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Heterogeneous mean-field analysis of best-of-n decision making in networks with zealots

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Abstract. Humans and animals often choose between options with different qualities. When the decisions are not determined by one or a few individuals leading a group, a collective can achieve a consensus through repeated interactions among the individuals. Collective decision-making is widely studied in the context of opinion dynamics, showing that individual mechanisms of option selection and the underlying social network affect the outcome. Mathematical techniques, such as the heterogeneous mean-field (HMF) theory, have been developed to systematically analyse the collective behaviour of interconnected agents. Based on the HMF theory, we propose a mathematical model that looks at the combined effects of multiple elements bearing upon the collective decision dynamics, such as the individuals' cognitive load, the difference in the quality of the options, the network topology, and the location of the zealots in the network. The results of this study show that, in scale-free networks, when individuals employ specific opinion selection mechanisms, characterised by a low cognitive load, the zealots have the ability to steer the consensus towards the option with the lowest quality or to group indecision. This result is reversed when the interaction network is sparsely connected and quite homogeneous – that is, most nodes have few neighbours – and cognitively simple individuals make accurate collective decisions, mostly unaffected by zealots voting for the option with the lowest quality.

Keywords: Opinion dynamics · Best-of-n · Zealots · Heterogeneous mean-field

1 Introduction

Human beings are every day faced with the problem of choosing among different options. Limited information or noisy conditions can make such decisions even more difficult; a possible way to overcome the issue is to exploit social interaction. Collective decision-making (CDM) is hence characterised by the fact that once the decision is made it is no longer attributable to any individual of the group. Achieving a consensus is the result of multiple interactions in which individuals choose an option according to some opinion formation mechanisms that can be relatively simple. For example, a general agreement can be reached using social feedback, by which consensus emerges among individuals that select an option by copying the preferences of one or more group mates.

CDM is not exclusive to humans but is also observed in other social species [6]. For example, groups of baboons collectively decide in which direction to move [27]; flocks of birds collectively decide their motion direction [3]; and swarms of bees collectively decide where to build a new nest site [25, 21]. Investigating CDM is important for understanding the behaviour of many biological systems, and for enabling autonomy in artificial systems such as robots [20]. For example, swarms of robots are programmed with collective decision-making algorithms to cooperatively perform a variety of tasks [13, 28, 24, 30]. Therefore we can conclude that different scientific disciplines are interested in investigating CDM and unveiling the elements that influence and contribute to determining the outcome of various decision-making processes.

CDM problems have been studied with different methods such as experimental methods [6, 7], computational modelling and simulation methods [12], and social network analysis [4, 18]. These studies have focused on different issues such as: i) the effect of different individual opinion selection mechanisms, each of which is associated with different cognitive costs (e.g., linked to memory, perception, attention) [10, 22]; ii) the effect of the homogeneity/heterogeneity in the group behaviour (i.e. individuals have equal/different behaviours) [8, 23]); iii) the effects of different topologies of the interaction network between the individuals [14, 26]; iv) the effects of the cost/benefit trade-off associated to the selection of each option (e.g., the quality of the chosen option and the time spent selecting it) [17, 19].

The objective of this study is to develop a mathematical model to analyse the combined effects of multiple factors (i.e., the cognitive load, the option qualities, the network topology, and the location in the network of zealots voting for the inferior option) bearing upon the opinion dynamics. More precisely, we model an asymmetrical binary collective decision-making process in which both options have equal costs, but one option has better quality than the other. Moreover, we model the exchange of information among agents as happening on a finite-size network composed of N nodes and L undirected edges, i.e., each node represents an agent and an edge the interaction existing among two agents. We also consider that certain individuals use conformism rules through which they agree with the opinion of their peers (which we call susceptible agents), and the rest never change their opinion and are normally called zealots [8, 23] or stubborn agents [17]. In our study, we only consider zealots with an opinion in favour of the inferior option, with the lower quality. Finally, we study different behaviours of the susceptible agents with respect to their cognitive load, that in our model translates into different pooling errors when an agent processes the opinions of her neighbours. The cognitive load is considered relatively low when an individual simply copies the preference of a randomly selected neighbour among the agents within her first connections, this behaviour corresponds to the voter model [26]. The cognitive load progressively increases for social feedback mechanisms in which each agent has to sample a progressively higher number of peers within her network of connections in order to select an option, e.g., to apply the local majority rule [11].

