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Risk as Impediment to Privatization? The Role of Collective Fields in Extended Agricultural Households

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I. Introduction

When workers share an output that they collectively produce, they de facto pool production risks and their individual incomes are better dampened than if they produce individually. Collective production thus plays an insurance role. The downside of output sharing is the temptation to free ride on other workers' efforts, resulting in a suboptimal effort allocation. This classic trade-off between efficiency and risk sharing has been extensively analyzed in the literature on agricultural producer cooperatives. In this context, individualization (or privatization) of the production process is always depicted as detrimental to insurance. This conclusion rests on the strong assumption that collective production is the only risk-sharing technology. In particular, it ignores voluntary interpersonal transfers, which are a major source of insurance in poor countries. In this paper we directly address this shortcoming and shed new light on the trade-off between privatization and risk sharing in a context where coworkers may engage in individual transfers in addition to sharing collective production, as is the case in extended households.

Even if the existing literature on risk and mutual insurance in poor countries focuses largely on interpersonal transfers (Platteau 2006, 854–74), a few works have nonetheless focused on the insurance role of collective property rights over productive assets or resources. On careful look, however, these works do not address the issue of risk sharing per se but are concerned with social protection

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and redistribution. For instance, informal rotation systems based on turns are aimed at ensuring equitable access to resources when sites are heterogeneous from the viewpoint of quality and fertility (Alexander 1982; Bromley and Chavas 1989; Platteau 1991, 122–35; Platteau and Seki 2001; Platteau 2006, 829–46). Or the existence of common property resources is justified by the need to guarantee their livelihoods to poor people who lack adequate private assets (Hayami 1981; Jodha 1986; McKean 1986; Agarwal 1991; Dasgupta and Maler 1993; Beck 1994; Baland and Platteau 1996; Godoy et al. 2000; Pattanayak and Sills 2001; Wunder 2001). Baland and Francois (2005) have proposed a formal analysis of the latter problem. They argue that because of the superior insurance properties of common property resources that tend to provide income maintenance in low states, any feasible scheme of private transfers under private property cannot ex ante Pareto-dominate allocations under the commons, despite the efficiency gains from privatization (when markets are incomplete). Yet because their game extends over a single period, the mechanism that they describe is just a one-shot redistribution that provides a minimum income to the poor.

The main strand of economic literature that genuinely looks at the insurance function of collective assets is concerned with agricultural producer cooperatives. Thus, Putterman and Digiorgio (1985) and Carter (1987) have analyzed the role of collective fields as a way to redistribute income from lucky to unlucky members in a multiperiod framework. This effect is achieved because collective output (or at least part of it) is distributed equally among members (Putterman 1989). If collective farming is subject to free riding, a trade-off inevitably arises between efficiency and risk-sharing considerations. Complete subdivision of cooperative land has been shown by Carter to be suboptimal, while intermediate forms that preserve some degree of risk sharing may prove superior. It bears emphasis that in the existing models, no system of private reciprocal transfers is allowed to operate side by side with collective property. Carter's conclusion that complete privatization (of cooperative land) is not optimal therefore does not come as a surprise. Note that there is a perfect analogy between the context of producer cooperatives consisting of a combination of collective and private landholdings (or a combination of collective and private activities) and the context of large extended family farms in which a common field whose output is equally shared coexists with private plots farmed by individual members (the so-called mixed farms largely prevalent in West Africa, as attested in Udry 1996; Fafchamps 2001; Guirkinger and Platteau 2015). Other authors have analyzed the connection between land allocation and risk sharing in the context of medieval Europe (McCloskey 1976; Townsend 1993) and sub-Saharan Africa (Fafchamps 1992; Platteau 2004).

In this paper, we want to consider the question as to whether collective property can survive in the presence of voluntary reciprocal transfers aimed at providing insurance. Or, to put it in the converse way, once private transfers are possible, can we expect that the risk-pooling collective activity will vanish because of the efficiency loss that it gives rise to? The answer to that question is not evident because if the collective activity is subject to incentive problems, reciprocal transfer arrangements are vulnerable to a well-known commitment problem (Kimball 1988; Coate and Ravallion 1993; Kocherlakota 1996; Ligon et al. 2002). Indeed, there is the temptation for each member of an informal risk-pooling network to break his or her promise to help a fellow participant who has suffered from a negative shock. Toward the purpose of shedding light on the above issue, we model a family farm of the mixed type. We assume that the family farm institution manages land allocation and risk sharing. We define the rate of privatization as the share of the farmland allocated to private plots. Over these plots, holders have use rights plus the right to rent them out. In accordance with Guirkinger and Platteau's (2015) analysis of the patriarchal farm, we obtain that first-best efficiency is achieved on private plots, whereas production on the collective field is plagued by free riding. Like them, we also assume that collective output is equally shared and privatized land is equally apportioned among all members (so that privatization is equitable). Unlike them, however, but similar to Carter (1987) or Baland and Francois (2005), we posit the existence of an idiosyncratic shock and risk aversion on the part of family members.[1]

In this framework, we are able to show analytically that under some conditions, further privatization of the land leads to a win-win situation. Indeed, efficiency gains may be compounded by greater insurance benefits arising from increased decentralized income transfers. This (potentially) positive effect of privatization on insurance transfers is, to our knowledge, ignored in the existing literature. It is obtained when privatization raises the value of staying in the household in such a way that even under limited commitment lucky members are willing to transfer larger amounts to unlucky members. The key point is that the insurance lost as a result of forgone collective production is outweighed by increased voluntary transfers. This situation is more likely when the family size is sufficiently large, implying that the free-riding problem is serious enough. As a consequence, complete privatization is optimal in large families. The same win-win outcome is likely to obtain if members give sufficient importance to

[1] The model allows for both idiosyncratic and covariate shocks. Obviously, insurance arrangements of the risk pooling type are effective to deal with only idiosyncratic shocks. In our model, the focus is therefore on the latter type of risk. However, a form of covariate risk is introduced in order to highlight the robustness of the results.

future income flows or if the available exit opportunities are not very attractive. Stated the other way around, our results indicate that depending on the model parameters, the efficiency-insurance trade-off may well exist and persist even when interpersonal transfers are allowed. In other words, complete privatization is not necessarily optimal, and the outcome highlighted by Carter (1987) and Baland and Francois (2005) is a distinct possibility.

The remainder of the paper is structured as follows. In Section II, after describing the set of assumptions on which the model is based, we define the risk-sharing mechanisms and specify successively the pattern of labor allocation between collective and private production and the consumption levels in each state of nature. We then come to the heart of our problem and analyze the trade-off between production efficiency and risk sharing. In Section III, we set two benchmark cases. In the first one, the trade-off is absent owing to the perfect enforceability of income transfers, while in the second one, the trade-off is unavoidable owing to the absence of income transfers. Section IV explores the intermediate case where income transfers are possible but subject to a static limited commitment problem. In this setting, we analyze the impact of privatization on expected consumption and risk sharing with incentive compatible income transfers. We highlight the conditions under which the trade-off may vanish or persist, and we carry out numerical simulations to fully characterize the effects of key parameters of the model on the optimal privatization rate. Section V discusses the case of dynamic limited commitment, which is solved in appendix D. Section VI summarizes the central lessons from the analysis.

II. The Model Setup

A. The Land Tenure Regime and the Market Environment

We consider an extended household composed of n members $i \in N$. Each household member is endowed with L units of land of homogeneous quality and one unit of productive time, provided he is not sick in the period considered; otherwise, his time endowment is zero. The household total land endowment nL is divided between collective and individual fields. Let μ denote the fraction of the aggregate family landholdings that is individually apportioned. For ease of exposition, we call μ the privatization rate.[2] Individual fields are of

[2] Notice, however, that the existence of a complete set of property rights associated with land is unnecessary for our results to hold. In particular, we do not assume that land can be sold or serve as a collateral (there is no credit market in the model). We discuss later the status of private land in the case of household dissolution, namely the issue of whether an individual could leave the household along with his individual plot. For the time being, it is sufficient to point out that these considerations do not play a critical role in our model. Strictly speaking, we only assume that the individual who receives a private plot is allowed to fully appropriate its proceeds.

homogeneous size μL, and each household member farms his plot independently. The size of the collective field is therefore $(1 - \mu)nL$, and its proceeds are equally shared among the members. Because of prohibitive supervision costs, labor on the collective field is supplied noncooperatively. Agricultural technology is identical across collective and individual parcels. The production function is defined on labor e and land l and is written as $f(e, l)$. It is subject to constant returns to scale and exhibits the usual following properties:

$$f_e > 0,$$

$$f_l > 0,$$

$$f_{ee} < 0,$$

$$f_{ll} < 0,$$

$$f_{le} = f_{el} > 0.$$

The market environment is characterized by the absence of credit and insurance markets. As already mentioned, we are agnostic about the existence of a land sale market. We nonetheless assume that land can be rented out either inside or outside the household at a rental rate equal to the profit-maximizing return per unit of land. Finally, we assume that labor allocated to agricultural work has a constant opportunity cost w per unit of time. For instance, household members may choose to undertake off-farm activities at a constant marginal return w, which implies the availability of nonagricultural income opportunities on the local labor market, temporary migration, or handicraft activities. Technically speaking, a unit time endowment has to be allocated between three activities: work on the collective field e^C, work on the private plot e^I, and off-farm work $1 - e^C - e^I$.[3]

B. *Risk Structure and Preferences*

In each growing season t, a subset $H_t \subseteq N$ of size h_t is randomly drawn within the household. Household members belonging to this subset have one unit of productive time, while the others are unable to work for the current season. This setting typically represents the case of a health shock. Risks affecting ag-

[3] The two assumptions related to the land rental and labor markets are not critical, but they improve the model readability. It could be objected that in traditional rural settings, land is sometimes lent rather than rented, with the consequence that no rental is actually paid. Yet land exchanges of this type often imply implicit rewards (e.g., in the form of future reciprocal favors). We assume that the level of the reward is set at the profit-maximizing return per land unit. The assumption regarding the labor market seems quite reasonable in contexts where labor is sufficiently mobile and responds to migration opportunities.

ricultural yields will essentially differ from this example since decisions are then taken before uncertainty is realized. In the case of illness, however, labor decisions are taken after the shock strikes and thus under certainty, which makes our model more tractable. Moreover, because we want to focus on intrahousehold risk sharing, we need to concentrate on shocks that household members can share. The risk of illness offers a particularly appropriate example because it is at least partially uncorrelated between members. Moreover, health shocks have been shown to be one of the most important sources of risk confronting rural households (Gertler and Gruber 2002; Dercon and Hoddinott 2004). We cannot deny that nonhealth shocks may also exist that are realized only after households members have made their labor allocation decisions. Common shocks (such as drought or low market prices) have this characteristic, but it may also be true of some idiosyncratic shocks insurable at the household level. We nevertheless refrain from analyzing this case because it would make the model very complex and perhaps intractable.[4] In addition, intuitive reasoning suggests that our main conclusion would not be affected; as will become evident after we have solved our model, there is no reason to believe that the main ingredients of the trade-off between efficiency and risk sharing would change.

