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Zealots Attack and the Revenge of the Commons: Quality vs Quantity in the Best-of-n

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Abstract. In this paper we study the effect of inflexible individuals with fixed opinions, or zealots, on the dynamics of the best-of-n collective decision making problem, using both the voter model and the majority rule decision mechanisms. We consider two options with different qualities, where the lower quality option is associated to a higher number of zealots. The aim is to study the trade-off between option quality and zealot quantity for two different scenarios: one in which all agents can modulate dissemination of their current opinion proportionally to the option quality, and one in which this capability is only possessed by the zealots. In both scenarios, our goal is to determine in which conditions consensus is more biased towards the high or low quality option, and to determine the indifference curve separating these two regimes. Using both numerical simulations and ordinary differential equation models, we find that: i) if all agents can modulate the dissemination time based on the option quality, then consensus can be driven to the high quality option when the number of zealots for the other option is not too high; ii) if only zealots can modulate the dissemination time based on the option quality, while all normal agents cannot distinguish the two options and cannot differentially disseminate, then consensus no longer depends on the quality and is driven to the low quality option by the zealots.

1 Introduction

Collective decision making is a process whereby a population of agents makes a collective decision based only on local perception and communication. Originally inspired by the behavior of social insects [4, 2], collective decision making is considered an important problem connected to more elaborated collective behaviors in swarms robotics [28], such as site selection or collective motion [3].

The best-of-n problem [28] is a special case where agents have to chose the best option among n possible alternatives with potentially different qualities. The option quality may be known to swarm members [29], or may need to be discovered [23, 25, 24]. An option can be considered *best* because it minimizes the cost required to be evaluated or because its intrinsic quality is the highest [28]. In the latter case, a method to achieve the optimal collective decision is to let each agent advertise an option for a duration that is proportional to its quality, a mechanism called "modulation of positive feedback" [8, 30, 29].

In this paper we focus on the best-of-n problem with $n = 2$ options in presence of stubborn individuals, henceforth called *zealots*. Zealots are individuals that have a fixed opinion that never changes. We introduce differential option quality and differential zealot quantity in an antagonistic setting: the two options are associated to different values of quality and zealot quantities; and the number of zealots is higher for the option that has a lower quantity, hence it is not obvious which option will prevail. Two specific cases are compared: i) all agents are able to measure the quality of the two options and disseminate for a time proportional to the quality; ii) only zealots and not the normal agents are able to measure the quality of the different options, and disseminate for a time proportional to the quality of their opinion, while normal agents disseminate for a time that is independent from the quality. This last scenario is referring for example to the case where, in a swarm of robots used for monitoring task, only some of them (zealots) have additional sensors and they can perceive the quality of the two options. In this case, the number of zealots can be a design parameter or a constraint depending on the problem: fully equipped robots with many sensors are more expensive due to a larger payload.

Using computer simulations and ordinary differential equations (ODEs) models, we ask the following question: Is the swarm consensus state more biased towards the option represented by more zealots or the one represented by the highest quality? We are particularly interested in identifying the "indifference curve" separating the two regions identified by the consensus state being more biased towards one or the other option. We investigate whether the indifference curve behaves differently across two scenarios. Finally, we determine whether these results are affected by two decision mechanisms: the voter models, whereby agents change their opinion copying the opinion of a random neighbor, and the majority model where instead agents adopt the opinion of the local majority.

The remaining of the paper is organized as follows. In Section 2, we discuss the state of art. In Section 3, we describe the collective decision-making model utilized in this study. In Section 4 we discuss the results obtained. In Section 5, we conclude the paper and discuss future developments.

2 State of the Art

The best-of-n problem is inspired by biological studies of swarms of ants and bees [9, 15, 26]. As extensively discussed in [28], the quality and cost of the options can further characterize the nature of the best-of-n decision-making problem. In the current paper, quality and not cost is the main factor driving consensus.

Another important element that bears upon the decision-making dynamics is the presence of zealots within the swarm. The influence of zealots has been abundantly studied in physics, but introduced within swarms only recently. In the following, we will first review the few contributions focusing on zealots within swarms, and then review some of the work done within physics.

