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Malbor Asllani & Timoteo Carletti

Desynchronize abnormal neuron behaviour to control epileptic seizures

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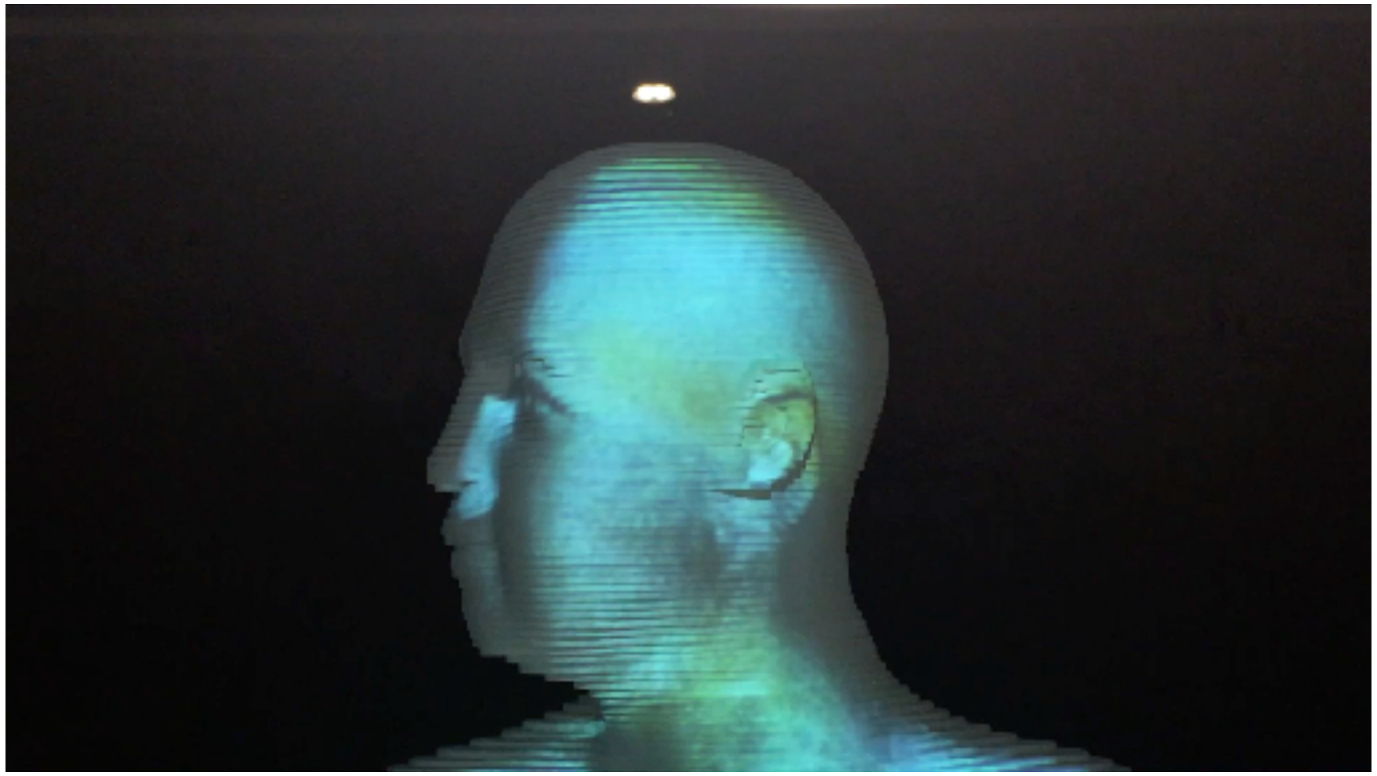
The Brain (i)