Household members are homogeneous in the sense that ex ante they face the same probability to be healthy in any season t. More specifically, a household member's time endowment is drawn from a Bernoulli distribution:

$$P(i \in H_t) = 1 - P(i \notin H_t) = \frac{\bar{h}}{n}, \forall\ i \in N, \forall\ t,$$

where $\bar{h}/n$ is the unconditional probability of being healthy, with $\bar{h}$ denoting the expected number of healthy members in the household. Our risk structure is very general in the sense that any type of correlation between individual draws is allowed. Let us briefly mention two extreme cases, namely perfectly covariate and perfectly idiosyncratic risks. The former would result from perfectly positive correlation between individual draws, in which case everyone has the same outcome in any given period. Household members are then unable to insure one another, and uncertainty is entirely borne at the household level. Perfectly

[4] Since labor decisions would be taken under uncertainty, the maximization problem would have to be specified in expected utility terms, which would make the equilibrium expressions extremely cumbersome. Another serious difficulty arises that concerns the specification of risk itself. In a context where the rate of privatization is continuous, implying that the size of private plots is also defined continuously, we need to spell out discrete units of land to which shocks apply. This will require assumptions that will be necessarily arbitrary. Otherwise, each discrete unit of land would be divisible in collective and individual plots, and as a result, agricultural shocks would be pooled and privatization would not increase individual exposure to shocks.

idiosyncratic risk obtains if the number of healthy household members remains constant over time, in which case perfect insurance can technically be achieved if risks are pooled within the household. As these extreme cases suggest, the covariate component of uncertainty is given by the probability distribution of the number of healthy household members h_t at the household level, which we define as

$$P(h_t = h) = p_h, \forall\ t,$$

with

$$\sum_{h=0}^{n} p_h = 1; E(h_t) = \sum_{h=0}^{n} p_h h = \bar{h}, \forall\ t.$$

At this stage, we leave the probability distribution of h unspecified, thereby allowing any correlation between individual draws.[5] However, we assume that draws are independent and identically distributed over time. Let us therefore drop the subscript t from now on. We are now able to specify the probability to be healthy conditional on the number of healthy members in the household, which is simply given by

$$P(i \in H | h) = 1 - P(i \notin H | h) = \frac{h}{n}, \forall\ i \in N.$$

Given the above risk structure, the household members' per-period expected utility within the household is given by

$$V = \sum_{h=0}^{n} p_h \left[\left(1 - \frac{h}{n}\right) u(c_0(h)) + \frac{h}{n} u(c_1(h)) \right], \tag{1}$$

where $c_1(h)$ denotes a member's consumption level if he belongs to H (is healthy) and $c_0(h)$ denotes his consumption if sick, when h household members are healthy.[6] Agents are risk averse. Their attitude toward risk is captured by the shape of the utility function: $u' > 0$; $u'' < 0$. Agents are infinitely lived and discount the future. The objective function of a household member i belonging to H for the current period therefore writes

$$U_{i \in H} = u(c_{i1}(h)) + \delta V_i,$$

where $\delta = \eta/(1 - \eta)$, with $\eta \in [0, 1]$ as the discount factor.

[5] The case of independent and identically distributed draws is obtained if h follows a binomial $(\bar{h}/n, n)$ distribution. On the other hand, if $h = \bar{h}$ with probability 1, risk is perfectly idiosyncratic and correlation between draws is negative.

[6] Exit options are defined in a subsequent section.

Note that in the above definition of V, we have implicitly assumed that all members have equal Pareto weights. In this setting, the first best is characterized by two conditions: (1) the marginal productivity of farm labor is equal to its opportunity cost w everywhere; (2) the consumption levels of healthy and sick members are equal in each given stage of nature: $\forall\ h, c_0(h) = c_1(h)$. The latter point implies that members are fully protected against idiosyncratic risks.

C. The Risk-Sharing Mechanisms and the Timing of the Game

In our framework, there are three channels through which sick members may be compensated for the shock they face. First, they are able to rent out their private plot of land. The size of the income shock is therefore normalized to the value of the productive time lost. Second, the output of the collective field is shared equally. As we make clear below, this sharing rule implies a form of transfer in kind from healthy to sick agents. More precisely, the transfer takes the form of labor applied to the collective field. Third, there are direct income transfers between household members. We let the transfer depend on the state of the world at the household level h. Let τ_h^{out} and τ_h^{in} denote the amounts transferred by healthy members and received by sick members, respectively. In each period, the insurance (direct income sharing) budget constraint is

$$h\tau_h^{\text{out}} \geq (n - h)\tau_h^{\text{in}}. \tag{2}$$

We assume that the budget constraint is binding in each period: no savings or borrowing is allowed. Note that precautionary savings at the household level would allow one to insure against the covariate component of uncertainty. We do not allow for this possibility and concentrate on insurance against idiosyncratic shocks, which relies on only intraperiod transfers. The second assumption is in accordance with our assumption of absent credit markets.

If perfect risk sharing is desirable for everyone ex ante, it might be in a healthy agent's interest to renege on his income transfer ex post. The benefit of doing so is simply given by the amount he was required to transfer. With regard to cost of such a deviation, we need to define the sanctions that the household can implement in case of deviation. We assume that two sanctions are used. First, an agent who deviates is excluded from the household. To assume that the threat of exclusion is credible is an extreme assumption, but the important thing to note is that the stronger the sanction, the more efficient the risk sharing. This actually tilts the argument against collective production as a way of enforcing risk sharing. Second, a member who deviates does not receive his share of collective production. This is a reasonable assumption since, taking place afterward, such a sanction is simple and easy to implement. The following time structure follows:

1. Nature draws a subset $H \subset N$ of size h. The household members belonging to H are endowed with one unit of productive time.
2. Household members $i \in H$ choose either to stay within the family farm and to abide by the insurance agreement $(\tau_1 \ldots \tau_{n-1})$ or to leave with the output of their private parcel at the end of the growing season.
3. Household members $i \in H$ noncooperatively allocate their work effort (e^C, e^I).
4. Household members who had chosen so in stage 2 leave the household forever with the output of their private parcel. The other members consume the sum of their private output and their share of collective output adjusted for the transfers they make or receive.

A series of points deserves a brief discussion here. First, there is no relevant distinction between being excluded and leaving the household voluntarily. Second, even if the household member leaves at the end of the season, the decision to leave is taken beforehand so that it can have an impact on the agent's optimal allocation of labor. Indeed, anticipating that he will not receive his share of the collective output, an agent who leaves should not take part in collective production. Third, in order to write the incentive compatibility condition on transfers, we need to define the agents' exit option—that is, their per-period reservation utility outside the household—which we call $\bar{V}$. The latter critically depends on prevailing rules of access to land. Three options are available here; (1) the departing member leaves without land, which happens if the departing member migrates or if land is under the corporate ownership of the family; (2) the departing member leaves with his total land endowment L; (3) the departing member leaves with his private plot μL. The latter possibility implies the existence of complete and well-defined property rights, which are not assumed in our model. Moreover, it will become clearer below that by generating a trade-off between production efficiency and risk sharing, case 3 would naturally tilt the argument against privatization. This is because privatization improves the household members' exit option, worsens the commitment issue, and hence impedes income sharing. Case 3 nevertheless remains relevant, and we therefore briefly explore it at the end of the paper. For the main analysis, we assume that $\bar{V}$ is exogenously given, which encompasses cases 1 and 2. In the next subsection, we specify labor allocation patterns and the implied consumption levels that will incorporate the three risk-sharing mechanisms available.

D. Labor Allocation

In this model, risk takes the form of a random shock that is supposed to affect the agents' ability to work. It follows that the agents' decision regarding the allocation of their time takes place once uncertainty is realized and is therefore unaffected by it. Healthy household members simply maximize their current consumption level, which writes

$$c_1 = \frac{1}{n} y^{\mathrm{C}} + y^{\mathrm{I}} + w(1 - e^{\mathrm{C}} - e^{\mathrm{I}}) - \tau_h^{\mathrm{out}}, \tag{3}$$

where y^{C} and y^{I} stand for agricultural production on the collective field and on the member's private plot, respectively. Throughout the text, the superscripts C and I will refer to activities on the collective and individual lands, respectively. According to our assumptions, y^{C} and y^{I} are given by

$$y^{\mathrm{C}} = f(E, (1 - \mu)nL), \quad \text{where } E \equiv \sum_{i \in H} e_i^{\mathrm{C}},$$

$$y^{\mathrm{I}} = f(e^{\mathrm{I}}, \mu L).$$

Equation (3) states that a healthy member's consumption is composed of a fraction $1/n$ of collective production, his entire private production, and off-farm income of w per residual unit of time (there are no savings). Besides, any healthy agent gives an income transfer τ_h^{out}. Recall that labor allocation is not contractible within the household and is hence chosen noncooperatively. Healthy agents maximize income (eq. [3]) with respect to the allocation of their work effort (e^{C}, e^{I}). It follows that in any Nash equilibrium, the allocation of time between the three activities is then given by the following arbitrage condition:[7]

$$\frac{1}{n} f_e^{\mathrm{C}} = f_e^{\mathrm{I}} = w. \tag{4}$$

As equation (4) illustrates, production on the collective field is plagued by free riding. As expected, given the equal sharing rule, labor is underprovided since its marginal productivity is n times higher than its opportunity cost w. Production on the collective field is therefore inefficient. In order to simplify ensuing notation and to compare the rent generated on the collective field to the rent obtained on private fields, the following lemma is useful.

LEMMA 1. Under constant returns to scale, if labor is applied so that its marginal productivity is equal to a constant k, total rent is proportional to the cultivated land area l; if $\tilde{e}(l, k)$ is such that $f_e(\tilde{e}, l) = k$, then

[7] The labor allocation must satisfy the constraint that $e^{\mathrm{C}} + e^{\mathrm{I}} \leq 1$. In the analysis, we concentrate on interior solutions where this constraint is satisfied. More precisely, we know only that at any interior solution, aggregate provision of labor is given by $1/n f_e(E^*, (1 - \mu)nL) = w$. There is a continuum of Nash equilibria satisfying this condition but with different distributions of effort among healthy household members. For simplicity, we assume that the symmetric equilibrium, in which $e_i^{\mathrm{C}*} = E^*/h, \forall\, i \in H$, is selected.

$$R(l,k) = f(\tilde{e}(l,k),l) - w\tilde{e}(l,k) \propto l$$
$$\Leftrightarrow R(l,k) = lR(1,k).$$

Proof. Provided in appendix A.

E. Consumption Levels

It follows from lemma 1 that the rent per unit of land area is constant. Let R^* and R^C denote the rent per unit of land on private plots and the collective field, respectively. Therefore, R^* and R^C are given by

$$R^* = R(1,w) = \frac{f(e^I, \mu L) - we^I}{\mu L},$$
$$R^C = R(1,nw) = \frac{f(E,(1-\mu)nL) - wE}{(1-\mu)nL},$$

where e^I and E are such that equation (4) is satisfied. Equation (4) implies that $R^C < R^*$. The rent on private land is indeed maximized with respect to labor application. Notice that because of a dilution effect, $\partial R^C/\partial n < 0$. Put differently, incentives to work worsen following an increase in the household size because the fraction of output that workers can appropriate is thereby reduced.

Let us now write the two consumption levels:

$$c_1 = \frac{1}{n}y^C + y^I + w(1 - e_i^C - e_i^I) - \frac{n-h}{h}\tau_h, \tag{5}$$

$$c_0 = \frac{1}{n}y^C + \tau_h + \mu L R^*. \tag{6}$$

The three types of transfers received by sick household members clearly appear in equation (6): (1) they receive a share of collective production, (2) they benefit from a pure income transfer τ_h, and (3) they are able to rent out their private plot at a rate R^*. Using the above expressions for R^* and R^C, we obtain the following expressions:

$$c_1 = L[(1-\mu)R^C + \mu R^*] + w - \frac{n-h}{h}\left(\frac{wE}{n} + \tau_h\right), \tag{7}$$

$$c_0 = L[(1-\mu)R^C + \mu R^*] + \left(\frac{wE}{n} + \tau_h\right). \tag{8}$$

The consumption levels are determined by the income generated by the two factors owned by members, namely land and labor. On the one hand, the rent associated to the land endowment is a weighted average of the collective and optimal rent levels, where weights are determined by the privatization rate. On the other hand, the value of labor is simply given by its opportunity cost w. By comparing equations (7) and (8), it can be seen that the extent of the shock faced by sick agents is precisely w. This is intuitive since what they lose is the value of their productive time. Finally, we can highlight the two transfers from the h healthy to the $n-h$ sick agents: τ_h is the pure income transfer received, while wE/n is a transfer in kind. The latter corresponds to the value of labor devoted to the collective field per household member.

