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# Dictionary learning in Extended Dynamic Mode Decomposition using a reservoir computer

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## 1 Introduction

The classic approach to study a dynamical system described by a map  $T : X \rightarrow X$  is based on the Poincaré representation, which focuses on *orbits*  $n \mapsto T^n(\xi)$  (where  $\xi \in X$  is an initial condition). An alternative approach is to lift the system to a functional space by focusing on *observables*  $f : X \rightarrow \mathbb{C}$  which evolve through the *Koopman operator*  $\mathcal{K}f = f \circ T$ . This alternative representation captures entirely the dynamics of the system through a linear (but infinite-dimensional) operator. This framework also allows data-driven techniques such as (Extended) Dynamic Mode Decomposition (EDMD), see [1] for review.

In practice, EDMD techniques are based on a finite-dimensional representation of the operator and require to choose a finite set of *dictionary functions* [2]. This choice is crucial, but has to be made *a priori*. Recently *Dictionary learning* methods for EDMD have been proposed to provide a set of functions that yields the best representation of the Koopman operator. In particular, [2] have considered a feed-forward network for this purpose. In this work, we propose to use a reservoir computer [3, 4] instead of a feed-forward network. This allows to train the dictionary with a dynamical network rather than with a static one. This approach can be used for spectral analysis, prediction, and possibly data-driven control.

## 2 Reservoir computer

Although conceptually simple, reservoir computing is powerful enough to compete with other algorithms on hard tasks such as channel equalization and phoneme recognition, amongst others (see [5, 6] for reviews). Reservoir computing is implemented by a recurrent neural network with fixed connections called *reservoir computer*. The internal states of the network are driven by a time dependent input, which is generated by the dynamical system of interest. The reservoir provides an output which is obtained through a linear combination of the internal states of the network. The weights of this linear combination are trained to minimize the least squares error between the output and a desired target, which is the state of the dynamical system at the next time step, in our case.

## 3 Dictionary learning in EDMD using a reservoir computer

Our approach is to combine the EDMD method with a reservoir computer. In contrast to the method proposed in [2], the optimization parameters are only the output weights  $W_{out}$ . The optimization problem is written as

$$\min_{W_{out}, K} \sum_{t=1}^{\tau-1} \|W_{out}S(t+1) - KW_{out}S(t)\|^2 + \lambda \|K\|_F \quad (1)$$

where  $S(t)$  is the vector of internal states of the reservoir at time  $n$ ,  $K$  is the finite-dimensional approximation of the Koopman operator and  $\lambda$  is a Tikhonov regularizer. The norm  $\|\cdot\|_F$  is the Frobenius norm.

The optimization is solved with two alternating steps:

1. Fix  $W_{out}$  and optimize  $K$  using the least squares method ;
2. Fix  $K$  and optimize  $W_{out}$  to minimize the norm of  $W_{out}S(n+1) - KW_{out}S(n)$  which is reduced to a Sylvester equation of the form  $AX + XB = C$ .

We will report on preliminary results and illustrate the methods in the context of spectral analysis and prediction.

## References

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