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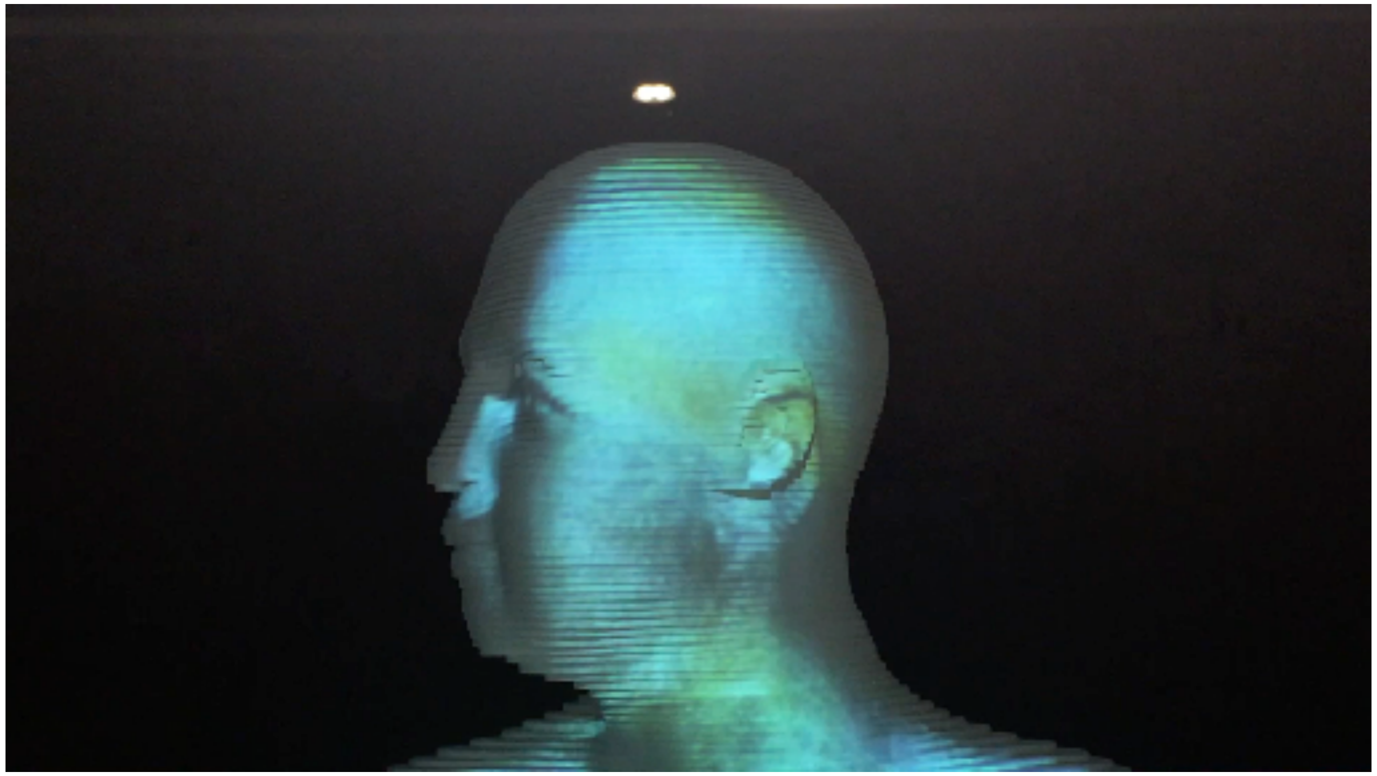


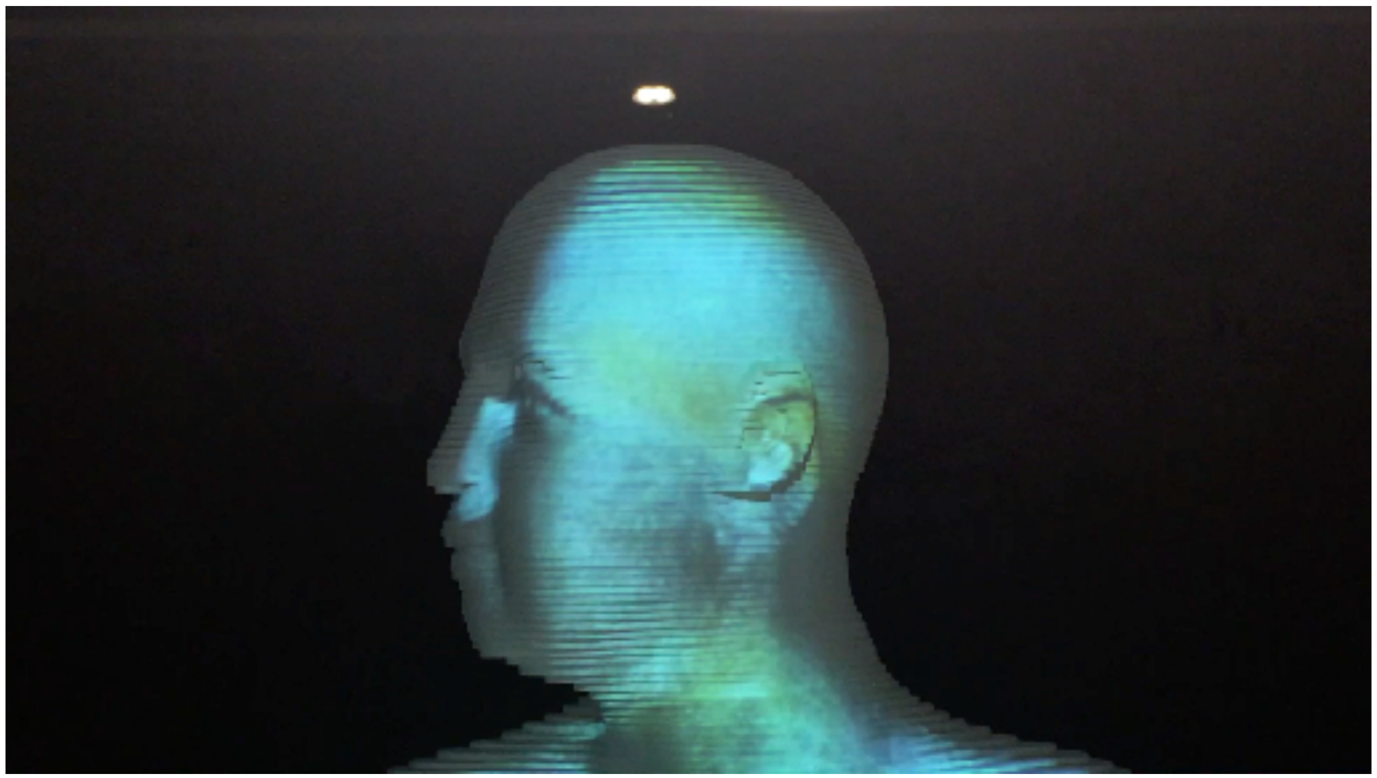
The Brain (i)