In the context of swarms, a recent study [21] illustrated the impact of zealots in the context of dynamic environments, where the option qualities can drastically change over time. Here, the presence of a small number of zealots enables the swarm to always select the option with the best quality even after abrupt changes, while without zealots, the swarm is not able to adapt and the consensus remains frozen. The authors in [5] introduced three types of malicious agents that can affect resilience of a swarm: contrarians, wishy-washy, and zealots. They performed a preliminary study on their effect on the best-of-n with four mechanisms: voter, majority, cross inhibition, and k-unanimity (q-voter). In [22], the authors also looked at the effects of malicious adversarial zealots in a data communication manipulation scenario, proposing a probabilistic decision-making rule to increase resilience. A very recent extension has been applied and evaluated the same scheme to a simulated swarm robotics scenario [14].

In the context of physics, the author in [6] introduced zealots in a model of pairwise social influence for opinion dynamics, and showed a rich phase diagram of the possible dynamics when only a small percentage of zealots is present. In the context of Internet social networks, the best placement of zealots that maximizes the impact on the consensus dynamics of the population is studied in [13]. The study shows that a small number of zealots can significantly influence the overall opinion dynamics and induce the entire population to reach a large consensus over disputed issues, such as Brexit. In [17], the authors studied the role of zealots in a social system using the naming game as decision mechanism. They show that even a very small minority can drive the opinion of a large population, if committed agents are more active than the others. However, this effect can be hindered if nodes with the same opinion are more connected with each other than with nodes with different opinion, producing a polarization inside the network.

The authors of [11, 18] studied the impact of zealots in a social network, considering different degrees of zealotry. The focus of [11] is studying the effect of zealotry on the convergence time of the system. In [18], despite having used the majority rule instead than the voter, the authors were able to find similar results as in [21, 7], in which introducing equal number of zealots on both option sides prevents the network from reaching a consensus state. Similarly, in [32], the presence of zealots is proven to prevent the formation of consensus, introducing instabilities and fluctuations in a binary voter model of a small-world network. A recent study illustrated in [1] aimed at studying the influence of zealots on "politically polarized" state vs consensus state and found that higher "influence of zealots" produces more polarization, shorter time to polarization, and conversely less consensus and longer to impossible time to consensus.

In [31], the authors showed the presence of a tipping point at which a minority of zealots is able to swing the initial majority opinion in a network. The study described in [16] focused on zealots with the voter model to perform peer-to-peer opinion influence, however differently from our work zealots were nodes of

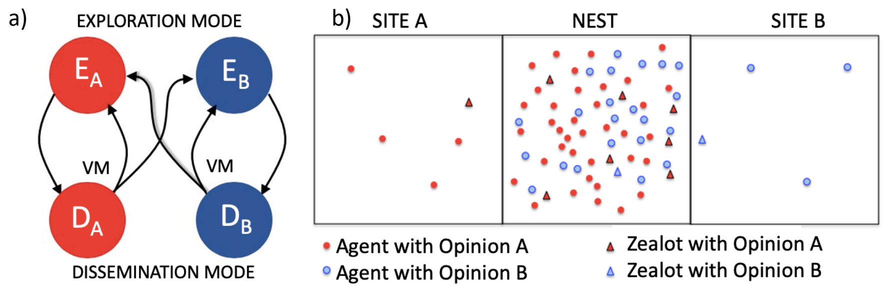

Fig. 1: (a) Probabilistic finite state machine. States represent dissemination and exploration states. Solid lines denote deterministic transitions, while dotted lines stochastic transitions. The symbol VM indicates the model (Voter/Majority) used at the end of the dissemination state. (b) The simulation arena.

a complex network. In [10], a scenario with zealots with the majority rule was studied. The outcome of the system was spontaneous symmetry breaking when zealot numbers were symmetrical for the two options, while consensus towards one option emerged even with minimal unbalance in the number of zealots. In these studies options did not have an intrinsic quality.

To summarize, zealotry has been abundantly studied in physics, typically in fixed interaction topologies, and only recently introduced in the context of swarms, in dynamic local interaction topologies. Compared to the latest work in swarms [21, 5, 22, 14], to the best of our knowledge, in this paper we study for the first time the interplay between different option quality and different zealot quantity, by extending the preliminary study in [19], in which the voter model only was considered and all the agents were able to disseminate differentially with quality.