The original contribution of this study is to illustrate how the interactions between i) the agents' cognitive load, ii) the interaction network topology, and iii) the location of zealots in the network, influence the decision-making process, i.e., consensus, or not, for the opinion with the best quality. Given the asymmetry in quality (i.e., one option is better than the other and therefore is shared more often [29]), we study under which conditions, the zealots (who only share opinions for the inferior option) manage to counterbalance the difference in qualities and drive the population toward a consensus on the lowest quality option. Our study shows that when the susceptible agents follow a simple behaviour with relatively high pooling errors, the zealots voting for the inferior option lead the population into either an indecision state or a consensus for the inferior option. However, our results also show that this result can be reversed when connectivity and heterogeneity of the interaction (social) network reduce (i.e., the network becomes more homogeneous with most nodes with few neighbours).

2 Method and Methodology

The aim of this section is to introduce the basic rules upon which the agents possibly update their opinion and then to build a mathematical model based on the heterogeneous mean-field assumption to unravel the role of some main model parameters, namely the fraction of zealots present in the population, their location in the network and the network topology.

2.1 Model description

Let us thus consider a group of N agents interacting in an undirected scale-free network [1, 16], where the probability for an agent to have k neighbours is given by $p_k \sim 1/k^\gamma$, with $\gamma > 2$. Let us recall that the closer γ is to 2 the more heterogeneous the degree distribution is, indeed nodes with a very large degree can be present because $\langle k^2 \rangle$ is unbounded; on the other hand, if $\gamma \gg 3$ very high degree nodes are very rare and the degree spread is well described by finite variance of the degree distribution. Assume also the network to be connected, to avoid to consider the trivial case of a population split into several non-communicating groups, and simple, namely among every couple of agents there is at most one communication channel. The network topology is thus encoded by the $N \times N$ adjacency matrix, $\mathbf{M}$, whose entries satisfy $M_{ij} = M_{ji} = 1$ if and only if agents i and j can exchange opinion, and 0 otherwise.

We classify agents, i.e., the nodes of the network into susceptible and zealots, the former change their opinion over time in response to social interactions while the latter are inflexible and never change their initial choice. In this work, we consider the scenario of the best-of-2 problem where each node holds an opinion that can take one of two different values, A or B, modelling the choice between two beliefs on a particular issue or topic. We also associate to each opinion the corresponding *quality*, i.e., $Q_A > 0$ and $Q_B > 0$. The quality defines the strength or the probability that the option is communicated to the neighbours [29, 23, 8]. Without lack of generality, for the rest of the work, we assume $Q_A = 1$, $Q_B \leq Q_A$ and hence $Q = Q_B/Q_A \leq 1$. We only consider the scenario where zealots hold an opinion in favour of the opinion with the lower quality (i.e., opinion B). In fact, it is less interesting to introduce zealots voting for the option with the highest quality (option A) because the group already votes more frequently for options with better quality and its is more frequently selected by the group. Here, we study the ability of the group to select the best option despite the presence of zealots voting for the inferior option.

To specify how a susceptible individual updates her belief based on the weighted opinions of her neighbours, we consider the model (1) from [22] which we display in Fig. 1 for some representative values of the parameter $\alpha \in [0, 1.5]$.

$$P_\alpha(x) = \begin{cases} \frac{1}{2} - \frac{1}{2}(1-2x)^\alpha & \text{if } 0 \leq x \leq \frac{1}{2} \\ \frac{1}{2} + \frac{1}{2}(2x-1)^\alpha & \text{if } \frac{1}{2} < x \leq 1\,. \end{cases} \tag{1}$$

Let us observe that α is negatively correlated to the *cognitive load*: as α increases the agents makes more pooling error. More precisely, for $\alpha = 0$, the function P_0 models agents that make no errors and change their opinion based on the weighted average of all their neighbours, i.e., agents adopt a majority rule. This requires a larger cognitive load than when $\alpha > 0$ in which case agents make errors. In the case of $\alpha = 1$, the function P_1 models agents that update their opinion by copying the one of a randomly selected neighbour, namely this behaviour is the (weighted) voter model. Our model generalises thus two prominent models of opinion dynamics, the (weighted) voter model [26, 29] and the (weighted) majority model [12, 11, 2]. For generic values of $\alpha > 0$ and $\alpha \neq 1$, the proposed model allows us to explore behaviours with intermediate levels of cognitive cost and pooling error.