III. The Trade-Off between Production Efficiency and Risk Sharing: Two Benchmark Cases

A. Full Commitment

We are now set to analyze the impact of privatization on production efficiency. This is the aim of the following lemma.

LEMMA 2. The expected aggregate household income is increasing in privatization:

$$\frac{\partial}{\partial \mu} \sum_{h=0}^{n} p_h \frac{Y(h)}{n} > 0.$$

Proof. In the absence of savings, aggregate household income is simply given by aggregate consumption, which writes

$$Y(h) = hc_1 + (n-h)c_0 = hw + nL[\mu R^* + (1-\mu)R^C], \quad (9)$$

where use has been made of equations (7) and (8). Recalling that h is a random variable, the derivative of expected aggregate income is then

$$\sum_{h=0}^{n} p_h \frac{\partial Y(h)}{\partial \mu} = (R^* - R^C)nL > 0.$$

This result is straightforward. For each unit of land withdrawn from the collective field and reallocated to private parcels, the rent increases by the difference between R^* and R^C. Privatization strengthens the incentives to work, thereby improving labor allocation. In the absence of economies of scale, the usefulness of collective production can come from only insurance considerations. However, if pure income transfers are perfectly enforceable, they should be preferred to collective production as a risk-coping device. This result appears in the next proposition, which establishes the first-best arrangement, understood as the optimal rule governing production and risk sharing within the household.

PROPOSITION 1: *Full commitment*. Under noncooperative labor allocation, the optimal institution in terms of land tenure and income transfers $(\mu^{FC}, \tau_1^{FC} \ldots \tau_{n-1}^{FC})$ is characterized by

1. Complete privatization: $\mu^{FC} = 1$.
2. Perfect insurance against idiosyncratic risk: τ_h^{FC} are such that $c_1 = c_0$, $\forall\ h \in \{0, \ldots n\}$. More precisely, $\tau_h^{FC} = wh/n$.

Proof. Maximizing expected utility (eq. [1]) with respect to any given transfer τ_h and using equations (7) and (8), the first-order condition is as follows:

$$\frac{\partial V}{\partial \tau_h} = p_h\left(1 - \frac{h}{n}\right)[u'(c_0(h)) - u'(c_1(h))] = 0$$

$$\Leftrightarrow c_1 = c_0 \Leftrightarrow \tau_h^*(\mu) = \frac{w}{n}(h - E^*).$$

Then, for each h and for a given privatization rate, there is an optimal income transfer $\tau_h^*(\mu)$ that equalizes the two consumption levels. Substituting for this optimal insurance scheme $(\tau_1^* \ldots \tau_{n-1}^*)$ in the objective function gives

$$V = \sum_{h=0}^{n} p_h u\left(\frac{Y(h)}{n}\right),$$

where the household's aggregate income $Y(h)$ is given by equation (9). Since, according to lemma 2, the aggregate household income Y increases with privatization, V immediately appears to be itself increasing in privatization:

$$\frac{\partial V}{\partial \mu} = \sum_{h=0}^{n} p_h u'\left(\frac{Y(h)}{n}\right)\frac{1}{n}\frac{\partial Y(h)}{\partial \mu} > 0.$$

The privatization rate is therefore at a corner $\mu^{FC} = 1$. Finally, $\tau_h^{FC} = \tau_h^*(1) = wh/n$, since no effort is spent on the collective field ($E = 0$).

Proposition 1 implies that if perfect risk sharing through income transfers is feasible, land should be completely privatized to maximize household income. To be more precise, the full commitment equilibrium is characterized by the combination of (1) a labor allocation that maximizes aggregate income and (2) equal levels of consumption between household members. In other words, no trade-off arises between efficiency and risk sharing. Two additional points are worth mentioning. First, as intuition would suggest, the optimal insurance transfer received, τ_h^{FC}, is increasing in the number of healthy members and in the value of labor w (since the shock is then higher). Second, while risk sharing through transfers offers a perfect coverage against idiosyncratic shocks, the household remains obviously uninsured against fluctuations of its aggregate income $Y(h)$.

B. No Commitment

The aim of this subsection is to highlight that, absent income transfers, a trade-off between production efficiency and risk sharing automatically arises. Our setting thus reproduces the result of Carter (1987) as a particular case.

Let κ_h denote the transfer from healthy to sick household members, which in this case consists of the value of labor on the collective field only:

$$\kappa_h = \frac{n-h}{h}\frac{wE}{n}.$$

Insurance being provided through the distribution of the collective production only, privatization will necessarily result in a reduction of the transfer: $\partial\kappa_h/\partial\mu < 0, \forall\ h$. This is because if the size of the collective parcel is reduced, the equilibrium level of collective labor also decreases: $\partial E/\partial\mu < 0$. We can then derive the following proposition.

PROPOSITION 2: *Impact of privatization on expected consumption and risk sharing in the absence of income transfers* ($\tau_h = 0, \forall\ h$).

1. Privatization increases the expected consumption of household members:

$$\frac{\partial}{\partial\mu}\sum_{h=0}^{n} p_h \frac{Y(h)}{n} > 0.$$

2. Privatization increases the gap between the consumption levels of healthy and sick household members in each state of the world:

$$\frac{\partial}{\partial\mu}(c_1 - c_0) \geq 0, \forall\ h.$$

3. Consequently, there is a trade-off between production efficiency and risk sharing.

Proof. In the absence of savings, aggregate household income is simply given by aggregate consumption, which writes

$$Y(h) = hc_1 + (n-h)c_0 = hw + nL[\mu R^* + (1-\mu)R^{C}], \quad (10)$$

where use has been made of equations (7) and (8). Recalling that h is a random variable, the derivative of expected aggregate income is then

$$\frac{\partial E_h(Y)}{\partial\mu} = \sum_{h=0}^{n} p_h \frac{\partial Y(h)}{\partial\mu} = (R^* - R^{C})nL > 0.$$

The first point directly follows.

Concerning the second point, from equations (8) and (7), it is evident that $\tau_h = 0$ implies

$$c_1 - c_0 = w\left(1 - \frac{E}{h}\right), \tag{11}$$

which is increasing in μ as $\partial E/\partial\mu < 0$.

Finally, the trade-off results from the preceding points. The first point simply states that privatization increases farm efficiency by strengthening the incentives to work and improving labor allocation. For each unit of land withdrawn from the collective field and reallocated to private parcels, the rent increases by the difference between R^* and R^C. Point 2 then establishes that in the absence of income transfers, privatization necessarily comes at the expense of risk sharing because the gap between the consumption of the sick and the healthy increases with privatization (eq. [11]). A trade-off therefore arises between risk and expected consumption.

Naturally, if pure income transfers would exist and were perfectly enforceable, they would be preferred to collective production as a risk-coping device: complete privatization would lead to the first best. What happens in the intermediate case where income transfers exist but are constrained by limited commitment? Section IV.B takes up this question.

IV. The Trade-Off between Production Efficiency and Risk Sharing: Static Limited Commitment

A. Income Transfers under Static Limited Commitment

We now write the incentive compatibility condition on transfers ($\tau_1 \ldots \tau_{n-1}$). In stage 2 of the game, for a given state of the world h, healthy household members have to decide whether to stay within the household. It will be optimal for them to pay their transfer and to stay provided that

$$u(c_d) + \delta\bar{V} \leq u(c_1) + \delta V,$$

where c_d is their current consumption level if they deviate. The definition of c_d is given by

$$c_d = w + \mu LR^*. \tag{12}$$

This consumption level is achieved if the departing member allocates his labor force optimally between farm activities on his private land and off-farm activities. He therefore reneges on the two types of transfers, namely the pure income transfer and the labor contribution to the collective field. The second-best vector of income transfers ($\tau_1 \ldots \tau_{n-1}$) must achieve the highest possible level of risk sharing while satisfying the following incentive compatibility conditions:

$$u(c_{\mathrm{d}}) - u(c_1(\tau_h)) - \delta(V - \bar{V}) \leq 0, \forall\ h \in \{0, \ldots n\}. \tag{13}$$

It is noteworthy that neither c_{d} (see eq. [12]) nor V depends on a particular state of the world h. Indeed, V is the expected utility over all possible realizations of h. It follows that the incentive compatibility constraint simply imposes a minimum consumption level $\tilde{c}_1$, such that condition (13) holds with equality. In other words, to deter deviation, the household should at least provide healthy members with a consumption of $\tilde{c}_1$. This in turn imposes a maximum amount of monetary transfer. We define κ as the highest incentive compatible amount of transfers in both income and labor and $\tilde{\tau}_h$ as the maximal incentive compatible amount of pure income transfer:

$$\kappa = \frac{n-h}{h}\left(\frac{wE}{n} + \tilde{\tau}_h\right) \Leftrightarrow \tilde{\tau}_h = \frac{h}{n-h}\kappa - \frac{wE}{n}, \tag{14}$$

such that

$$u(c_{\mathrm{d}}) - u(c_1(\kappa)) - \delta(V - \bar{V}) = 0. \tag{15}$$

Thus, κ measures the aggregate willingness to transfer of a healthy household member. We are now set to characterize the second-best vector of incentive compatible transfers.

PROPOSITION 3: *Income transfers under limited commitment*. The second-best vector of income transfers $(\tau_1^{\mathrm{SB}} \ldots \tau_{n-1}^{\mathrm{SB}})$ is such that

$$\begin{aligned}\tau_h^{\mathrm{SB}} = \min\{\tilde{\tau}_h, \tau_h^*\} &= \tilde{\tau}_h \text{ if } h < \tilde{h}, \\ &= \tau_h^* \text{ otherwise,}\end{aligned}$$

where $\tilde{\tau}_h$, τ_h^*, and $\tilde{h}$ are given by

$$\tau_h^* = \frac{w}{n}(h - E), \tag{16}$$

$$\tilde{\tau}_h = \frac{h}{n-h}\kappa - \frac{wE}{n},$$

$$\tilde{h} = \left(1 - \frac{\kappa}{w}\right)n. \tag{17}$$

Proof. The incentive compatibility condition on income transfers will be binding if the number of healthy household members is lower than $\tilde{h} =$

$(1 - \kappa/w)n$; otherwise, perfect insurance will be incentive compatible. In other words, the optimal income transfer will be incentive compatible provided that

$$\tau_h^* < \tilde{\tau}_h \Leftrightarrow h > \tilde{h}.$$

As a result, the function $\tau_h^{SB}(h)$ described above defines the highest level of transfers that do not violate the incentive compatibility constraint.[8]

This proposition states that there are two types of states of the world: states where the incentive compatibility condition is binding and risk sharing is incomplete and states where the condition is not binding and risk sharing is therefore complete. In this model, complete risk sharing simply amounts to having the income transfer equal to τ_h^*, which is such that the consumption levels are equalized across healthy and sick agents ($c_1 = c_0$, where c_1 and c_0 are given by eqq. [7] and [8]) The result is very intuitive. Indeed, when there are few healthy workers, insurance needs are important and cannot be met by the agents' willingness to transfer, which is constant across states. On the contrary, when h is high, the required transfer is reduced and is therefore more likely to be incentive compatible.

It follows that the level of risk sharing achieved within the household is positively correlated with its aggregate income. When there are few workers, this income is lower and risk sharing is incomplete. Household members are then more vulnerable to idiosyncratic shocks when the shock is important at the household level or, equivalently, when more people are hit at the same time. This result is fully compatible with previous work on risk sharing (Coate and Ravallion 1993). Risk sharing is clearly impeded by limited commitment, especially if the covariate shock is important.