3 The Model

In the best-of-n problem, a swarm of agents has to reach a collective decision among n possible alternatives. In this paper, the $n = 2$ opinions considered are labelled A and B and have intrinsic quality values ρ_A and ρ_B. The best collective decision is made if consensus is for the option with highest quality: formally, a large majority $M \leq N(1-\delta)$ of agents agrees on the same option, where δ is a small number chosen by the experimenter. $\delta = 0$ corresponds to *perfect consensus*. Variants of the best-of-n are: the two options may have differential access times or costs [28], option quality may change over time [21, 19], or the swarm may have a heterogeneous nature [21]. In the latter, a special case consists in the swarm composed by two different types of agents: zealots, agents with a fixed unchangeable opinion A or B; and normal agents, initialized with opinion A or B, but able to change their opinion by applying a decision mechanism that relies on the observation of other agents in local proximity.

Table 1: Model parameters used in simulations

Parameter	Description	Values
N	swarm size	{100, 1000}
ρ_A	site A quality	1
ρ_B	site B quality	$\{1, 1.05, 1.10, .., 2\}$
σ_B	proportion of zealots with opinion B to N	$\{0, 0.0125, 0.025, 0.05\}$
σ_A	proportion of zealots with opinion A to N	$\{0, 0.05, ..0.5\}$

3.1 The Simulation Model

Similarly to [20], the behaviour of the agents is controlled by the probabilistic finite state machine (FSM) shown in Figure 1a. The FSM has four possible states: dissemination state of opinion A (D_A), dissemination state of opinion B (D_B), exploration state of opinion A (E_A), and exploration state of opinion B (E_B). Agents are located in a rectangular arena divided in a central part called the *nest* and lateral (left and right) parts called the *sites*, each associated to A or B, respectively (see Figure 1b). All agents are initialized inside the nest, and move toward the site associated with their opinion to explore that option, for an exponentially distributed amount of time (sampled independently per agent) with mean time q, independent of the current opinion. After exploration, agents have measured the site quality and travel back to the nest after having switched to the dissemination state associated with their current opinion (D_A if they were in E_A, D_B if they were in E_B).

In the dissemination state at the nest, to meet the well-mixed criterion as much as possible [12], agents perform a correlated random walk. Each agent locally broadcasts his opinion continuously, and this message is sensed by other agents in local proximity that are in the process of applying the decision mechanism (before transitioning back to the exploration state). The time spent by the agent disseminating its opinion is exponentially distributed with mean proportional to the site quality they have last visited $g \cdot \rho_i, i \in \{a, b\}$. We considered two different cases in this paper. In the first, both normal agents and zealots with opinion A disseminate proportional to ρ_A, and both normal agents and zealots with opinion B disseminate proportional to ρ_B. In the second case, only zealots disseminate proportional to quality (ρ_A or ρ_B), while normal agents disseminate independently from the quality proportionally to $\rho = 1$. This second case is novel in this paper and was introduced to determine whether modulation of positive feedback is effective through zealots only.

At the end of dissemination, normal agents and zealots behave in two different ways. Normal agents can change their opinion based on the opinions of other agents within a specified spatial radius (in our simulations set to 10 units). The voter model or the majority rule is applied: In the case of voter model, the agent switches its opinion to the one of a random neighbors within the interaction radius [30]; while in majority rule, the agent switches its opinion to the one of the majority of its neighbors ($G = 2$ neighbors [29]).

3.2 ODEs Model

We adapted the model proposed in [21] which extends the ones in [30, 29]. The variables e_A, e_B, d_A, d_B model the sub-population of agents exploring site A, exploring site B, disseminating in the nest opinion A and disseminating in the nest opinion B, respectively. The variables modeling sub-populations of zealots are constant. They are denoted with $e_{AS}, e_{BS}, d_{AS}, d_{BS}$. The total proportion of agents with opinion A and B are respectively $x_A = e_A + d_A + e_{AS} + d_{AS}$ and $x_B = e_B + d_B + e_{BS} + d_{BS}$. The total number of agents is conserved $x_A + x_B = 1$.