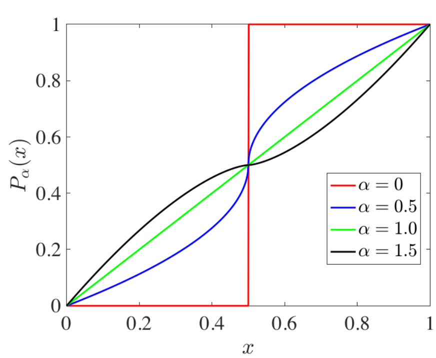

Fig. 1. The function $P_\alpha(x)$ for several values of α.

The system evolves asynchronously: each time step an agent i is randomly selected with a uniform probability and makes a social interaction. If the agent is a zealot nothing happens; otherwise if the selected agent is susceptible, she updates her opinion as a function of the weighted fractions of local opinions

$$n^{\#}_{i,A} = \frac{Q_A n_{i,A}}{Q_A n_{i,A} + Q_B n_{i,B}} \quad \text{and} \quad n^{\#}_{i,B} = \frac{Q_B n_{i,B}}{Q_A n_{i,A} + Q_B n_{i,B}} \,, \tag{2}$$

which are based on the number of i's neighbours $n_{i,A}$ and $n_{i,B}$, with opinion A and B, respectively, and the options qualities Q_A and Q_B. Note that we trivially have $n^{\#}_{i,A} + n^{\#}_{i,B} = 1$, $\forall i$.

Let k_i be the degree of node i, namely the number of agent i's neighbours, thus $n_{i,A} + n_{i,B} = k_i$. Then by recalling $Q = Q_B/Q_A$ we can rewrite Eq. (2) as

$$n^{\#}_{i,A} = \frac{n_{i,A}/k_i}{(1-Q)n_{i,A}/k_i + Q} \quad \text{and} \quad n^{\#}_{i,B} = 1 - n^{\#}_{i,A} \,. \tag{3}$$

Assume the selected i-th agent holds opinion A (resp. opinion B), then with probability $P_\alpha(n^{\#}_{i,B})$ (resp. $P_\alpha(n^{\#}_{i,A})$), she can change her opinion to B (resp. A). Let us also observe that because of the functional form of (1) and because $n^{\#}_{i,A} + n^{\#}_{i,B} = 1$, we can conclude that $P_\alpha(n^{\#}_{i,A}) + P_\alpha(n^{\#}_{i,B}) = 1$. The process continues by iteratively selecting one agent at a time and by updating its opinion; eventually the system reaches a stationary state.

2.2 A mathematical model with option's quality and zealots

The objective of this subsection is to propose a simple mathematical model defined by an ordinary differential equation (ODE) allowing us to study the evolution of group opinion, but also to unravel the role of the involved parameters, the cognitive load, the ratio of the opinion qualities $Q = Q_B/Q_A$, the fraction of zealots, and the network topology γ.

To make some analytical progress we rely on the heterogeneous mean-field assumption (HMF) [15, 5], namely we hypothesise that nodes with the same degree are dynamically equivalent and their evolution can be described by using the degree conditional probability $p(k'|k)$, namely the probability that a node with degree k is connected to another node of degree k'. Therefore, nodes are grouped into degree classes, more precisely we define A_k (resp. B_k), as the number of nodes with degree k and opinion A (resp. opinion B). To distinguish between susceptible agents with opinion B and zealots, let us introduce Z_k to denote the number of zealots with opinion B and degree k. Therefore, letting N_k to denote the total number of nodes with degree k, we have:

$$A_k + Z_k + S_k = N_k \,, \tag{4}$$

where S_k denotes the number of susceptible agents having opinion B and degree k. Eventually we introduce the fraction of agents having opinion A and degree k, $a_k = A_k/N_k$, and similarly the fraction of susceptible having opinion B with degree k by $b_k = S_k/N_k$ and by $\zeta_k = Z_k/N_k$ the fraction of zealots with degree k. Therefore, for all k,

$$a_k + b_k + \zeta_k = 1\,. \tag{5}$$

The goal of the HMF is to derive an ODE ruling the evolution of a_k and b_k. By starting from an idea recently developed in [22], we improve it with the addition of zealots to eventually obtain