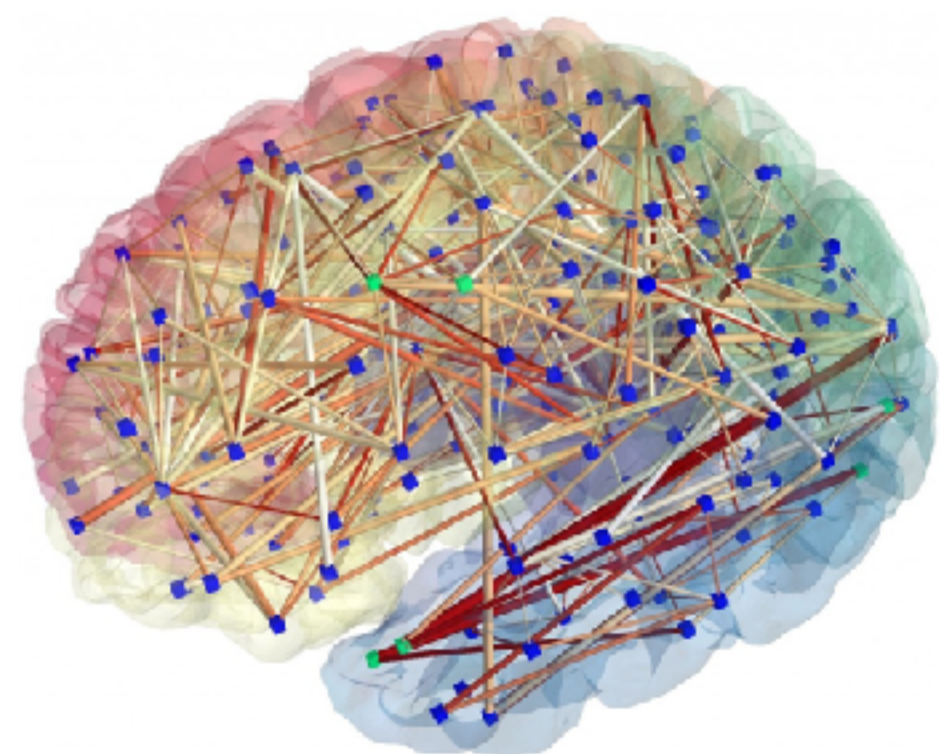
The Virtual Brain Project

CONNECTOME
COORDINATION FACILITY

HBP Human Brain Project

…





Synchronization is a key issue to achieve the normal behaviour.

The Brain (ii)

In neurodegenerative diseases, such as Parkinson or epilepsy, **abnormal synchronization** induces undesired effects such as tremors and epileptic seizures.

Goal: to Reduce/control **abnormal synchronization** to avoid (lighten) such undesired effects.

- Administration of oral drugs (partially effective Parkinson's disease but inefficient for nearly 1/3 of epileptic patients)
- Clinical methods (neurostimulation to modulate the neuronal activity to desynchronise the phase dynamics of neurons).
  - Deep Brain Stimulation (DBS), microelectrodes are inserted in the basal ganglia.
  - Transcranial Magnetic Stimulation (TMS), an external magnetic field interferes with the neuronal activity

Take home message

Our goal is to propose and study of a novel **minimally invasive neurostimulation** procedure principally **oriented** to **suppress** the **abnormal synchronization**.

It could thus be potentially used to reduce focal epileptic seizures or to deal with other neurological diseases.

We need:

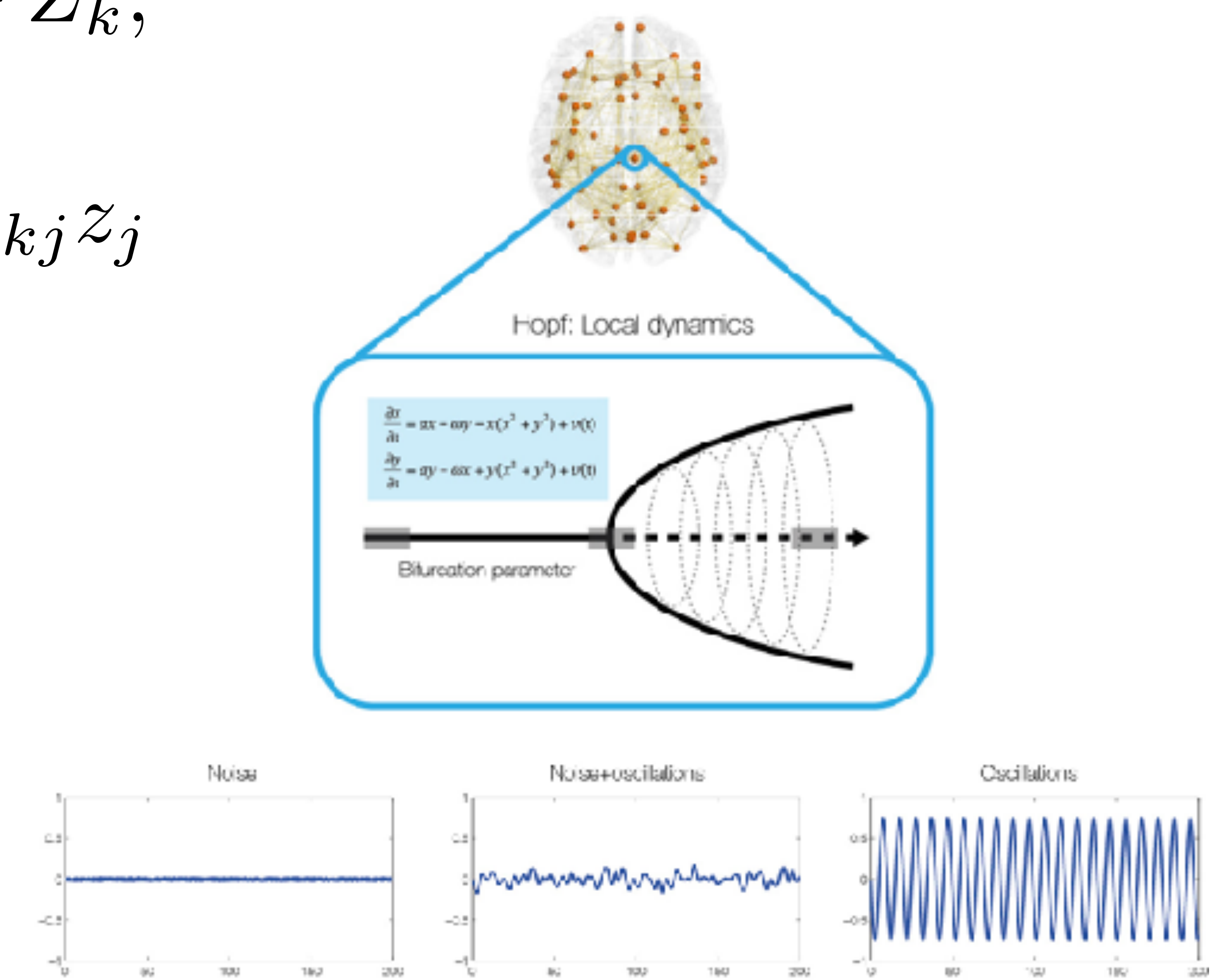
- a model;
- a control strategy to reduce synchronisation;
- an operational implementation of such strategy.