It is now useful to write the expected utility of household members that is obtained by incorporating the second-best vector of income transfers:

$$V(\tau_h^{SB}) = \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \left[\left(1 - \frac{h}{n}\right) u(c_0) + \frac{h}{n} u(c_1) \right] + \sum_{\lfloor \tilde{h} \rfloor + 1}^{n} p_h u\left(\frac{Y}{n}\right), \quad (18)$$

where

$$c_0 = L[(1-\mu)R^C + \mu R^*] + \frac{h}{n-h}\kappa,$$
$$c_1 = L[(1-\mu)R^C + \mu R^*] + w - \kappa.$$

[8] Note that $\tilde{h}$ being a continuous function, the last constrained state (where limited commitment is binding) is given by $\lfloor \tilde{h} \rfloor$, i.e., the entire part of $\tilde{h}$. For the same reason, the first unconstrained state is $\lfloor \tilde{h} \rfloor + 1$.

Bear in mind that the willingness to transfer of the contributing members (κ) is itself a function of expected utility V (see its definition in eq. [15]). A higher expected utility within the household indeed provides an incentive to transfer. It follows that equation (18) does not give an explicit expression of expected utility. What we have is an implicit definition of the objective function $V(\mu)$, written as follows:

$$\Psi(\mu, V) = \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \left[\left(1 - \frac{h}{n}\right) u(c_0) + \frac{h}{n} u(c_1) \right] + \sum_{\lfloor \tilde{h} \rfloor + 1}^{n} p_h u\left(\frac{Y}{n}\right) - V = 0.$$

In this configuration, the first-order condition with respect to the privatization rate μ is given by $\Psi_\mu = 0$, where Ψ_μ is the partial derivative of Ψ with respect to μ (details provided in app. B). The term V is therefore taken as a constant in computing the following derivatives. Moreover, the sign of the marginal utility of privatization $dV/d\mu$ is given by the sign of Ψ_μ. This is because in

$$\frac{dV}{d\mu} = -\frac{\partial\Psi/\partial\mu}{\partial\Psi/\partial V} = -\frac{\Psi_\mu}{\Psi_V},$$

it can be shown that $\Psi_V < 0$ (for a technical discussion, see app. B).

The household is assumed to implement the second-best vector of income transfers and to select the privatization rate that maximizes expected utility. In the following, we explore the implications of this optimization program.

B. *Analytical Results*

We now analyze the most interesting case of limited commitment. In other words, we want to determine the impact of privatization on household production and risk sharing when incentive compatible income transfers are allowed.

LEMMA 3. With incentive compatible income transfers, the (constrained) optimal privatization rate should satisfy the following Kuhn-Tucker conditions:

$$\begin{aligned} \Psi_\mu &= \frac{\partial V}{\partial\mu} + \frac{\partial V}{\partial\kappa}\frac{\partial\kappa}{\partial\mu} = 0 \text{ and } \mu^* < 1, \\ &= \frac{\partial V}{\partial\mu} + \frac{\partial V}{\partial\kappa}\frac{\partial\kappa}{\partial\mu} > 0 \text{ and } \mu^* = 1. \end{aligned} \tag{19}$$

PROPOSITION 4: *Impact of privatization on expected consumption and risk sharing with incentive compatible income transfers.*

1. Privatization increases the expected consumption level of household members:

$$\frac{\partial}{\partial \mu} \sum_{h=0}^{n} p_h \frac{Y(h)}{n} > 0.$$

Holding constant the willingness to transfer of contributing members (κ), the direct effect of privatization on expected utility is positive:

$$\frac{\partial V}{\partial \mu} = E(u')(R^* - R^C)L > 0.$$

2. In equilibrium, the impact of privatization on risk sharing is indeterminate: κ, the willingness to transfer of a healthy household member, might either decrease or increase with privatization:

$$\frac{\partial \kappa}{\partial \mu} < 0 \Leftrightarrow \frac{u'(c_d)}{u'(c_1)} > 1 - \frac{R^C}{R^*},$$

where κ and c_d are given by equations (15) and (12), respectively.

Proof. The proof of the first point of proposition 2 applies to point 1 here.

Regarding the second point, recall that

$$\begin{aligned} c_1 &= w + L[(1 - \mu)R^C + \mu R^*] - \kappa, \\ c_d &= w + \mu L R^*. \end{aligned}$$

Applying the implicit function theorem to equation (15) and holding V constant (see app. B), we find

$$\frac{\partial \kappa}{\partial \mu} = -\frac{u'(c_d)}{u'(c_1)} L R^* + (R^* - R^C)L,$$

so that

$$\frac{\partial \kappa}{\partial \mu} < 0 \Leftrightarrow \frac{u'(c_d)}{u'(c_1)} > 1 - \frac{R^C}{R^*}. \tag{20}$$

The first part of proposition 4 states that under limited commitment, holding the level of transfers (κ) constant, privatization continues to have a positive impact on household production. Privatization therefore increases expected utility: $\partial V / \partial \mu > 0$. This term is the first term that appears in the first-order condition (eq. [19]). The second term $\partial V / \partial \kappa$ is positive. Indeed, when income sharing is incomplete, an increase in the amount transferred by healthy household members reduces the difference between the consumption levels of healthy and sick members, thereby reducing the extent of idiosyncratic risk. The third term, namely $\partial\kappa/\partial\mu$, precisely represents the impact of privatization on risk sharing. As highlighted in the second point of proposition 4, this effect is ambiguous under limited commitment. In other words, we cannot rule out the possibility that an increase in privatization raises the level of risk sharing. This is why, in lemma 3, we allow for corner solutions with respect to the optimal privatization rate μ^* (the whole family landholding is distributed in the form of private parcels).

A positive impact of privatization on risk sharing is not readily understandable. Indeed, the mechanical effect by which privatization reduces the size of the collective production remains. Put differently, the transfer in kind—measured by the labor productivity of healthy members on the collective field—is unambiguously smaller. However, income transfers may increase so much as to offset this effect. The potential increase in income transfers would be caused by the change in the incentives to transfer. To understand this effect, let us analyze condition (20) in detail.

To begin with, notice that the left-hand side is lower than 1 since $c_\mathrm{d} > c_1$. Recall that κ is determined by the incentive compatibility condition (15), which we reproduce here:

$$u(c_\mathrm{d}) - u(c_1(\kappa)) = \delta(V - \bar{V}).$$

In the light of this condition, one can immediately infer that $c_\mathrm{d} > c_1 \Leftrightarrow V > \bar{V}$, which must be true for the ex ante participation constraint to be satisfied. In other words, since we allow household members to leave the household once they are informed about their type (healthy or sick), we must also allow them to leave ex ante, so that we must have $V > \bar{V}$. A member should be better off inside the household ex ante. Since exclusion implies a future sanction, in the current period, insurance transfers can drive c_1 below c_d so that a departing member would consume more outside the household in the current period. Looking back at condition (20), it can be seen that $\partial\kappa/\partial\mu$ is more likely to be negative if c_d is close to c_1, that is, if V is close to $\bar{V}$. Loosely speaking, income transfers are more difficult to enforce if the exit option is too high. In this instance, privatization reduces risk sharing, implying that the role of collective production as an insurance mechanism becomes more important. Besides, the incentive compatibility condition also tells us that c_d is closer to c_1 if δ is low. Therefore, privatization reduces risk sharing when the degree of the members' patience is low. The reasoning is exactly the same as before. The threat of exclusion is stronger if agents are patient, which helps to enforce the income transfers.

Let us now focus on the effect of n on inequality (20). Because of the incentive dilution effect, household size increases the wedge between the private benefit of collective production (proportional to $1/n$) and the private cost and therefore decreases the rent on the collective field R^{C} (R^{*} is independent of n). A decrease in R^{C} tightens inequality (20) because it increases the right-hand side and decreases the left-hand side. The latter effect is driven by the decrease in c_1, which decreases the ratio $u'(c_d)/u'(c_1)$. The intuition is straightforward: in large households, collective production entails important efficiency losses, which pushes toward privatization. Stated differently, collective production is less costly as an insurance device in small households where privatization is then more likely to reduce the extent of risk sharing.

In conclusion, the analysis of the partial effect of privatization on the incentives to transfer reveals that a household is more likely to maintain some form of collective production (μ^{*} is more likely to be interior) if (1) its size n is small, (2) the household members' exit option $\bar{V}$ is high, or (3) the discount factor δ is low (agents are impatient). To fully characterize the impact of these parameters on the optimal privatization rate, we need to take into account additional partial effects and specify functional forms. In the discussion below, we assume a constant absolute risk aversion utility function. In appendix C, we present the complete comparative statics for $\bar{V}$ and δ. The proposition below summarizes the results obtained.

PROPOSITION 5: *Patience, exit opportunities, and the optimal privatization rate*. Under constant absolute risk aversion, the optimal privatization rate (1) decreases with $\bar{V}$ and (2) increases with δ.

Proof. Provided in appendix C.

To provide some intuition about this result, let us examine the partial effects of $\bar{V}$ on the first-order condition of the problem (the same analysis applies to δ). Using an envelope argument, appendix C establishes that

$$\operatorname{sign}\left(\frac{d\mu}{d\bar{V}}\right) = \operatorname{sign}\left(\frac{\partial^2 V}{\partial\mu\partial\kappa}\frac{\partial\kappa}{\partial\bar{V}} + \frac{\partial^2 V}{\partial\kappa^2}\frac{\partial\kappa}{\partial\mu}\frac{\partial\kappa}{\partial\bar{V}} + \frac{\partial V}{\partial\kappa}\frac{\partial^2\kappa}{\partial\mu\partial\bar{V}}\right).$$

The last term of this sum ($\partial^2\kappa/\partial\mu\partial\bar{V}$) captures the partial effect on incentive compatible transfers analyzed above, which is negative since higher exit opportunities tighten the incentive compatibility constraint: $\partial^2\kappa/\partial\mu\partial\bar{V} < 0$ (note that $\partial V/\partial\kappa > 0$, insurance transfers increase the expected utility of current consumption, everything else held constant). The first two terms are second-order effects related to the concavity of the utility function. The first one pertains to the impact of κ on the efficiency gain induced by privatization. The term $\partial^2 V/\partial\mu\partial\kappa$ is negative since decreasing marginal utility implies that better off individuals are less sensitive to an increase in expected consumption. It is multiplied by the negative effect of $\bar{V}$ on incentive compatible transfers, so that overall this first partial effect goes in the opposite direction to the last term. Conversely, the second partial effect reinforces the last one because for an interior solution, $\partial\kappa/\partial\mu < 0$ (see eq. [20]). This second partial effect captures the increasing cost of lower transfers when marginal utility is decreasing. We have now signed each term separately:

$$\overbrace{\frac{\partial^2 V}{\partial\mu\partial\kappa}}^{<0}\overbrace{\frac{\partial\kappa}{\partial\bar{V}}}^{<0}+\overbrace{\frac{\partial^2 V}{\partial\kappa^2}}^{<0}\overbrace{\frac{\partial\kappa}{\partial\mu}}^{<0}\overbrace{\frac{\partial\kappa}{\partial\bar{V}}}^{<0}+\overbrace{\frac{\partial V}{\partial\kappa}}^{>0}\overbrace{\frac{\partial^2\kappa}{\partial\mu\partial\bar{V}}}^{<0}.$$

In the case of constant absolute risk aversion, it is easy to show that the first term is dominated by the last ones, so that increasing exit opportunities reduces privatization. In the case of decreasing absolute risk aversion, we cannot unambiguously sign the expression, but the simulation results presented in the next section show that the negative impact of $\bar{V}$ on privatization continues to be obtained with a decreasing absolute risk aversion utility function.