The system of 8 ODEs with 8 state variables is given by:

$$\dot{d_A} = -\frac{1}{\rho_{AN} g} d_A + \frac{1}{q} e_A \qquad \dot{d_{AS}} = -\frac{1}{\rho_A g} d_{AS} + \frac{1}{q} e_{AS} \quad (1)$$

$$\dot{d_B} = -\frac{1}{\rho_{BN} g} d_B + \frac{1}{q} e_B \qquad \dot{d_{BS}} = -\frac{1}{\rho_B g} d_{BS} + \frac{1}{q} e_{BS} \quad (2)$$

$$\dot{e_A} = -\frac{1}{q} e_A + \frac{p_{AA}}{\rho_{AN} g} d_A + \frac{p_{BA}}{\rho_{BN} g} d_B \qquad \dot{e_{AS}} = -\frac{1}{q} e_{AS} + \frac{1}{\rho_A g} d_{AS} \quad (3)$$

$$\dot{e_B} = -\frac{1}{q} e_B + \frac{1 - p_{AA}}{\rho_{AN} g} d_A + \frac{1 - p_{BA}}{\rho_{BN} g} d_B \qquad \dot{e_{BS}} = -\frac{1}{q} e_{BS} + \frac{1}{\rho_B g} d_{BS} \quad (4)$$

Equations on the left column describe the dynamics of normal agents, while equations on the right column describe the dynamics of zealots. In Equation 1-left, the proportion of agents disseminating opinion A increases because of agents returning from the exploration of A at rate $\frac{1}{q}$, and decreases because of agents terminating dissemination at rate $\frac{1}{\rho_{AN} g}$. Similarly, equation 2-left describe the rate of increase of the number of agents disseminating opinion B. In Equation 3-left the number of agents exploring site A decreases because of agents finishing exploration at rate $\frac{1}{q}$, and increases because of two contributions: i) agents that had previously opinion A and kept the same opinion after the application of the voter/majority model and ii) agents that had previously opinion B but switch to A as a result of the voter/majority model. Similarly, Equation 4-left describes how agents exploring site B vary. The rates p_{AA}, p_{AB}, p_{BA}, and p_{BB} describe the probabilistic outcome of the two decision mechanisms and are described next. Note that qualities in the left column equations are indicated with ρ_{AN} and ρ_{BN} as placeholders. These correspond to the site qualities $\rho_{AN} = \rho_A$, $\rho_{BN} = \rho_B$ when all agents disseminate differentially, while $\rho_{AN} = \rho_{BN} = \rho = 1$ when only zealots disseminate differentially. The dynamic of zealots is described in a very similar way by the equations on the right column. The only difference consists in the impossibility that a zealot to change its opinion after any interaction, thus the terms that depend on the decision mechanisms are omitted. For the zealot case, the dissemination always takes place proportional to ρ_A and ρ_B.

Regarding the decision mechanism, for the voter model the probability that the outcome of the decision is A (resp. B) is the probability that, when observing a random agent disseminating, that random agent is disseminating A (resp. B). This is given by the ratio of agents disseminating A with respect to the

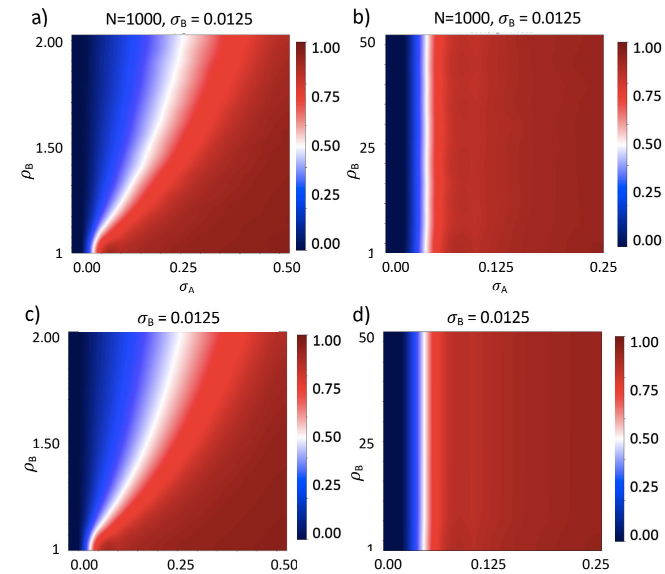

Fig. 2: Consensus heatmaps for the voter model in simulations (first row) and with ODEs (second row), for all agents performing differential dissemination (a and c) and for only performing differential dissemination (b and d). In all cases $\sigma_B = 0.0125$, and $N = 1000$ in the simulations. The colour scale represents the consensus for A. Dark blue colors indicate perfect consensus to the best opinion B, dark red colors indicate perfect consensus to the worst opinion, A, while the white color shows the indifference curve (consensus state around 0.5).