A brain model

Neurons modelled as nonlinear oscillators (Stuart-Landau model)

$$\dot{z}_k = (a_k + \imath\omega_k - |z_k|^2)z_k + Z_k,$$

$$\text{where} \quad Z_k = \frac{K}{N}\sum_{j=1}^{N} A_{kj} z_j$$



ω_k natural frequency

a_k bifurcation parameter
(<0 stable eq, >0 limit cycle)

K coupling parameter

Deco, G., Kringelbach, M. L., Jirsa, V. K. and Ritter, P., *The dynamics of resting fluctuations in the brain: metastability and its dynamical cortical core*, Sci. Rep., **7**, 3095/s41598-017-03073-5 (2017)

A simplified brain model

Let $z_k = \rho_k e^{\iota \phi_k}$ and assume $\rho_k \sim \rho_j$ for all k and j (and $a_k = 1$) then

$$\dot{\phi}_k = \omega_k + \frac{K}{N} \sum_{j=1}^{N} A_{kj} \sin(\phi_j - \phi_k)$$

Kuramoto model

A simplified brain model

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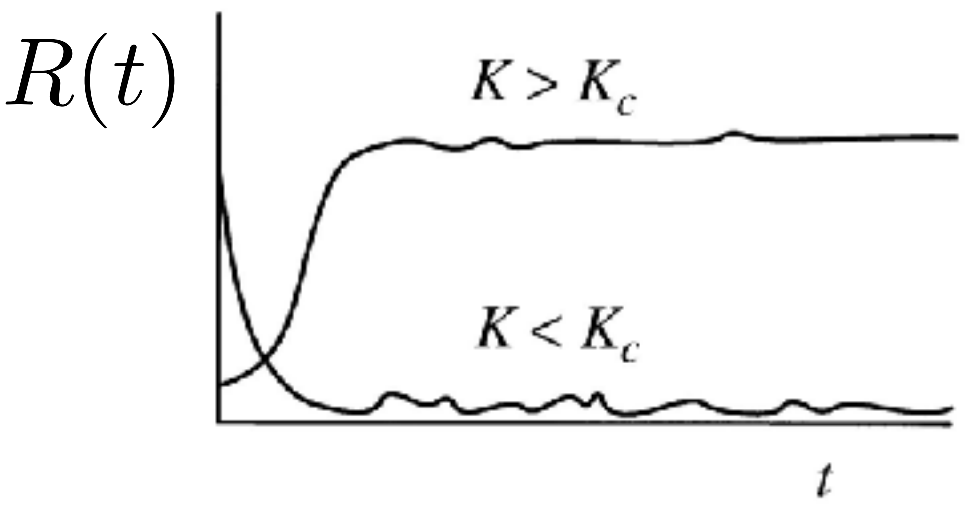
Kuramoto model