$$\frac{d\langle a\rangle}{dt} = -\langle a\rangle + \sum_k q_k(1-\zeta_{k+1})\sum_{\ell=0}^{k+1}\binom{k+1}{\ell}\langle a\rangle^{k+1-\ell}\left(1-\langle a\rangle\right)^{\ell} P_\alpha\left(\frac{k+1-\ell}{k+1-\ell+\ell Q}\right)\,, \tag{6}$$

where we defined $\langle a\rangle = \sum_k q_k a_{k+1}$, being q_k the probability for a node to have an excess degree k, namely

$$q_k = \frac{(k+1)\,p_{k+1}}{\langle k\rangle} \quad \forall k \geq 0\,,$$

with $\langle k\rangle = \sum_k kp_k$ the average node degree. Eq. (6) contains the relevant parameters of the model, the zealots (ζ_k), the model of opinion dynamics (P_α), the opinion quality (Q) and the network topology (q_k). The aim of the next section is to determine the equilibria of such equation and determine their stability, hence the system fate. Before to do this let us observe that knowing $\langle a\rangle(t)$ from (6) one can obtain the evolution of a_k for all k by using the following equation [22]:

$$\frac{da_k}{dt} = -a_k + (1-\zeta_k)\sum_{l=0}^{k-1}\binom{k}{l}\langle a\rangle^{k-l}\left(1-\langle a\rangle\right)^{l} P_\alpha\left(\frac{k-l}{k-l+lQ}\right)\,.$$

2.3 Equilibria and stability of the analytical model

The equilibria of the system are obtained by setting to zero the right hand side of (6). Let us thus define the function

$$f_\alpha(x) := -x + \sum_k q_k(1-\zeta_{k+1})\sum_{\ell=0}^{k+1}\binom{k+1}{\ell}x^{k+1-\ell}\left(1-x\right)^{\ell} P_\alpha\left(\frac{k+1-\ell}{k+1-\ell+\ell Q}\right)\,, \tag{7}$$

hence by denoting $\langle a^*\rangle$ a system equilibrium, we have by definition

$$f_\alpha(\langle a^*\rangle) = 0\,.$$

A direct inspection of (7) allows to prove that $f_\alpha(0) = 0$, hence $\langle a^*\rangle = 0$, i.e., absence of agents with opinion A, is an equilibrium of the system. On the other hand, $f_\alpha(1) = -\sum_k q_k\zeta_{k+1} \neq 0$, hence the presence of zealots (with opinion B) prevents the system from converging to a population where only agents A will exist. Finally the existence of nontrivial solution $0 < \langle a^*\rangle < 1$ to the equation $f(\langle a^*\rangle) = 0$ will determine a coexistence of opinions A and B in the network.

The stability of the above-mentioned equilibria can be determined by looking at the derivative of the function f_α evaluated on the same equilibria. Such analysis will be presented in the following section where we also discuss the impact of the main model parameters.

3 Results

In this section, we present the results obtained for the analytical model described in Section 2.2. As already mentioned, we focus on the impact of the parameter α, the network topology, hereby summarised into the exponent γ of the power law, and the social influence of the zealots. More precisely, regarding the zealot analysis, we are interested in both their relative abundance and their position in the network, namely if they sit onto high-degree (hubs) or small-degree (leaves) nodes. To place zealots in hubs, we set $\zeta_k = 1$ for all $k \geq k_M$, for some sufficiently large k_M, this accounts to add into the model an average number of zealots equal to $Z_{tot} = \sum_{k \geq k_M} N_k \sim \sum_{k \geq k_M} N c_\gamma / k^\gamma$, where c_γ is a normalisation constant such that $\sum_k p_k = 1$ and N is the total number of nodes in the network. When we assume zealots to lie on leaves nodes and to fair compare this condition with the previous one, we consider the same number of zealots, that we set into the hubs by assuming $\zeta_{k_{min}} = Z_{tot}/N_{k_{min}}$, where k_{min} is a small enough degree; more precisely:

$$\zeta_{k_{min}} = \frac{Z_{tot}}{N_{k_{min}}} \sim \frac{Z_{tot}}{N p_{k_{min}}} = k_{min}^\gamma \sum_{k \geq k_M} \frac{1}{k^\gamma} \sim \frac{k_M}{\gamma - 1} \left(\frac{k_{min}}{k_M} \right)^\gamma .$$

Note that the above strategy does not imply adding an infinite number of zealots, indeed in any network realisation, e.g., by using the configuration model, there is a finite number of nodes with a degree larger than k_M and thus Z_{tot} is also a finite quantity. These finite-size effects can be studied in future research.