total number of agents disseminating: $p_{AA} = p_{BA} = \frac{d_A+d_{AS}}{d_A+d_{AS}+d_B+d_{BS}}$ (resp. $p_{BB} = p_{AB} = \frac{d_B+d_{BS}}{d_A+d_{AS}+d_B+d_{BS}}$.).

For the majority model, where each agent switches its opinion to the one hold by the majority of its G neighbors, the two probabilities are simply given by the cumulative sum of probabilities distributed according to a hypergeometric distribution modeling how many neighbors have each of the two opinions [29]. As in [29], we used: $p_{AA} = \sum_{\frac{G}{2}}^{G+1} \frac{G!}{r!(G-r)!} p^r (1-p)^{G-r}$ and $p_{BA} = \sum_{0}^{\frac{G}{2}} \frac{G!}{r!(G-r)!} p^{G-r} (1-p)^r$.

4 Experimental Evaluation

The experiments were conducted using a simulation tool originally developed by [30]. The simulated arena is a rectangular, two-dimensional space. The collision of the agents is not modeled, however, previous results show that real robot experiments could be accurately reproduced [27].

In each experiment, σ_A (resp. σ_B) is the proportion of zealots committed to A (resp. B). In every run, we first initialize the zealots according to σ_A

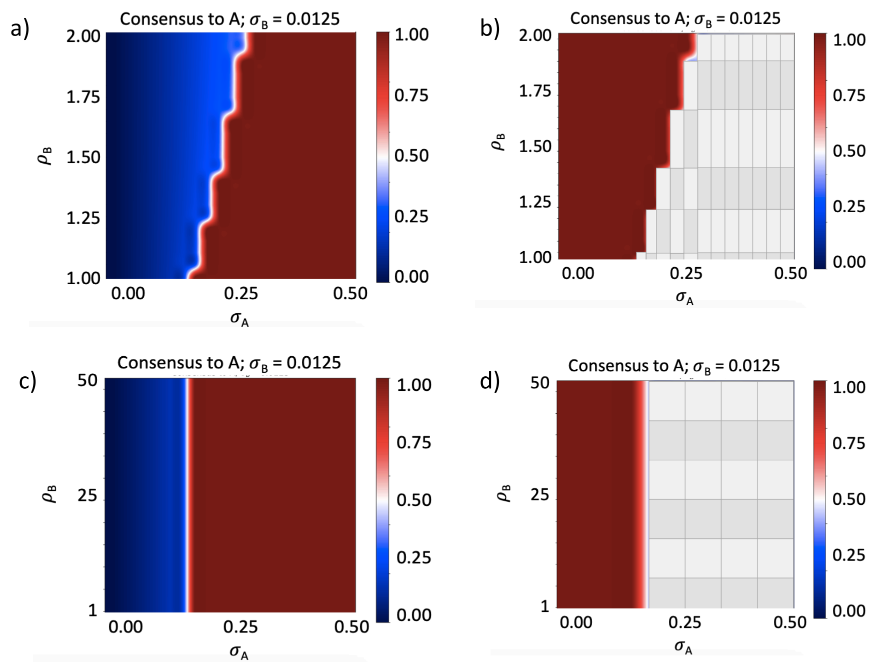

Fig. 3: Consensus heatmap obtained from ODE solution of majority model ($\sigma_B = 0.0125$). Two cases are considered: all agents disseminate for a time proportional to the quality of the option (Panel a and b) or only zealots disseminate for a time proportional to the quality of the option (Panel c and d). The colour scale represents N_A/N. Blue cells indicate perfect consensus (agreement to the best opinion, B). Red cells mean consensus to the worst opinion, A. Tiled cells in (b) and (d) indicate the lack of a second stable equilibrium.

and σ_B. Afterwards, we set 50% of the remaining (normal) agents to opinion A and the remaining (normal) agents to opinion B. We fix $N = 1000$ agents and $\sigma_B = 0.0125$, as preliminary [19] as well as current study shows that these parameters do not affect the results. The nest size to 316×316 and two sites have the same size of the nest. As zealots need to be more numerous for the option with the lower quality, we set $\sigma_A \geq \sigma_B$ and $\rho_A \leq \rho_B$. Table 1 reports all parameter values.