C. *Simulation Results*

Proposition 4 explores the mechanics underlying the effects of privatization out of equilibrium. It shows that while privatization systematically improves efficiency, it may either decrease or increase the scope for risk sharing. Figures 1 and 2 illus-

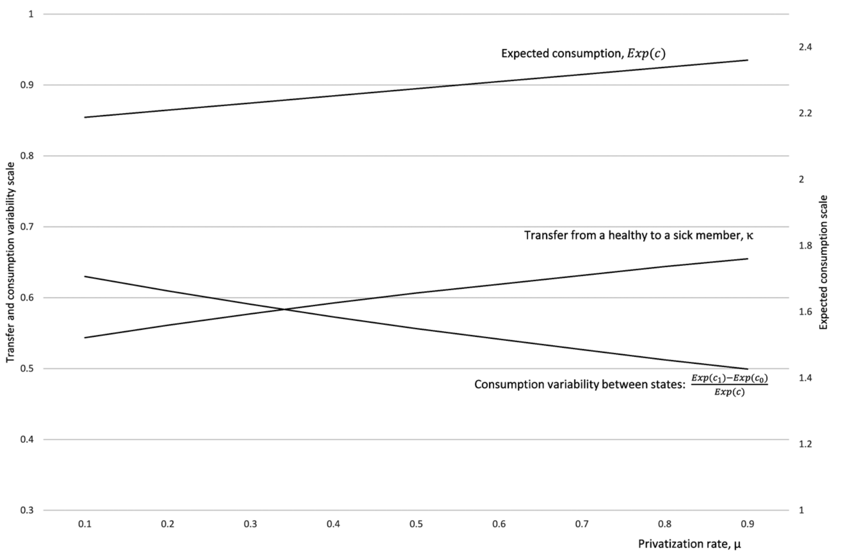

Figure 1. Efficiency and risk sharing as a function of privatization: the win-win case.

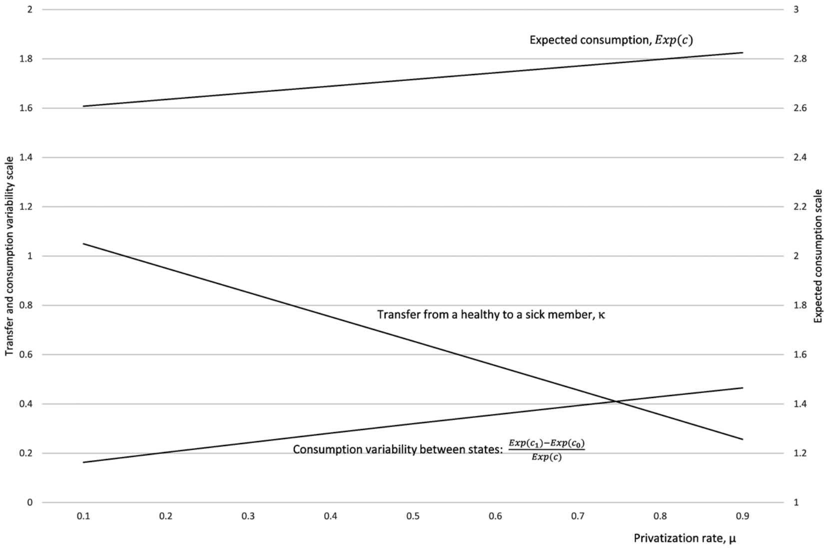

Figure 2. Efficiency and risk sharing as a function of privatization: trade-off between efficiency and risk sharing.

trate these possibilities with the following functional forms. The utility is assumed logarithmic, $u(x) = \ln(x)$, so that it exhibits decreasing absolute risk aversion and unitary constant relative risk aversion. The production function is a constant return to scale Cobb-Douglas function, $f(e, l) = e^{\beta} l^{1-\beta}$. Finally, we take a binomial distribution of h, $h \sim B(\bar{h}/n, n)$, and thus assume that shocks are independently and identically distributed.[9] Figure 1 presents a case where there is no trade-off between efficiency and risk sharing in the privatization process. Both expected consumption and transfers from healthy to sick (κ) are monotonically increasing in the rate of privatization (μ). As a result, the variability of expected consumption across states is decreasing in the rate of privatization. Full privatization would unambiguously obtain in this case.[10] In contrast, figure 2 depicts a classic trade-off between efficiency and risk sharing: whereas expected consumption increases with μ, the transfer from healthy individuals decreases and consumption variability increases.[11]

Turning to comparative static results, proposition 5 establishes the role of key parameters (discount rate and exit option) on the optimal rate of privatiza-

[9] For each privatization rate, κ is found by grid search using the incentive compatibility constraint.
[10] The following parameters are used to generate fig. 1: $\delta = 0.7$, $L = 5$, $w = 3$, $\bar{h}/n = 0.6$, $n = 3$, $\bar{V} = 0.5$.
[11] The following parameters are used to generate fig. 2: $\delta = 0.8$, $L = 9.78$, $w = 2.5$, $\bar{h}/n = 0.595$, $n = 2$, $\bar{V} = 0.94$. With these parameters, further simulation reveals that the optimal privatization rate is 0.80.

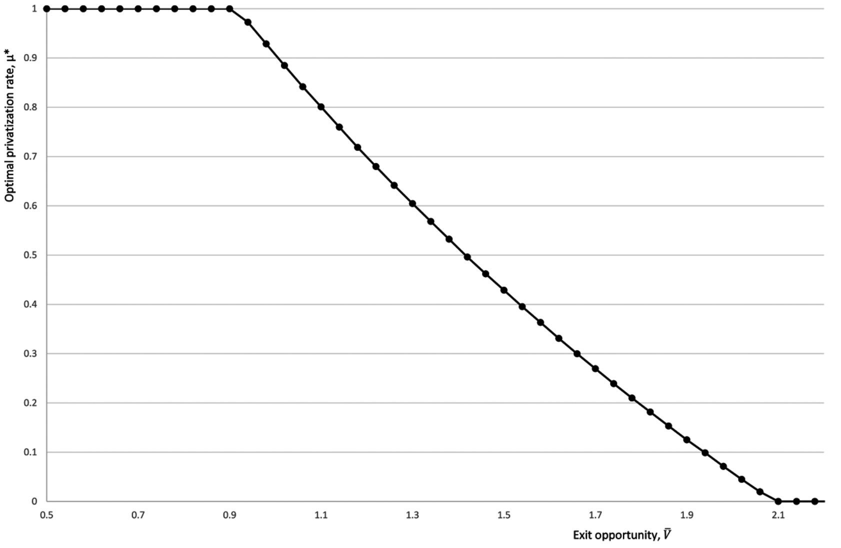

Figure 3. Exit opportunity and optimal privatization rate.

tion in the case of constant absolute risk aversion. In the following simulations, we relax this assumption and confirm that the result holds with a decreasing absolute risk aversion utility function. Figures 3 and 4 plot the optimal privatization rate for a range of values for $\bar{V}$ and δ.[12] The figures clearly show that an increase in the value of the exit option decreases the optimal privatization rate, while an increase in patience has the opposite effect. In fact, if members exhibit very little patience or have good exit opportunities, the optimal allocation is a purely collective farm.

D. Remark

The issue of a precise definition of the exit option $\bar{V}$ in terms of land ownership has been discussed earlier. Let us briefly come back to this question in light of our results, which have been derived for the case of exogenous $\bar{V}$. The implication of this assumption in terms of land ownership is that the departing household member leaves either with his total land endowment L or without land, since in these two instances the exit option is independent of the privatization rate. One could imagine that the holder of what we call a private parcel actually enjoys complete property rights over it, in which case the departing member would leave with a parcel of size μL. Under this alternative assump-

[12] The following parameters are used to generate fig. 3: $L = 10$, $w = 2.5$, $\bar{h}/n = 0.6$, $n = 2$, $\delta = 0.2$. For fig. 4, the parameters are $L = 9.78$, $w = 3.5$, $\bar{h}/n = 0.595$, $n = 2$, $\bar{V} = 0.88$.

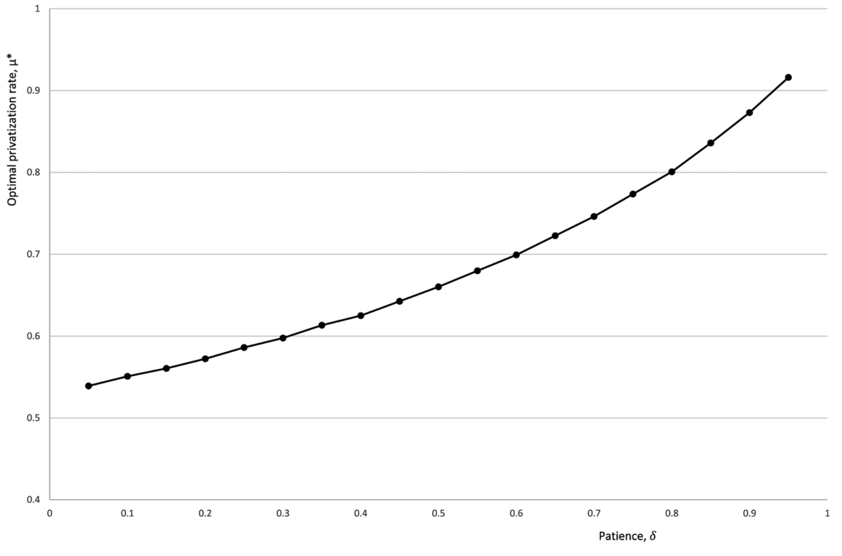

Figure 4. Patience and optimal privatization rate.

tion, we would have a positive effect of privatization on the exit option: $\partial \bar{V}/\partial\mu > 0$. It is easy to see that this situation would be detrimental to risk sharing since it would negatively affect the willingness to transfer, κ. The trade-off between production efficiency and risk sharing would then be more likely to arise.

V. Dynamic Limited Commitment

In the static limited commitment model, history does not matter in the sense that there is no real dynamic incentive at work within the household. While it is true that in the repeated model that we have used, future payoffs keep the household together, the history of the shocks that affect each and every member of the household does not influence relative positions in the risk sharing arrangement. In the static model, the amounts of current transfers depend only on how many members are healthy in the current period. By contrast, in the canonical dynamic limited commitment model of Ligon et al. (2002), current transfers evolve as a function of past shocks and may even differ between healthy members. Adopting such a dynamic framework brings more flexibility in the risk-sharing arrangement and probably better reflects the way households behave in reality. In particular, the recipient of a transfer may promise a future compensation to the donor, who would then be incited to transfer more during the current period. In other words, the incentive compatibility constraint is relaxed, and risk sharing is consequently improved. It is therefore important to assess the impact of dynamic incentives on the results that we have obtained with the static limited commitment model.

Given the complexity of the analytics, we refrain from presenting here an adapted version to our problem of the dynamic model of Ligon et al. (2002). The interested reader will find it in appendix D. A key point is that the terms of the trade-off between efficiency and risk sharing remain essentially unchanged: the same three effects are at play. On one side, privatization improves production efficiency. On the other side, it generates two opposite effects on incentives to share risk. The positive effect is related to the efficiency effect: the incentive compatibility constraint is relaxed because of the higher future value of staying within the household. As for the negative effect, it results from the better immediate exit option available to healthy members. Now, because dynamic limited commitment allows for more risk sharing through income transfers, the insurance role of the collective field is correspondingly reduced. As a consequence, the optimal rate of privatization of the household farm is expected to be higher.

The question at the heart of this paper is whether the trade-off between efficiency and risk sharing in a mixed farm (where collective and private plots coexist) may disappear in the presence of voluntary income transfers. We have shown in Section IV that the answer is positive. This conclusion is reinforced when dynamic incentives are allowed for. This is not surprising because, looked at from the standpoint of the incentive compatibility constraint, the case of dynamic limited commitment is in-between the full and the static limited commitment cases.