Order parameter

$$Re^{\iota \Psi} = \frac{1}{N} \sum_{j=1}^{N} e^{\iota \phi_j}$$



A simplified brain model

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Kuramoto model

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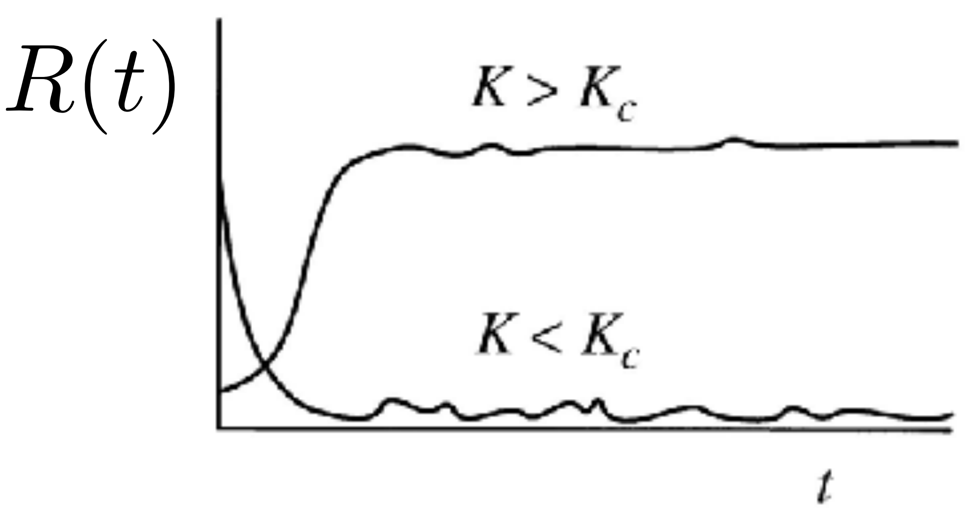
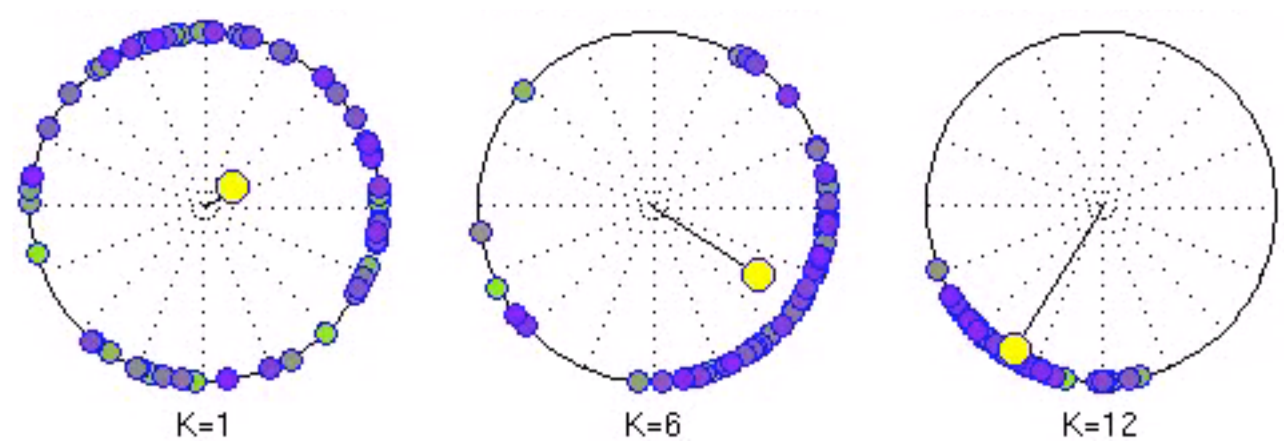


Non Synchronized state

Partially Synchronized state

Synchronized state



Nil, partial and full phase-locking in an all-to-all network of Kuramoto oscillators. Phase-locking is governed by the coupling strength K and the distribution of intrinisic frequencies ω. Here, the intrinsic frequencies were drawn from a normal distribution (M=0.5Hz, SD=0.5Hz). The yellow disk marks the phase centroid. Its radius is a measure of coherence.

Links between chaos and synchronization (i)

The Kuramoto model can be embed in a (2N-dim) Hamiltonian system.

$$H(\phi,\mathbf{I}) = \sum_i \omega_i I_i - \frac{K}{N}\sum_{i,j} A_{ij}\sqrt{I_iI_j}(I_j - I_i)\sin(\phi_j-\phi_i) \equiv H_0(\mathbf{I}) + V(\boldsymbol{\phi},\mathbf{I})$$

On the invariant "Kuramoto" torus, the dynamics of H is the same of the Kuramoto model

$$\mathcal{T}^K := \{(\mathbf{I},\boldsymbol{\phi}) \in \mathbb{R}^N_+ \times \mathbb{T}^N : I_i = 1/2\ \forall i\}$$

Moreover, the Kuramoto oscillators are in a **synchronous** state **if and only if** the Kuramoto torus is (transversally) **unstable**.

