Institutional Repository - Research Portal
Dépôt Institutionnel - Portail de la Recherche

researchportal.unamur.be

University of Namur

RESEARCH OUTPUTS / RÉSULTATS DE RECHERCHE

Dictionary learning in Extended Dynamic Mode Decomposition using a reservoir computer

Gulina, Marvyn; Mauroy, Alexandre

Published in:
Book of Abstracts

Publication date:
2019

Document Version
Peer reviewed version

Link to publication

Citation for pulished version (HARVARD):
Gulina, M & Mauroy, A 2019, Dictionary learning in Extended Dynamic Mode Decomposition using a reservoir computer. in P Patrinos, I Markovsky, W Michiels, J Swevers & S Waldherr (eds), *Book of Abstracts : 38th Benelux Meeting on Systems and Control.* KU Leuven, pp. 42.

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 31. Oct. 2025

Dictionary learning in Extended Dynamic Mode Decomposition using a reservoir computer

Marvyn Gulina and Alexandre Mauroy
Namur Institute for Complex Systems (naXys) and Department of Mathematics,
University of Namur, Rempart de la vierge 8, B-5000 Namur, Belgium
Email: marvyn.gulina@unamur.be and alexandre.mauroy@unamur.be

1 Introduction

The classic approach to study a dynamical system described by a map $T : X \to X$ is based on the Poincaré representation, which focuses on *orbits* $n \mapsto T^n(\xi)$ (where $\xi \in X$ is an initial condition). An alternative approach is to lift the system to a functional space by focusing on *observables* $f : X \to \mathbb{C}$ which evolve through the *Koopman operator* $\mathscr{K} f = f \circ T$. This alternative representation captures entirely the dynamics of the system through a linear (but infinite-dimensional) operator. This framework also allows data-driven techniques such as (Extended) Dynamic Mode Decomposition (EDMD), see [1] for review.

In practice, EDMD techniques are based on a finite-dimensional representation of the operator and require to choose a finite set of *dictionary functions* [2]. This choice is crucial, but has to be made *a priori*. Recently *Dictionary learning* methods for EDMD have been proposed to provide a set of functions that yields the best representation of the Koopman operator. In particular, [2] have considered a feed-forward network for this purpose. In this work, we propose to use a reservoir computer [3, 4] instead of a feed-forward network. This allows to train the dictionary with a dynamical network rather than with a static one. This approach can be used for spectral analysis, prediction, and possibly data-driven control.

2 Reservoir computer

Although conceptually simple, reservoir computing is powerful enough to compete with other algorithms on hard tasks such as channel equalization and phoneme recognition, amongst others (see [5, 6] for reviews). Reservoir computing is implemented by a recurrent neural network with fixed connections called *reservoir computer*. The internal states of the network are driven by a time dependent input, which is generated by the dynamical system of interest. The reservoir provides an output which is obtained through a linear combination of the internal states of the network. The weights of this linear combination are trained to minimize the least squares error between the output and a desired target, which is the state of the dynamical system at the next time step, in our case.

3 Dictionary learning in EDMD using a reservoir computer

Our approach is to combine the EDMD method with a reservoir computer. In contrast to the method proposed in [2], the optimization parameters are only the output weights W_{out}. The optimization problem is written as

$$\min_{W_{out},K} \sum_{t=1}^{\tau-1} \|W_{out}S(t+1) - KW_{out}S(t)\|^2 + \lambda \|K\|_F \qquad (1)$$

where $S(t)$ is the vector of internal states of the reservoir at time n, K is the finite-dimensional approximation of the Koopman operator and λ is a Tikhonov regularizer. The norm $\|\cdot\|_F$ is the Frobenius norm.

The optimization is solved with two alternating steps:

1. Fix W_{out} and optimize K using the least squares method ;
2. Fix K and optimize W_{out} to minimize the norm of $W_{out}S(n+1) - KW_{out}S(n)$ which is reduced to a Sylvester equation of the form $AX + XB = C$.

We will report on preliminary results and illustrate the methods in the context of spectral analysis and prediction.

References

[1] M. Budišić, R. Mohr, and I. Mezić, 2012. Chaos **22**, 047510 (2012).

[2] Q. Li, F. Dietrich, E. Bollt, I. Kevrekidis, 2017. Chaos **27**, 103111 (2017)

[3] H. Jaeger, GMD Report **148** (2010).

[4] P. Antonik, M. Gulina, J. Pauwels, S. Massar, 2018. Phys. Rev. E **98**, 012215 (2018)

[5] M. Lukǒsevičius and H. Jaeger, 2009. Comp. Sci. Rev. **3**, 127 (2009).

[6] M. Lukǒsevičius, H. Jaeger, and B. Schrauwen, 2012. Künst. Intell. **26**, 365 (2012).