4.1 Results with the Voter Model

In Fig.2, we report the heatmaps obtained from simulations and ODE corresponding to the two cases where all the agents disseminate proportionally to the quality (panels a and c) and where only zealots disseminate proportionally to the quality (panels b and d). The simulations results (panels a and b) are reproduced very well by the ODE predictions (c and d, respectively). When all the agents are aware or can measure the qualities of the two options, the consensus to the best option B, represented in blue color, can still be reached despite the increasing number of zealots of the opposite opinion. Only for very high number of zealots (larger than 30% of the total agents), consensus is driven to the worst option A. The indifference curve here is diagonal and depends on both parameters ρ_B and σ_A. The quality of the best option B has a predominant effect with respect to the quantity of zealots for the worst option A.

On the contrary, if only zealots can measure the quality and disseminate differentially, consensus is never driven to the best option B, except for the case where the number of zealots of the worst option is the same or less than the number of zealots of the best option. In other words, the indifference curve is in this case vertical and only depends on the parameter σ_A.

4.2 Results with the Majority Rule

Given the very good results obtained from ODE that accurately reproduce the multi-agents simulations, we used ODEs to study how the system behaves when using the majority rule as decision mechanism. These are shown in Fig.3. Panels a and b show the case where all agents are disseminating proportionally to the quality, while panels c and d show the case where only zealots are disseminating proportionally to the quality. Fig.3 reports only the stable equilibria. In both cases, two different regimes can be observed. For every value σ_A, a stable equilibria appears (left panels), while a second stable equilibrium exists only for low values of σ_A (right panels). This additional stable equilibrium for the worst option A can be explained by the faster and less accurate dynamics of the majority rule [29]. Looking to the first stable equilibirium (left panels), results are similar to those of voter model: If all agents disseminate differentially, we observe a dependency on ρ_B, while if only zealots disseminate differentially the results depend only on σ_A. However, compared to voter decision mechanism, the majority rule seems more resilient to the quantity of zealots A: When all agents disseminate differentially, the system is more resilient to σ_A for lower values of ρ_B, while for higher values of ρ_B the voter and the majority behave in a similar way; additionally, also when only zealots disseminate differentially the system can converge to the best option for higher values of σ_A using the majority compared to the voter.

We also visualize the bifurcation diagram (Fig. 4) for the case where all agents disseminate proportionally to the quality (left column) and the case where zealots only disseminate proportionally to the quality (right column). Every row represents a different value of $\rho_B = 1, 1.5, 2$ respectively. The consensus state for A, denoted by x_A, is studied for increasing σ_A. Here, we confirm the presence of two stable equilibria for low values of σ_A. At some point, a saddle-node bifurcation occurs, and only one stable equilibrium survives. However, we observe that the position of the saddle-node bifurcation moves to the right with respect to σ_A only for the case where all agents disseminate differentially, and not for the case where only zealots do so. We believe these dynamics are interesting because this potentially means that the system is irreversible: if initially the