Fig. 2 summarises our main results. We fix the values of $Q = Q_B/Q_A = 0.9$, the power law exponent γ, and the zealot location in the network, and we numerically compute the zeros of the function f_α for values of $\alpha \in [0, 2]$ to obtain the equilibria of the system. Once the equilibria have been found, we evaluate the derivative of f_α and we determine its sign, if $f'_\alpha(\langle a^* \rangle) > 0$ then the equilibrium $\langle a^* \rangle$ is unstable and marked with a red points in Fig. 2, on the other hand if $f'_\alpha(\langle a^* \rangle) < 0$ then the equilibrium $\langle a^* \rangle$ is stable and we represent it in green. The three top panels refer to the strategy consisting of setting the zealots in the leaves (here $k_{min} = 1$), and the three bottom panels refer to the opposite strategy with the zealots in the hubs, $k_M = 100$. Moving from left to right we increase γ, passing from $\gamma = 2.5$ (left panels a) and d)), $\gamma = 3.0$ (middle panels b) and e)) and $\gamma = 3.5$ (right panels c) and f)).

Several conclusions can be drawn from those results. For large enough α, the system always sets into a state where opinions A and B coexist, the closer to 0.5 the larger α; this behaviour is independent of where the zealots are placed in the network or the network topology, i.e., γ. Hence a too-large pooling error α by the agents (which corresponds to a very small cognitive load) prevents the group from choosing the opinion with the highest quality.

For intermediate values of the pooling error, e.g., close to $\alpha = 1$, the impact that zealots have on the opinion dynamics depends on the interaction network topology. For scale-free networks with strong degree heterogeneity, e.g., $\gamma = 2.5$, the location where zealots are placed has a strong impact on the system fate.

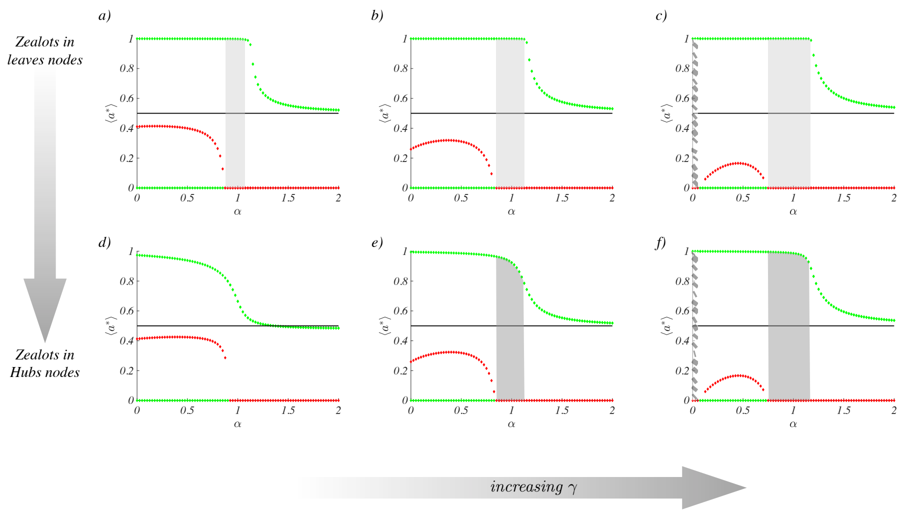

Fig. 2. Bifurcation diagrams of the HMF model. We report the equilibria $\langle a^* \rangle$ of Eq. (6) as a function of α; stable equilibria, i.e., associated to $f'_\alpha(\langle a^* \rangle) < 0$, are coloured in green while unstable ones, i.e., associated to $f'_\alpha(\langle a^* \rangle) > 0$, in drawn red. The top panels correspond to zealots set into leaves nodes while the bottom panels to the strategy of placing the zealots into the hubs. Panels a) and d) correspond to $\gamma = 2.5$, panels b) and e) $\gamma = 3.0$, panels c) and f) to $\gamma = 3.5$. The remaining parameters have been fixed to $k_{min} = 1$, $k_M = 100$ and $Q = 0.9$.