VI. Conclusion

When a risk-pooling collective mechanism is available side by side with a private activity providing high efficiency but no insurance, we expect an efficiency-insurance trade-off to exist. Removal of the collective mechanism in the context of incomplete (insurance and credit) markets would therefore be suboptimal, as has been illustrated in the case of agricultural producer cooperatives.

If private transfers are feasible inside the household, it is not clear whether this trade-off subsists and therefore what the implication is in terms of the desirability of full privatization. Indeed, to the extent that risk can be shouldered through voluntary reciprocal transfers, we cannot rule out the possibility that further privatization will enhance both insurance and efficiency, thus creating a win-win outcome. This paper has precisely shown that, indeed, the trade-off between efficiency and insurance can disappear when agents are allowed to make income transfers. Complete privatization may thus become optimal when some conditions are satisfied. Among these conditions is a sufficiently large size of the social unit managing the collective mechanism (in our particular setup, the family farm) so that free-riding impedes collective production enough, or the low attractiveness of exit options available to household members.

In reality, since the above two conditions favorable to land division and privatization may not coexist—reduced mortality leads to larger households, but growing market integration raises the level of exit options—the empirical prediction following from our model seems to be strongly ambiguous. However, market integration may involve other effects than just creating new outside income opportunities. In particular, it typically gives rise to new patterns of demand for agricultural products, and the new higher value-added products (e.g., vegetables, fruits) often require a shift to more care-intensive production techniques. Because the use of such techniques is especially vulnerable to the free-riding problem (it results in so-called management diseconomies of scale), we should expect collective production to become even less efficient when farm output mixes are tilted in favor of the new products demanded by urban consumers.

The central lesson from our theoretical foray is the following: it cannot be assumed that collective production is justified as soon as insurance markets are incomplete and agents are risk averse. When private transfers are possible, the efficiency-insurance trade-off is no more certain to exist, and land tenure individualization might bring both efficiency and insurance benefits. Conversely, it is not because intrahousehold private transfers are possible that complete division of a family landholding among members is necessarily optimal.

Appendix A

A1. Proof of Lemma 1

Suppose $\tilde{e}(l, k)$ is such that $f_e(\tilde{e}, l) = k$. Under this allocation rule, the rent generated on a field of size l is then given by

$$R(l, k) = f(\tilde{e}(l, k), l) - w\tilde{e}(l, k).$$

We will proceed in two steps. We will show that under constant returns to scale, $\tilde{e}(l, k)$ is proportional to l and that, consequently, $f(\tilde{e}(l, k), l))$ is also proportional to l.

First step. $\tilde{e}(l, k) \propto l$. First, under constant returns to scale, we have

$$f(e, l) = lf\left(\frac{e}{l}, 1\right)$$

$$\Rightarrow f_e(e, l) = lf_e\left(\frac{e}{l}, 1\right)\frac{1}{l} = f_e\left(\frac{e}{l}, 1\right).$$

It follows that

$$f_e(\tilde{e}, l) = k \Rightarrow f_e\left(\frac{\tilde{e}}{l}, 1\right) = k \Leftrightarrow \frac{\tilde{e}}{l} = \psi(k).$$

This tells us that $\tilde{e}/l$ is given by a function $\psi(k)$, which depends on k only. Hence, $\tilde{e}(l, k) = l\psi(k)$ and is thus proportional to l.

Second step. The preceding step allows us to write $\tilde{e}(l, k) = l\tilde{e}(1, k)$. Combining this relationship with the property of constant returns to scale,

$$f(\tilde{e}(l, k), l) = lf_l(\tilde{e}(1, k), 1).$$

It follows that

$$\begin{aligned} R(l, k) &= f(\tilde{e}(l, k), l) - w\tilde{e}(l, k) = l[f_l(\tilde{e}(1, k), 1) - w\tilde{e}(1, k)] \\ &= lR(1, k)\,. \end{aligned}$$

Appendix B

B1. First-Order Condition with Respect to Privatization under Limited Commitment

As we mention in the text, because κ appears in both c_0 and c_1 and because κ is a function of V (see eq. [15]), the function to maximize $V(\mu)$ is implicitly given by

$$\Psi(\mu, V) = \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h\left[\left(1 - \frac{h}{n}\right)u(c_0) + \frac{h}{n}u(c_1)\right] + \sum_{\lfloor \tilde{h} \rfloor + 1}^{n} p_h u\left(\frac{Y}{n}\right) - V = 0. \tag{B1}$$

Making use of the implicit function theorem, the first-order condition with respect to the privatization rate is then as follows:

$$\frac{dV}{d\mu} = -\frac{\Psi_\mu}{\Psi_V} = 0 \Leftrightarrow \Psi_\mu = 0.$$

Therefore, the first-order condition simply requires that $\Psi_\mu = 0$. At this stage, one may realize that by an application of the implicit function theorem, the numerator Ψ_μ corresponds to the partial derivative of Ψ with respect to μ while maintaining V constant. This is why V is held constant in the analysis of the first-order condition given by equation (19).

For the purpose of making comparative statics, however, we need to check that Ψ_μ has the same sign as $dV/d\mu$. This will be the case if and only if $\Psi_V < 0$, which is shown below. The idea is to show that even if the objective is implicitly defined, the numerator Ψ_μ contains the usual information in the sense that it has the sign of the expected utility gain from a marginal increase in the argument, namely the privatization rate μ.

$$\begin{aligned}\Psi_V &= \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \left[\left(1 - \frac{h}{n}\right) u'(c_0) \frac{\partial c_0}{\partial \kappa} + \frac{h}{n} u'(c_1) \frac{\partial c_1}{\partial \kappa} \right] \frac{\partial \kappa}{\partial V} - 1 \\ &= \delta \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} \left(\frac{u'(c_0)}{u'(c_1)} - 1 \right) - 1,\end{aligned}$$

where use has been made of the following relationships:

$$\frac{\partial c_0}{\partial \kappa} = \frac{h}{n-h}; \frac{\partial c_1}{\partial \kappa} = -1; \frac{\partial \kappa}{\partial V} = \frac{\delta}{u_1'}.$$

The latter equation comes from an application of the implicit function theorem on equation (15). Define[13]

$$f(V) = \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \left[\left(1 - \frac{h}{n}\right) u(c_0) + \frac{h}{n} u(c_1) \right] + \sum_{\lfloor \tilde{h} \rfloor + 1}^{n} p_h u\left(\frac{Y}{n}\right).$$

What we want to obtain is

$$\Psi_V < 0 \Leftrightarrow \delta \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} \left(\frac{u'(c_0)}{u'(c_1)} - 1 \right) = f'(V) < 1.$$

The implicit definition of the objective (21) simply becomes $f(V) = V$. Graphically, V is then located at the intersection between $f(V)$ and the 45° line. We now show that the shape of $f(.)$ implies that at this point $f'(V) < 1$, which is precisely what we want to demonstrate. Notice that $f(.)$ is strictly concave:

$$f''(V) = \delta \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} \frac{u_0'' \frac{h}{n-h} u_1' + u_0' u_1''}{u_1'^2} \frac{\partial \kappa}{\partial V} < 0.$$

[13] Recall that V is among the determinants of κ.

It follows that $f(V)$ and V intersect at most twice and that V will be the highest at the second intersection, where $f'(V) < 1$. Since at this second intersection point the utility V is necessarily higher than at the first one, it will be selected by the household. This completes the proof that $\Psi_V < 0$, implying that $dV/d\mu$ has the same sign as Ψ_μ.

Appendix C

C1. Proof of Proposition 5

In this appendix, we provide the developments of the comparative statics exercise. According to appendix B, the marginal impact of privatization on per-period expected utility has the following form:

$$\frac{dV}{d\mu} = -\frac{\Psi_\mu}{\Psi_V},$$

where the function Ψ is defined by equation (B1). One can then make use of the implicit function theorem to find that the marginal impact of some exogenous parameter θ on an interior solution for the privatization rate μ^* has the sign of

$$\frac{d^2V}{d\theta d\mu} = -\frac{\Psi_{\theta\mu}\Psi_V - \Psi_\mu\Psi_{\theta V}}{\Psi_V^2}.$$

Since the first-order condition implies that $\Psi_\mu = 0$, the sign of $d\mu^*/d\theta$ is simply given by the sign of $-\Psi_{\theta\mu}\Psi_V$. As we have shown in appendix B, $\Psi_V < 0$. As a consequence, we need to determine only the sign of $\Psi_{\theta\mu}$. In this appendix, our attention is restricted to a couple of parameters $(\bar{V}, \delta)$, which alter the incentives to transfer and hence the value of κ but have nothing to do with production efficiency. Put differently, $\bar{V}$ and δ only affect Ψ_μ indirectly through κ. We can then use the chain rule and write the derivative of expression (19) with respect to $\theta \in \{\bar{V}, \delta\}$ as[14]

[14] The term $\partial^2\kappa/\partial\kappa\partial\mu$ may be confusing because κ appears in the denominator. Recall that the function κ $(\mu; \bar{V}, \delta)$ is implicitly defined (see eq. [15]). The implicit function theorem allows us to calculate $\partial\kappa/\partial\mu$. However, κ still appears in the latter expression through c_1. Actually, one would desire to have $\partial\kappa/\partial\mu$ as a function of μ, $\bar{V}$, and δ only, which is impossible precisely because of the implicit definition of κ. While we calculate $\partial^2\kappa/\partial\theta\partial\mu$ $(\theta \in \{\bar{V}, \delta\})$, we then need to take their indirect impact into account. Applying the chain rule, we have indeed that $\partial^2\kappa/\partial\theta\partial\mu = (\partial^2\kappa/\partial\kappa\partial\mu)(d\kappa/d\theta)$.

$$\Psi_{\theta\mu} = \left(\frac{\partial^2 V}{\partial\kappa\partial\mu} + \frac{\partial^2 V}{\partial\kappa^2}\frac{\partial\kappa}{\partial\mu} + \frac{\partial V}{\partial\kappa}\frac{\partial^2\kappa}{\partial\kappa\partial\mu}\right)\frac{d\kappa}{d\theta}.$$

It follows that $d\mu^*/d\theta$ has the sign of $d\kappa/d\theta$ if and only if

$$\Lambda = \frac{\partial^2 V}{\partial\kappa\partial\mu} + \frac{\partial^2 V}{\partial\kappa^2}\frac{\partial\kappa}{\partial\mu} + \frac{\partial V}{\partial\kappa}\frac{\partial^2\kappa}{\partial\kappa\partial\mu} > 0.$$

This is generally true, as we demonstrate below. This case is the most intuitive. Indeed, this tells us that if for some exogenous reason the contributing members' willingness to transfer is reduced, then the privatization rate should decrease to allow the household to insure its members through a higher collective production. In other words, privatization increases with the equilibrium value of its members' willingness to transfer κ. This is intuitive because, in this case, the household can substitute income transfers to collective production, which is costly in terms of production efficiency, to achieve a certain level of risk sharing. Let us analyze the sign of Λ. Afterward, we will turn to the analysis of $d\kappa/d\theta$, $\theta \in \{\bar{V}, \delta\}$.

Bearing in mind that the function V is given by equation (18), the first term of Λ is[15]

$$\frac{\partial^2 V}{\partial\kappa\partial\mu} = (R^* - R^{\mathrm{C}})L\sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n}\left[u''(c_0) - u''(c_1)\right].$$

The other useful elements are

$$\frac{\partial V}{\partial\kappa} = \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n}[u'(c_0) - u'(c_1)],$$

$$\frac{\partial^2 V}{\partial\kappa^2} = \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n}\left[u''(c_0)\frac{h}{n-h} + u''(c_1)\right],$$

$$\frac{\partial\kappa}{\partial\mu} = -\frac{u'(c_{\mathrm{d}})}{u'(c_1)}LR^* + (R^* - R^{\mathrm{C}})L,$$

$$\frac{\partial^2\kappa}{\partial\kappa\partial\mu} = -\frac{u'(c_{\mathrm{d}})}{u'(c_1)}\frac{u''(c_1)}{u'(c_1)}LR^*.$$

[15] See $\partial V/\partial\mu$ as given in proposition 4.

