Hu, *Phys. Lett. A* **270**, 288 (2000).
[67] S.N. Gninenko, *Phys. Rev. D* **91**, 015004 (2015), arXiv:1409.2288 [hep-ph].