Putting the zealots into the leaves does not prevent the groups from selecting the best option (Fig. 2a), instead when zealots sit in the hubs, the susceptible agents are unable to make consensus decisions and remain polarised between the two options (in Fig. 2d the system for $\alpha \approx 1$ converges to the stable equilibrium $\langle a^* \rangle \sim 0.5$). The situation changes when the network heterogeneity decreases (i.e. for higher γ). Here, regardless of the location of the zealots in the network (leaves or hubs), the stable equilibrium is $\langle a^* \rangle \sim 1$, representing a consensus decision for the best option. This can be further appreciated by comparing the grey rectangles in the top and bottom panels of Fig. 2, which have the same horizontal size. The cause of this effect – to be investigated in future research – can be due to the rare presence of hubs in networks with large γ.

We also observe a strong impact of γ on the system outcome for small pooling error α, however having zealots in different locations (leaves or hubs) does not impact the system's equilibria. Indeed for $\alpha \approx 0$ we can observe that the unstable equilibrium branch (red dots) is below 0.5, this means that an initial population with few A agents, e.g., 40% of A and 60% of B, is capable to converge to a consensus toward A, and this result holds true despite the presence of zealots and their placement. This effect amplifies with increasing γ, e.g., for

$\gamma = 3.0$ the faction of A can be as small as $\sim 30\%$, and for large enough γ, i.e., scale-free networks with a relatively homogeneous degree distribution, any initial arbitrarily small fraction of agents with opinion A will be able to prevail and spread in the whole population (see the tiny dashed rectangles in panels c and f associated to $\gamma = 3.5$, where the equilibrium $\langle a^* \rangle = 0$ is unstable and thus the system converges to the only remaining possible equilibrium $\langle a^* \rangle \sim 1$).

To obtain a more global view of the complex interplay of the parameters, we studied the equilibrium $\langle a^* \rangle$ as a function of α and γ for a fixed value of $Q = 0.9$ (see Fig. 3). Moreover in each considered case we studied the impact of the strategy of placing the zealots on the leaves nodes (top panels) or on the hubs (bottom panels). In the two panels on the left, we colour-code the equilibrium reached by the system (yellow high values of $\langle a^* \rangle$ close to 1 and blue $\langle a^* \rangle$ close to 0), starting from an initial population with half agents holding opinion A and half opinion B. One can observe a striking difference between the top panel Fig. 3a), where zealots sit into leaves, and the bottom panel Fig. 3b), where zealots sit into hubs. In the first case, the equilibrium $\langle a^* \rangle$ is almost independent of γ and the system exhibits two main behaviours: for $\alpha \lesssim 1$ the whole group converges to a consensus to A, while for $\alpha \gtrsim 1$ the population is deadlocked at indecision with two similar-sized groups of agents with opinion A and B that coexist. In the second case, when zealots are placed into the hubs (Fig. 3c): the population converges to a consensus for the opinion with the lower quality when $\alpha \sim 1$ and $\gamma \lesssim \gamma_*$, where $\gamma_* \sim 2.33$. To better visualise this qualitative difference in the dynamics, we report on the right panels the bifurcation diagram with the three equilibria $\langle a^* \rangle$ as a function of the cognitive load for $\gamma = 2.2$, which is lower than the critical $\gamma_* \sim 2.33$. In the top panel, Fig. 3b), with zealots set into the leaves, the population converges to an almost consensus (large majority) for option A for $\alpha \lesssim 1.1$. On the other hand, in the bottom panel, Fig. 3d), with zealots set into hubs, for $\alpha \sim 1$ the group chooses the opinion with the lower quality. These results show that a population of agents using the (weighted) voter model as decision-making behaviour can be driven to adopt the opinion with the lower quality by zealots placed into hubs of a sufficiently heterogeneous scale-free network, i.e., $\gamma < \gamma_*$.