Assembling those terms, we end up with

$$\begin{aligned}
&\operatorname{sign}\{\Lambda\}\\
&= \operatorname{sign}\left[(R^* - R^{\mathrm{C}})L\sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n-h} u_0'' - \frac{u_{\mathrm{d}}'}{u_1'} LR^* \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} u_0' \left(\frac{u_0''}{u_0'}\frac{h}{n-h} + \frac{u_1''}{u_1'}\right)\right]\\
&= \operatorname{sign}\left\{ - \underbrace{\left[(R^* - R^{\mathrm{C}})L - \frac{u_{\mathrm{d}}'}{u_1'} LR^*\right]}_{=\frac{\partial\kappa}{\partial\mu}<0} \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n-h} u_0' \eta(c_0) \right.\\
&\qquad \left. - \frac{u_{\mathrm{d}}'}{u_1'} LR^* \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} u_0' (\eta(c_0) - \eta(c_1)) \right\},
\end{aligned}$$

where $\eta(c) = -u''(c)/u'(c)$ is the coefficient of absolute risk aversion. Therefore, under constant absolute risk aversion,

$$\operatorname{sign}\{\Lambda\} = \operatorname{sign}\left(- \frac{\partial\kappa}{\partial\mu} \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n-h} u_0' \eta(c_0) \right),$$

and $\Lambda > 0$. Indeed, in light of the first-order condition with respect to the privatization rate (eq. [19]), one may realize that at any interior solution, $\partial\kappa/\partial\mu < 0$. Alternatively, under decreasing absolute risk aversion, $\eta(c_0) - \eta(c_1) > 0$, and we have an effect in the opposite direction. In our simulation results, we have tested a large set of parameters values and have precisely assumed that agents' preferences were characterized by decreasing absolute risk aversion. The expression $\Lambda > 0$ appears to be a robust result because we never encountered a negative sign for Λ in any of the tested parameters combinations.

We now turn to the analysis of $d\kappa/d\theta$, $\theta \in \{\bar{V}, \delta\}$. It is worth reproducing here the implicit definition of κ:

$$\chi = u(c_{\mathrm{d}}) - u(c_1(\kappa)) - \delta(V^*(\bar{V}, \delta) - \bar{V}) = 0, \tag{C1}$$

where $V^*(\bar{V}, \delta)$ is indirect expected utility, that is, V evaluated at the optimal privatization rate and for equilibrium behaviors.[16] Equation (C1) therefore gives us a function of the form $\kappa(\bar{V}, \delta, V^*(\bar{V}, \delta))$. We thus have

$$\frac{d\kappa}{d\bar{V}} = \frac{\partial\kappa}{\partial\bar{V}} + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\bar{V}},$$
$$\frac{d\kappa}{d\delta} = \frac{\partial\kappa}{\partial\delta} + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\delta},$$

where

$$\frac{\partial V^*}{\partial\bar{V}} = \frac{\partial V^*}{\partial\kappa}\frac{\partial\kappa}{\partial\bar{V}},$$
$$\frac{\partial V^*}{\partial\delta} = \frac{\partial V^*}{\partial\kappa}\frac{\partial\kappa}{\partial\delta}.$$

Indeed, V^* becomes affected by $\theta \in \{\bar{V}, \delta\}$ only through their impact on incentives to transfer and hence on κ. Substituting and rearranging, we obtain

$$\frac{d\kappa}{d\bar{V}} = \frac{\partial\kappa}{\partial\bar{V}}\left(1 + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\kappa}\right),$$
$$\frac{d\kappa}{d\delta} = \frac{\partial\kappa}{\partial\delta}\left(1 + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\kappa}\right).$$

Finally, applying the implicit function theorem on equation (C1), the partial derivatives are given by

$$\frac{\partial\kappa}{\partial V^*} = -\frac{\partial\kappa}{\partial\bar{V}} = \frac{\delta}{u_1'} > 0,$$
$$\frac{\partial\kappa}{\partial\delta} = \frac{V^* - \bar{V}}{u_1'} > 0.$$

[16] While calculating the first-order condition with respect to μ, we were allowed to treat V as a constant by an application of the implicit function theorem (see app. B). Here, however, we cannot do that anymore as we are precisely taking the derivative of the first-order condition. The equilibrium value of V indeed contains the parameters of interest. Nevertheless, their indirect impact through μ^* can be neglected by an envelope argument.

It follows that the total derivative is equal to the partial derivative multiplied by a coefficient

$$\left(1 + \frac{\partial \kappa}{\partial V^*}\frac{\partial V^*}{\partial \kappa}\right) > 1.$$

Indeed, recall that $\partial V^*/\partial \kappa > 0$. The reason is that risk sharing is incomplete in constrained states of the world. Therefore, an increase in the aggregate transfer κ increases expected utility. This multiplicative coefficient is due to a feedback effect by which a partial effect of a parameter θ on κ becomes reinforced by its effect on indirect utility V^*, which itself affects κ.

We then conclude that

$$\frac{d\kappa}{d\bar{V}} < 0; \frac{\partial \kappa}{\partial \delta} > 0.$$

Combining this with the fact that $\Lambda > 0$ leads the result.

Appendix D

D1. The Dynamic Limited Commitment Model

In this appendix, we develop the dynamic limited commitment version of our model. Because this framework is more general and encompasses the static limited commitment model, this requires us to adapt the notation. In particular, in the dynamic limited commitment model, a state of nature is defined not only by the number of healthy (vs. sick) household members but also by the identity of the sick and healthy members. Let us define $s_{it} \in \{0, 1\}$ as the health shock hitting agent i at time t. The expression $s_t = (s_{1t}, \ldots, s_{nt})' \in \Sigma = \{0, 1\}^n$ is the vector of shocks at time t and characterizes the state of nature. Moreover, p_s is the probability of drawing a particular state $s \in \Sigma = \{0, 1\}^n$.[17] For simplicity, we assume that s_t is independent and identically distributed over time. The history of states at the beginning of period t is captured by

$$S_t = \{s_{t-1}, \ldots, s_0\}.$$

Let $V_{i,t}(S_t)$ denote expected utility at time t, given history S_t, before the state of nature s_t is realized. We will refer to $V_{i,t}(S_t)$ as ex ante utility:

$$V_{i,t}(S_t) = \sum_{s\in\Sigma} p_s[u_i(c_{i,t}(s, S_t)) + \delta V_{i,t+1}(s, S_t)].$$

[17] Notice that the probability distribution is more general that the one used in the static version. Formally, $p_h = \Sigma_{s\in S_h} p_s$, where $s \in S_h$ are such that $\Sigma_{i\in N} s_i = h$.

Also, we define $U_{i,t}(s, S_t)$ as ex post utility, that is, the level of expected utility at time t after the realization of s_t:

$$U_{i,t}(s, S_t) = u_i(c_{i,t}(s, S_t)) + \delta V_{i,t+1}(s, S_t).$$

Hence

$$V_{i,t}(S_t) = \sum_{s\in\Sigma} p_s U_{i,t}(s, S_t).$$

Finally, in each period, any agent i has an ex post exit option of $\bar{U} = u(c_{\mathrm{d}}(\mu)) + \delta\bar{V}$, where

$$c_{\mathrm{d}}(\mu) = \mu L R^* + w.$$

In order to determine the second-best risk sharing and land tenure arrangements, we need to solve the following dynamic programming problem: the arrangement should maximize the ex ante utility of agent n in any period t, which depends on his/her own consumption in the current period $c_{i,t}$ and on the level of promised utilities to the other household members $V_{1,t+1}(s, S_t), \dots,$ $V_{n-1,t+1}(s, S_t)$.[18] In the absence of storage, the resource constraint imposes that agent n's own consumption is given by the difference between the household current aggregate production $Y(s; \mu)$ and the consumption of all other household members, where

$$Y(s; \mu) = nL[(1 - \mu)R^{\mathrm{C}} + \mu R^*] + h(s)w,$$

with $h(s) = \Sigma_{i\in N} s_i$. As in the static model, we assume that the land tenure arrangement (namely the privatization rate μ) is fixed over time. Concerning the risk-sharing arrangement, it is allowed to evolve over time because of the dynamic nature of the model. More precisely, it consists of a level of consumption in the current period $c_{i,t}$ and of a level of promised utility $V_{i,t+1}$ for each possible state $s \in \Sigma$, given a history S_t, for all agents with the exception of agent n.

Therefore, the optimization program writes

$$\max_{\mu,((c_{i,t}(s,S_t),V_{i,t+1}(s,S_t))_{s\in\Sigma})_{i\in N\setminus\{n\}}} V_{n,t},$$

where

$$V_{n,t} = \sum_{s\in\Sigma} p_s \left[u_n\left(Y(s; \mu) - \sum_{i=1}^{n-1} c_{i,t}(s, S_t) \right) + \delta V_{n,t+1}(V_{1,t+1}(s, S_t), \dots, V_{n-1,t+1}(s, S_t); \mu) \right],$$

18 The arrangement is second best, given that labor allocation decisions are noncooperatively made.

subject to a series of constraints:

$$\lambda_{i,t}(S_t): \sum_{s\in\Sigma} p_s[u_i(c_{i,t}(s, S_t)) + \delta V_{i,t+1}(s, S_t)] \geq V_{i,t}(S_t), \quad \forall\ i \in N\setminus\{n\}, \tag{D1}$$

$$p_s\phi_{i,t}(s, S_t): u_i(c_{i,t}(s, S_t)) + \delta V_{i,t+1}(s, S_t) \geq u_i(c_{\mathrm{d}}(\mu)) + \delta\bar{V}, \forall\ s \in \Sigma, \forall\ i \in N\setminus\{n\}, \tag{D2}$$

$$p_s\phi_{n,t}(s, S_t): u_n\left(Y(s;\mu) - \sum_{i=1}^{n-1} c_{i,t}(s, S_t)\right) + \delta V_{n,t+1}(V_{1,t+1}(s, S_t), \ldots, V_{n-1,t+1}(s, S_t);\mu) \geq u_n(c_{\mathrm{d}}(\mu)) + \delta\bar{V}, \forall\ h \in S. \tag{D3}$$

Equation (D1) is the promise-keeping constraint, with Lagrange multiplier $\lambda_{i,t}(S_t)$ (notice that $\lambda_{i,t}(S_t)$ can be interpreted as a Pareto weight on agent i at time t). This constraint states that at time t, the risk-sharing arrangement must give to each agent a level of ex ante utility, which must be (at least) equal to the level that was agreed on in $t-1$. Under limited commitment, the risk-sharing and land tenure arrangements must be such that at any point in time, an agent is willing to stay within the household and to abide by the agreement. In particular, equation (D2) is the incentive compatibility condition for income transfers. This constraint captures the incentives that an agent might have to renege on his/her promise to make an income transfer to others when required by the risk-sharing arrangement. Moreover, $\phi_{i,t}(s, S_t)$ is the Lagrange multiplier on this constraint, and p_s is the probability of being faced with the decision of staying or leaving in state of nature s. Equation (D3) is the incentive compatibility condition of agent n. This constraint must be written separately, given that it depends on the following choice variables $((c_{i,t}(s, S_t), V_{i,t+1}(s, S_t))_{s\in\Sigma})_{i\in N\setminus\{n\}}$.

Let us now derive the first-order conditions of this problem. Let $L_t(S_t)$ denote the Lagrangian of the maximization problem.