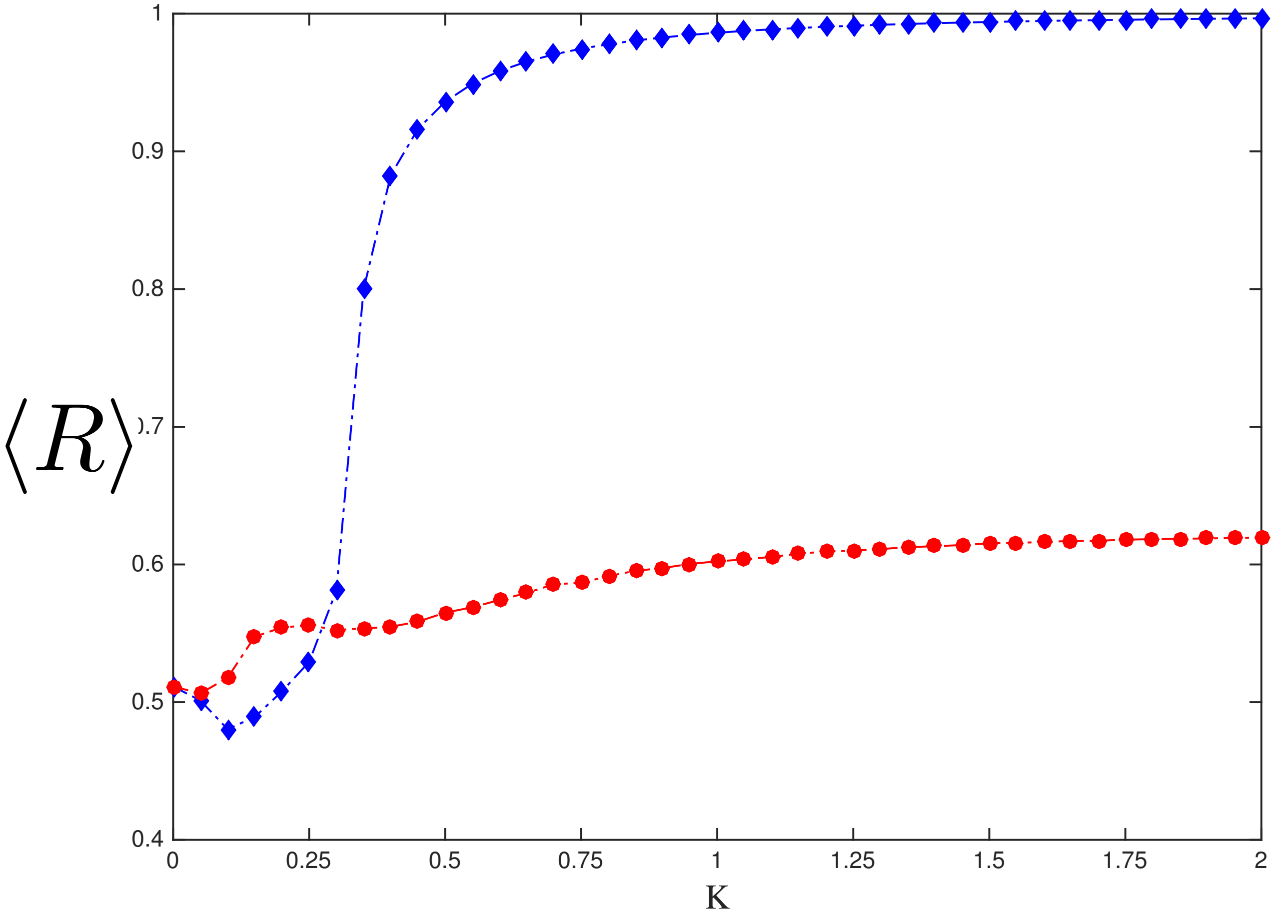
Witthaut, D. and Timme, M., *Kuramoto dynamics in Hamiltonian systems*, Phys. Rev. E, **90**, 032917 (2014).

Links between chaos and synchronization (ii)

$$H^{ctrl}(\phi, \mathbf{I}) = H_0(\mathbf{I}) + V(\phi, \mathbf{I}) + f_V(\phi, \mathbf{I})$$

$$f_V(\phi, \mathbf{I}) = \mathcal{O}(K^2)$$

Using the **Hamiltonian Control theory** one can modify the Hamiltonian system by adding a **small** term capable to **increase** the **stability** of the invariant Kuramoto torus



Vittot, M., *Perturbation theory and control in classical or quantum mechanics by an inversion formula*, J. Phys. A: Math. Gen. **37**, 6337 (2004).

Gjata, O., Asllani, M., Barletti L. and Carletti, T., *Using Hamiltonian control to desynchronize Kuramoto oscillators*, Phys. Rev. E, **95**, 022209 (2017)

Effective control

$$f_V(\phi, \mathbf{I})$$

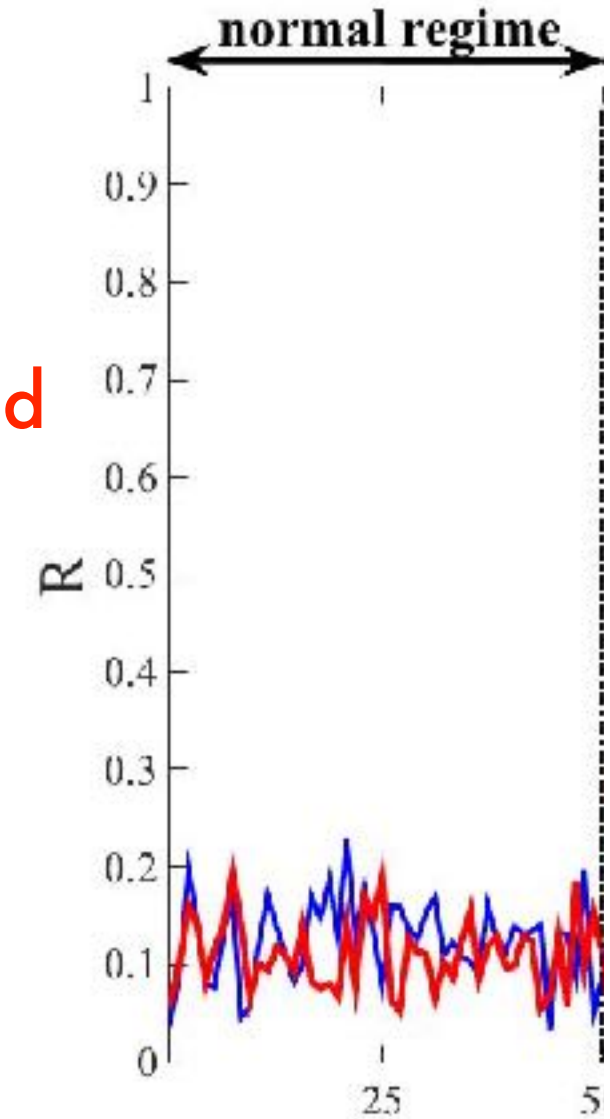
- Control only M << N nodes and add a tunable parameter γ
- Consider a local field generated by the controlled nodes

