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Gulina, Marvyn; Mauroy, Alexandre

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Marvyn Gulina and Alexandre Mauroy
Namur Institute for Complex Systems (naXys) and Department of Mathematics,
University of Namur, Rue de Bruxelles 61, B-5000 Namur, Belgium
Email: marvyn.gulina@unamur.be and alexandre.mauroy@unamur.be

1 Spectral identification of networks

Spectral network identification aims at estimating the eigenvalues of the network Laplacian matrix from data. This framework allows to infer global network information (e.g., mean node degree, bounds on minimal and maximal degrees, etc.) from local observations at a small number of nodes. It was initially developed for networks of diffusive-coupled units [3], [4], and has not be used for applications that require other types of coupling. In this work, we fill this gap by considering spectral identification in the case of pulse-coupled oscillators, which is the prototypical model in neuroscience.

2 Pulse-coupled oscillators

We consider a network of N identical weakly pulse-coupled phase oscillators $\theta_i \in \mathbb{S}$ described by the dynamics

$$\dot{\theta}_i = \omega + \varepsilon \sum_{j=1}^{N} \sum_{k \in \mathbb{N}} \boldsymbol{A}_{ij} Z(\theta_i(t))\, \delta(t - \tau_j^{(k)}), \quad i = 1, \ldots, N \quad (1)$$

where ω is the natural frequency, $\boldsymbol{A}_{ij}$ are the entries of the network adjacency matrix $\boldsymbol{A}$, $Z : \mathbb{S} \to [0, 2\pi[$ is the so-called phase response curve (PRC), and $\{\tau_j^{(k)}\}_{k \in \mathbb{N}}$ is a sequence of firing (i.e., spiking) times such that $\theta_j(\tau_j^{(k)}) = 0$. Moreover, we assume that the coupling is weak, i.e., $\varepsilon \ll 1$, and that the oscillators synchronize, i.e., $\lim_{t \to \infty} \theta_j(t) = \theta^*(t)$ for all j (with $\dot{\theta}^* = \omega$).

We measure spiking times series $\{\tau_j^{(k)}\}_{k \in \mathbb{N}}$ for a few nodes j and we aim at estimating (some information on) the eigenvalues of the network Laplacian matrix $\boldsymbol{L}$ (i.e., $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$, where $\boldsymbol{D}$ is the matrix of in-degrees).

3 Stroboscopic map and weak coupling

Denote $\bar{\theta}_i[k] := \theta_i(kT)$ and define the stroboscopic map $\boldsymbol{S} : \mathbb{S}^N \to \mathbb{S}^N$ such that

$$\bar{\theta}_i[k+1] = \boldsymbol{S}_i(\bar{\theta}_1[k], \ldots, \bar{\theta}_N[k]).$$

Under the weak coupling assumption, we can use averaging techniques to approximate the Jacobian matrix of the stroboscopic map by

$$\boldsymbol{J} = \boldsymbol{I} + \varepsilon Z'(0)\, \boldsymbol{L}. \quad (2)$$

It follows that we can recover the spectrum of L from the spectrum of J.

4 Numerical experiments

In practice, the procedure consists of three main steps:

1. Estimate the sequence of phases from measured spiking time sequences.
2. Estimate the eigenvalues of the Jacobian matrix by using the DMD algorithm [2] with the approximated phases.
3. Identify the eigenvalues of the Laplacian matrix using equation (2).

This procedure will be illustrated on synthetic spiking time series generated by the dynamics (1). In particular, we will report on preliminary results such as estimating the number of clusters in networks of varying sizes using the spectral gap of the identified Laplacian spectrum.

These early findings open the door to applications in neuroscience, e.g., performing the spectral identification on pulse-coupled FitzHugh – Nagumo neurons [1].

References

[1] R. FitzHugh, "Impulses and physiological states in theoretical models of nerve membrane," *Biophysical Journal*, vol. 1, no. 6, pp. 445–466, Jul. 1961. DOI: 10/dmw38k.

[2] P. J. Schmid, "Dynamic mode decomposition of numerical and experimental data," *Journal of Fluid Mechanics*, vol. 656, pp. 5–28, Jul. 2010, ISSN: 1469-7645. DOI: 10/frnm7m.

[3] A. Mauroy and J. Hendrickx, "Spectral identification of networks using sparse measurements," *SIAM Journal on Applied Dynamical Systems*, vol. 16, no. 1, pp. 479–513, Jan. 2017, ISSN: 1536-0040. DOI: 10/f9336p.

[4] M. Gulina and A. Mauroy, "Spectral identification of networks with generalized diffusive coupling," *IFAC-PapersOnLine*, vol. 55, no. 30, pp. 492–497, 2022. DOI: 10/grjq2t.