4 Conclusion

In this paper, we presented the results of a study focused on a best-of-n collective decision-making problem, with $n = 2$ options of different quality, and a heterogeneous population comprising a majority of the agents that have a conformist behaviour and change their opinion based on the social feedback and a minority of agents – referred to as zealots – who never change their opinion. The interactions among the agents happen over a social network whose nodes are the agents and the edges are the possible interaction links. We analyse the opinion dynamics for populations of agents with voter-like behaviours. We consider a continuum of behaviours characterised by the pooling error α that agents make when processing social information; making more errors reduces the agent's cog-

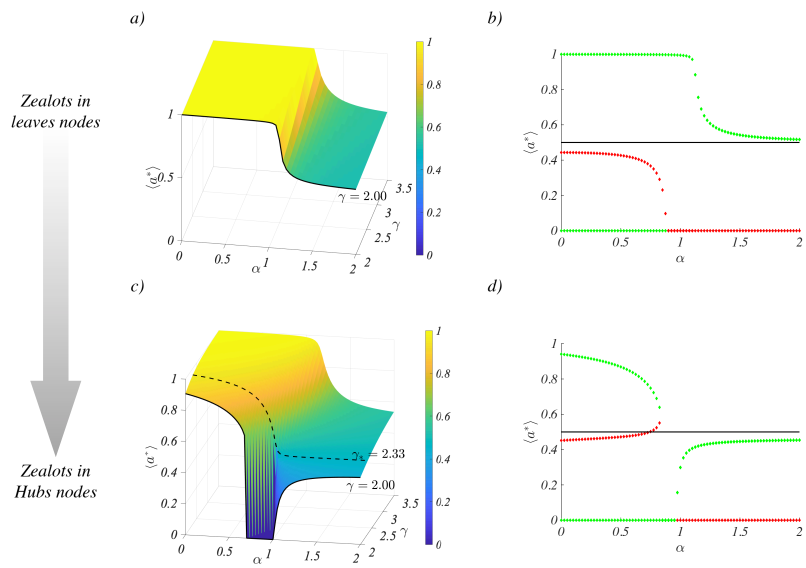

Fig. 3. Bifurcation diagrams of the HMF. We report the equilibrium $\langle a^* \rangle$ given by (6) as a function of (α, γ) for a fixed value of $Q = 0.9$ (left panels), and the same equilibrium where we also fix $\gamma = 2.2 < \gamma_*$ (right panels). Top panels correspond to zealots set into leaves nodes while the bottom panels to the strategy of placing the zealots into the hubs. The remaining parameters have been fixed to $k_{min} = 1$ and $k_M = 100$.

nitive load. Our model extends [22] and generalises through a single function a number of known voter-like models, such as the (weighted) voter model [26, 29] and the majority model [12, 11, 9].

We build our mathematical model of the collective decision process using the heterogeneous mean-field (HMF) theory. The determination of the system equilibria and their stability allowed us to study the combined effect of the model parameters, characterised by the cognitive load, the opinion quality, the network topology, and the location of zealots in the network. In particular, we have studied populations of agents with a given cognitive load (pooling error α) that interact on a scale-free network. In our analysis, we varied both the network topology and the location (leaves or hubs) where zealots—all holding the lowest quality opinion—are placed. The results have shown that the combined effect of these factors generated an articulated landscape characterised by different outcomes of the collective decision process. In case agents employ a high level of cognitive load, the collective decision follows the one of the majority (i.e., democratic decisions) with a bias for the best alternative that grows as the network degree distribution becomes more homogeneous (i.e., high γ). When the cognitive load is minimal, the group is unable to make a decision due to high pooling errors. The most interesting outcomes are obtained when the parameter $\alpha \sim 1$, corresponding to populations of agents employing decision-making mechanisms requiring a medium level of cognitive load, similar to the (weighted) voter model. In this case, zealots placed in the hubs, i.e., nodes with a large degree, are able to drive the entire population away from the consensus for the best quality option and lock it into indecision or to consensus for the inferior option. When zealots are placed in the leaves (i.e., nodes with a small degree) rather than in hubs, the population shows different decision dynamics and zealots are not able to interfere with the selection of the best option. This phenomenon is amplified by the parameter γ, i.e., the one ruling the heterogeneity of the scale-free network, in terms of node degree distribution. Large enough γ allows the better quality opinion to spread in the whole population even if initially a relatively small minority of agents has such opinion (Fig. 2). In the future, we aim to generalise these results by exploring a larger range of parameters and finding unifying patterns. In particular, we intend to study, in addition to the location in the network, how the quantity of the zealots and the option qualities influence the decision-making dynamics and how all these parameters inter-play with each other in determining the opinion dynamics of the population.

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