Effective control

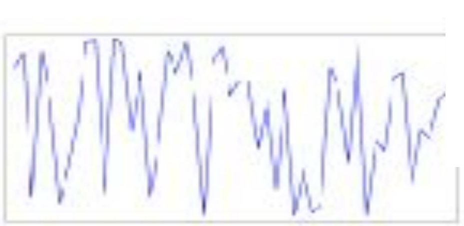
Result for the Kuramoto $K \sim 0.1 < 0.4 \sim K_{crit}$

original system

controlled system



K



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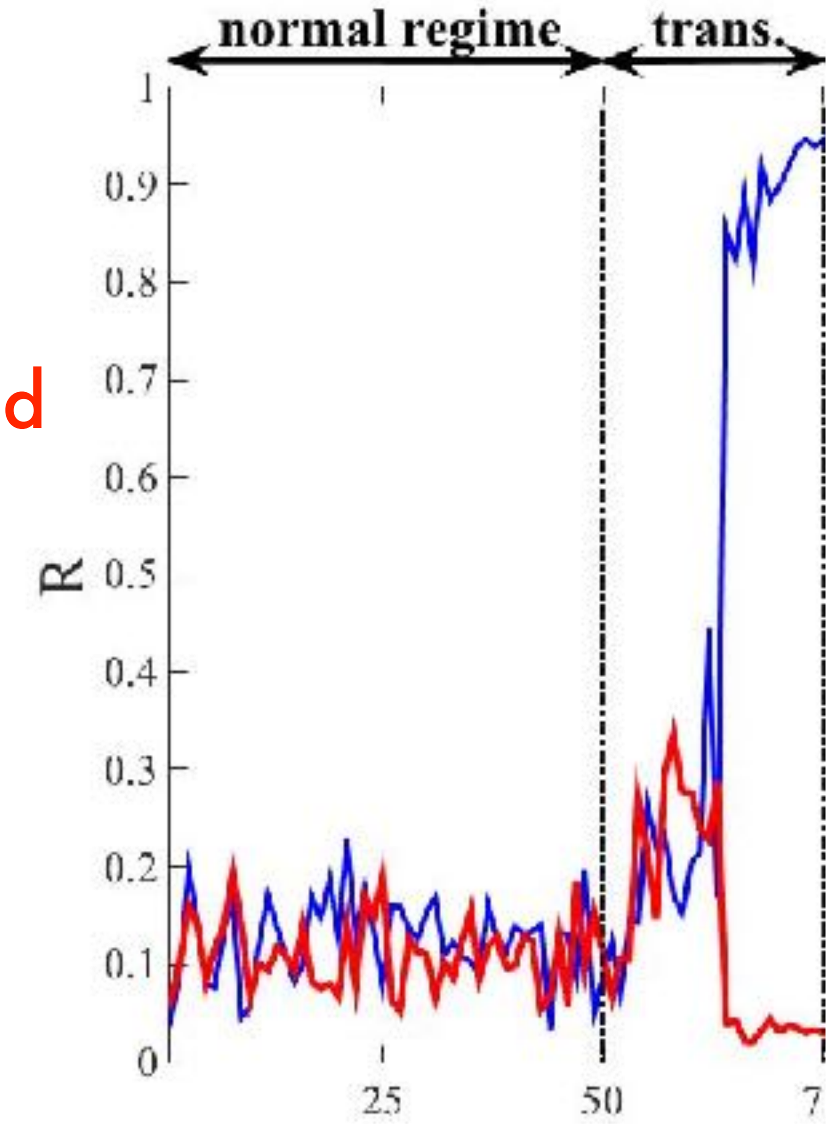