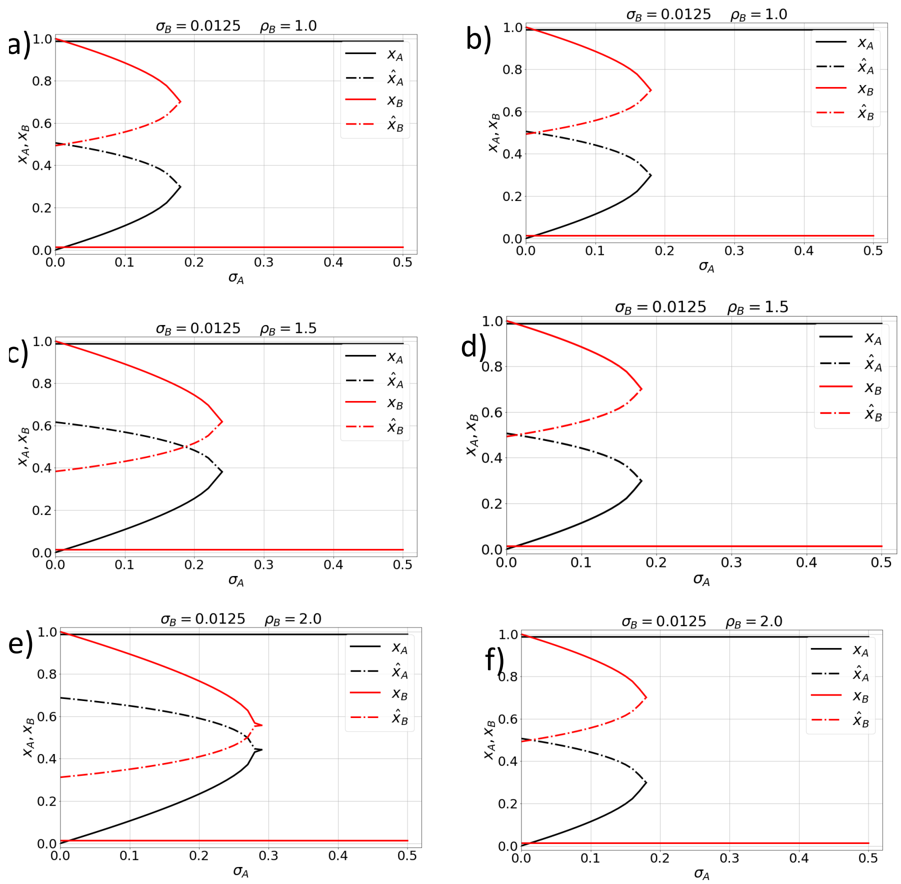

Fig. 4: Bifurcation diagram for majority model with all agents disseminating differentially (left column) and with only zealots disseminating differentially (right column) for different values of ρ_B: $\rho_B = 1$ (first row), $\rho_B = 1.5$ (second row), $\rho_B = 2$ (third row). $\sigma_B = 0.0125$ in all plots. Stable equilibria are represented by a continuous line, while unstable equilibria are represented by a dashed line and indicated with an $\hat{}$.

number of zealots A is low, consensus will very likely be for B. However, if σ_A increases, consensus will abruptly change to A after the bifurcation. From that point onwards, reducing again σ_A will not recover consensus to B but the system will be locked in the A consensus state even for progressively lower and lower values of σ_A.

5 Conclusions

In this paper the well established model of best-of-n model is investigated by focusing on the interplay between zealots and quality. We focus on the antagonistic scenario in which the number of zealots is higher for the option that has the lowest quality. Two specific cases are considered: i) both normal agents and zealots can measure the quality of the two options; ii) only zealots can measure the quality of the two options.

The main findings of this paper are: i) if both zealots and normal agents have a different dissemination time determined by the quality of their opinion, the quality has the capability to drive the consensus to the best option, provided that the number of stubborn of the worst opinion is not too high; ii) if only zealots disseminate for a time proportional to the quality of their opinion, the consensus is driven only by the number of zealots. In this case, the quality never prevails and the consensus is to the option with higher number of zealots. From a social perspective, these results show that if only an élite knows how good different alternatives are, or have means to measure this information, the consensus cannot be driven to the better quality if the number of zealots supporting the worse quality is higher than the number of zealots supporting the best quality. This means that zealots can be explicitly designed to manipulate the opinion of the population. On the contrary, it is of paramount importance, at least in our models, that the whole population has the means to assess the quality of the alternatives, because this is the only way to be resilient, up to a given extent, to zealot manipulations and to achieve the best social good.

In the future, we would like to further analyze the dynamics, especially those with the majority model that manifested interesting irreversible dynamics. We would like to relate this model with others such as those based on cross inhibition [25]. Additionally, from the engineering perspective, we would like to understand whether it is possible to design a resilient mechanism for the normal individuals to be resilient to zealots even when they cannot measure the quality, in order to revert the results obtained with zealot-only differential dissemination. These can be useful in swarm robotics applications whereby sensors necessary to estimate quality are expensive and can only be produced for a minority of the individuals.

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