D1.1. First-Order Condition with Respect to Consumption

The first-order condition with respect to the consumption level of agent i in state s writes

$$\frac{\partial L_t(S_t)}{\partial c_{i,t}(s, S_t)} = p_s\,[-(1 + \phi_{n,t}(s, S_t))u_n'(c_{n,t}(s, S_t; \mu)) + (\lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t))u_i'(c_{i,t}(s, S_t))] = 0$$

$$\Leftrightarrow \frac{u_n'(c_{n,t}(s, S_t; \mu))}{u_i'(c_{i,t}(s, S_t))} = \frac{\lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t)}{1 + \phi_{n,t}(s, S_t)}.$$

Similarly for agent j,

$$\frac{u_n'(c_{n,t}(s, S_t; \mu))}{u_j'(c_{j,t}(s, S_t))} = \frac{\lambda_{j,t}(S_t) + \phi_{j,t}(s, S_t)}{1 + \phi_{n,t}(s, S_t)}.$$

Combining the first-order conditions with respect to the consumption levels of any pair of agents $\{i, j\}$ in a given state s, we obtain

$$\frac{u_j'(c_{j,t}(s, S_t))}{u_i'(c_{i,t}(s, S_t))} = \frac{\lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t)}{\lambda_{j,t}(S_t) + \phi_{j,t}(s, S_t)}, \forall\ i, j \in N. \qquad \text{(D4)}$$

D1.2. First-Order Condition with Respect to Promised Utilities

The first-order condition with respect to the level of expected utility promised to agent i in $t + 1$ after state s writes

$$\frac{\partial L_t(S_t)}{\partial V_{i,t+1}(s, S_t)} = p_s\delta\left[(1 + \phi_{n,t}(s, S_t))\frac{\partial V_{n,t+1}(s, S_t)}{\partial V_{i,t+1}(s, S_t)} + \lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t)\right] = 0$$

$$\Leftrightarrow -\frac{\partial V_{n,t+1}(s, S_t)}{\partial V_{i,t+1}(s, S_t)} = \frac{\lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t)}{1 + \phi_{n,t}(s, S_t)},$$

where

$$-\frac{\partial V_{n,t+1}(s, S_t)}{\partial V_{i,t+1}(s, S_t)} = \lambda_{i,t+1}(s, S_t),$$

by the definition of the Lagrange multiplier. Therefore,

$$\lambda_{i,t+1}(S_{t+1}) = \frac{\lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t)}{1 + \phi_{n,t}(s, S_t)}.$$

Similarly for agent j,

$$\lambda_{j,t+1}(S_{t+1}) = \frac{\lambda_{j,t}(S_t) + \phi_{j,t}(s, S_t)}{1 + \phi_{n,t}(s, S_t)}.$$

Combining both expressions gives

$$\frac{\lambda_{i,t+1}(S_{t+1})}{\lambda_{j,t+1}(S_{t+1})} = \frac{\lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t)}{\lambda_{j,t}(S_t) + \phi_{j,t}(s, S_t)}, \forall\ i, j \in N. \tag{D5}$$

Making use of equations (D4) and (D5), we obtain

$$\frac{\lambda_{i,t+1}(S_{t+1})}{\lambda_{j,t+1}(S_{t+1})} = \frac{\lambda_{i,t}(S_t) + \phi_{i,t}(s, S_t)}{\lambda_{j,t}(S_t) + \phi_{j,t}(s, S_t)} = \frac{u_j'(c_{j,t}(s, S_t))}{u_i'(c_{i,t}(s, S_t))}, \forall\ i, j \in N.$$

These relationships capture the dynamics of the risk-sharing arrangement. They can be interpreted as follows. Considering two household members, the first equality tells us that their relative Pareto weights are constant over time, unless the incentive compatibility condition is binding for at least one of them. As an illustration, suppose that it is binding for i and not for j; that is, $\phi_{i,t} > 0$ and $\phi_{j,t} = 0$. In this case, λ_i/λ_j increases, which reflects the fact that the relative position of i in the arrangement improves. In other words, in period t, agent i has probably faced a relatively favorable income draw, which makes him reluctant to share income with other unlucky agents. The optimal risk-sharing arrangement will relax (partly, since the constraint remains binding in equilibrium) his/her incentive compatibility condition by granting him/her a future reward. The second equality states that relative marginal utilities are determined by the initial ratio of Pareto weights, which is itself determined by the history of shocks, unless a constraint binds. Notice that insurance is considered complete as soon as the ratio of marginal utilities is constant over time. Limited commitment is, however, a source of imperfection. As expected, insurance is incomplete when an incentive compatibility condition is binding in equilibrium.

D1.3. First-Order Condition with Respect to Land Tenure

Let us now see how the second-best land tenure arrangement is determined jointly with the risk-sharing arrangement studied above. The first-order condition with respect to the privatization rate μ is as follows:

$$\begin{aligned}\frac{\partial L_t(S_t;\mu)}{\partial \mu} &= \sum_{s\in\Sigma} p_s(1 + \phi_{n,t}(s, S_t))\left[u_n'(c_{n,t}(s, S_t))\frac{\partial Y(s;\mu)}{\partial \mu} + \delta\frac{\partial V_{n,t+1}}{\partial \mu}\right] \\ &\quad + \sum_{i=1}^{n}\sum_{s\in\Sigma} p_s\phi_{i,t}(s, S_t)\left[-u_i'(c_{\mathrm{d}}(\mu))\frac{\partial c_{\mathrm{d}}(\mu)}{\partial \mu}\right],\end{aligned} \tag{D6}$$

where

$$\frac{\partial Y(s;\mu)}{\partial\mu} = nL(R^{*} - R^{\mathrm{C}}),$$

$$\frac{\partial c_{\mathrm{d}}(\mu)}{\partial\mu} = LR^{*}. \tag{D7}$$

The term $\partial V_{n,t+1}/\partial\mu$ can be found in the following way: First note that, by definition,

$$V_{n,t+1} = \sum_{s_{t+1}\in\Sigma} p_s\Bigg[u_n\Bigg(Y(s;\mu) - \sum_{i=1}^{n-1} c_{i,t+1}(s,S_{t+1})\Bigg)$$

$$+ \delta V_{n,t+2}(V_{1,t+2}(s,S_{t+1}),\ldots,V_{n-1,t+2}(s,S_{t+1});\mu)\Bigg].$$

Taking the derivative of this expression with respect to μ gives

$$\frac{\partial V_{n,t+1}}{\partial\mu} = \sum_{s_{t+1}\in\Sigma} p_s\left[u_n'(c_{n,t+1}(s,S_{t+1}))\frac{\partial Y(s;\mu)}{\partial\mu} + \delta\frac{\partial V_{n,t+2}}{\partial\mu}\right],$$

where

$$\frac{\partial V_{n,t+2}}{\partial\mu} = \sum_{s_{t+2}\in\Sigma} p_s\left[u_n'(c_{n,t+2}(s,S_{t+2}))\frac{\partial Y(s;\mu)}{\partial\mu} + \delta\frac{\partial V_{n,t+3}}{\partial\mu}\right].$$

Substituting and applying the same procedure recursively, we end up with

$$\frac{\partial V_{n,t+1}}{\partial\mu} = \frac{\partial Y(s;\mu)}{\partial\mu}EU'(S_{t+1}), \tag{D8}$$

where

$$EU'(S_{t+1}) \equiv E_{s_{t+1}}u_n'(c_{n,t+1}(s_{t+1},S_{t+1})) + \delta E_{s_{t+1}}E_{s_{t+2}}u_n'(c_{n,t+2}(s_{t+2},S_{t+2}))$$

$$+ \delta^2 E_{s_{t+1}}E_{s_{t+2}}E_{s_{t+3}}u_n'(c_{n,t+3}(s_{t+3},S_{t+3})) + \ldots \tag{D9}$$

Notice that by equation (D7), $\partial Y(s;\mu)/\partial\mu$ does not differ from one state s to another.

Substituting equation (D8) into equation (D6) gives

$$\frac{\partial L_t(S_t;\mu)}{\partial\mu} = \frac{\partial Y(s;\mu)}{\partial\mu}\sum_{s\in\Sigma}p_s(1+\phi_{n,t}(s_t,S_t))[u_n'(c_{n,t}(s_t,S_t)) + \delta EU'(S_{t+1})]$$
$$- \frac{\partial c_{\mathrm{d}}(\mu)}{\partial\mu}\sum_{i=1}^{n}\sum_{s\in\Sigma}p_s\phi_{i,t}(s,S_t)u_i'(c_{\mathrm{d}}(\mu)).$$

Making use of the fact that

$$EU'(S_t) = \sum_{s\in\Sigma}p_s[u_n'(c_{n,t}(s_t,S_t)) + \delta EU'(S_{t+1})], \tag{D10}$$

we can write

$$\frac{\partial L_t(S_t;\mu)}{\partial\mu} = \frac{\partial Y(s;\mu)}{\partial\mu}EU'(S_t)$$
$$+ \frac{\partial Y(s;\mu)}{\partial\mu}\sum_{s\in\Sigma}p_s\phi_{n,t}(s_t,S_t)[u_n'(c_{n,t}(s_t,S_t)) + \delta EU'(S_{t+1})]$$
$$- \frac{\partial c_{\mathrm{d}}(\mu)}{\partial\mu}\sum_{i=1}^{n}\sum_{s\in\Sigma}p_s\phi_{i,t}(s,S_t)u_i'(c_{\mathrm{d}}(\mu)).$$

Rearranging gives

$$\frac{\partial L_t(S_t;\mu)}{\partial\mu} = \frac{\partial Y(s;\mu)}{\partial\mu}EU'(S_t)$$
$$+\sum_{s\in\Sigma}p_s\phi_{n,t}(s_t,S_t)\left[(u_n'(c_{n,t}(s_t,S_t)) + \delta EU'(S_{t+1}))\frac{\partial Y(s;\mu)}{\partial\mu} - u_n'(c_{\mathrm{d}}(\mu))\frac{\partial c_{\mathrm{d}}(\mu)}{\partial\mu}\right] \tag{D11}$$
$$- \frac{\partial c_{\mathrm{d}}(\mu)}{\partial\mu}\sum_{i=1}^{n-1}\sum_{s\in\Sigma}p_s\phi_{i,t}(s,S_t)u_i'(c_{\mathrm{d}}(\mu)),$$

where

$$\frac{\partial Y(s;\mu)}{\partial\mu} = nL(R^* - R^{\mathrm{C}}) > 0,$$
$$\frac{\partial c_{\mathrm{d}}(\mu)}{\partial\mu} = LR^* > 0.$$

A careful inspection of expression (D11) allows us to retrieve the three distinct effects of privatization on risk sharing that we have highlighted in the context of the static model. The first term on the right-hand side is positive by equations (D9) and (D10) and lemma 2. It captures the effect according to which privatization makes noncooperative labor allocation decisions more efficient. The second term depicts the impact of privatization on the incentive compatibility condition of agent *n*. Two effects are at play. On the one hand, the first term in brackets is positive and indicates that higher production efficiency, $\partial Y(s;\mu)/\partial\mu > 0$, causes agent *n*'s continuation payoffs to increase, thereby relaxing his/her incentive compatibility condition and providing stronger incentives to abide by the risk-sharing arrangement. On the other hand, the second term in brackets is negative because of the fact that privatization increases the consumption level in the current period in case of deviation: $\partial c_{\mathrm{d}}(\mu)/\partial\mu > 0$. The third term on the right-hand side of equation (D11) tells us the same for all the other agents. It has to be noted that the first and second effects—which pertain to efficiency and to the link between efficiency and the incentive compatibility condition, respectively—actually concern all agents and not only agent *n*, as it might appear at first sight. This is due to the construction of the optimization problem, where the entire household surplus is attributed to agent *n*, subject to the constraint that some target utility levels are achieved by the others. In fact, the positive effect of privatization on efficiency and risk sharing applies to the whole household.

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