K

Result for the Kuramoto $K \sim 0.1 < 0.4 \sim K_{crit}$

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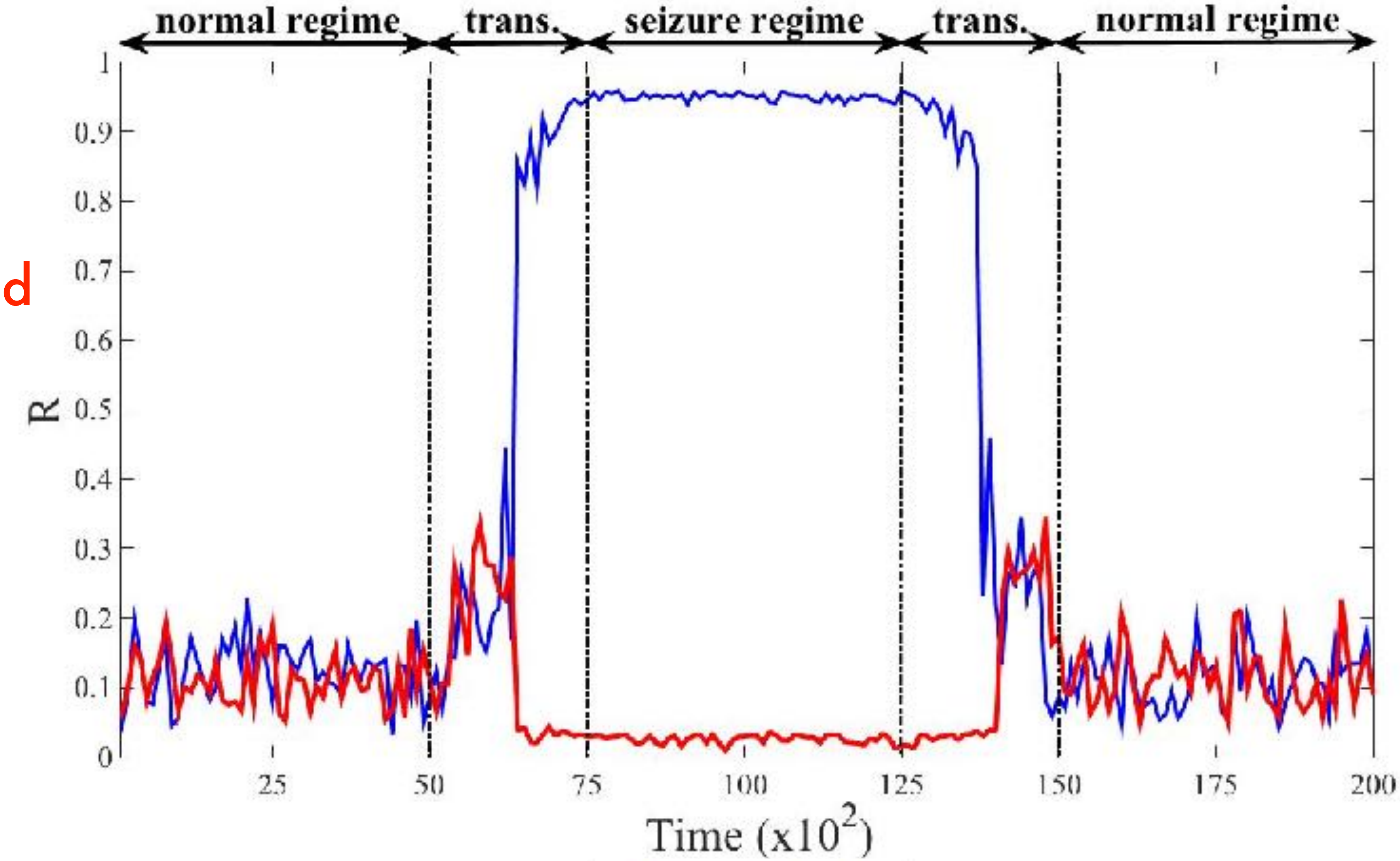
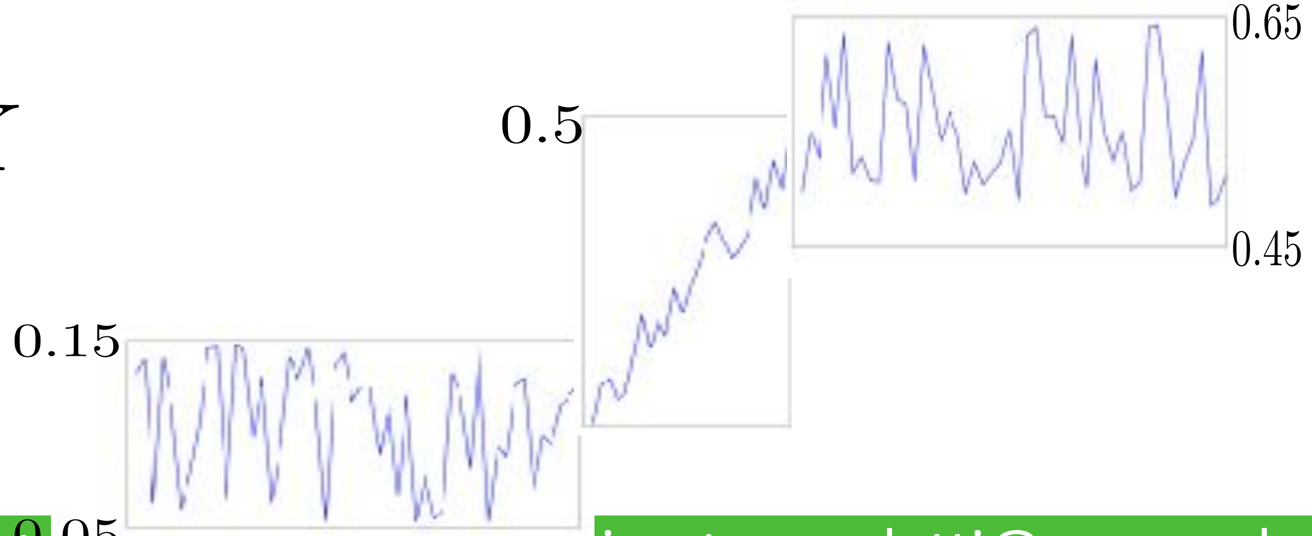
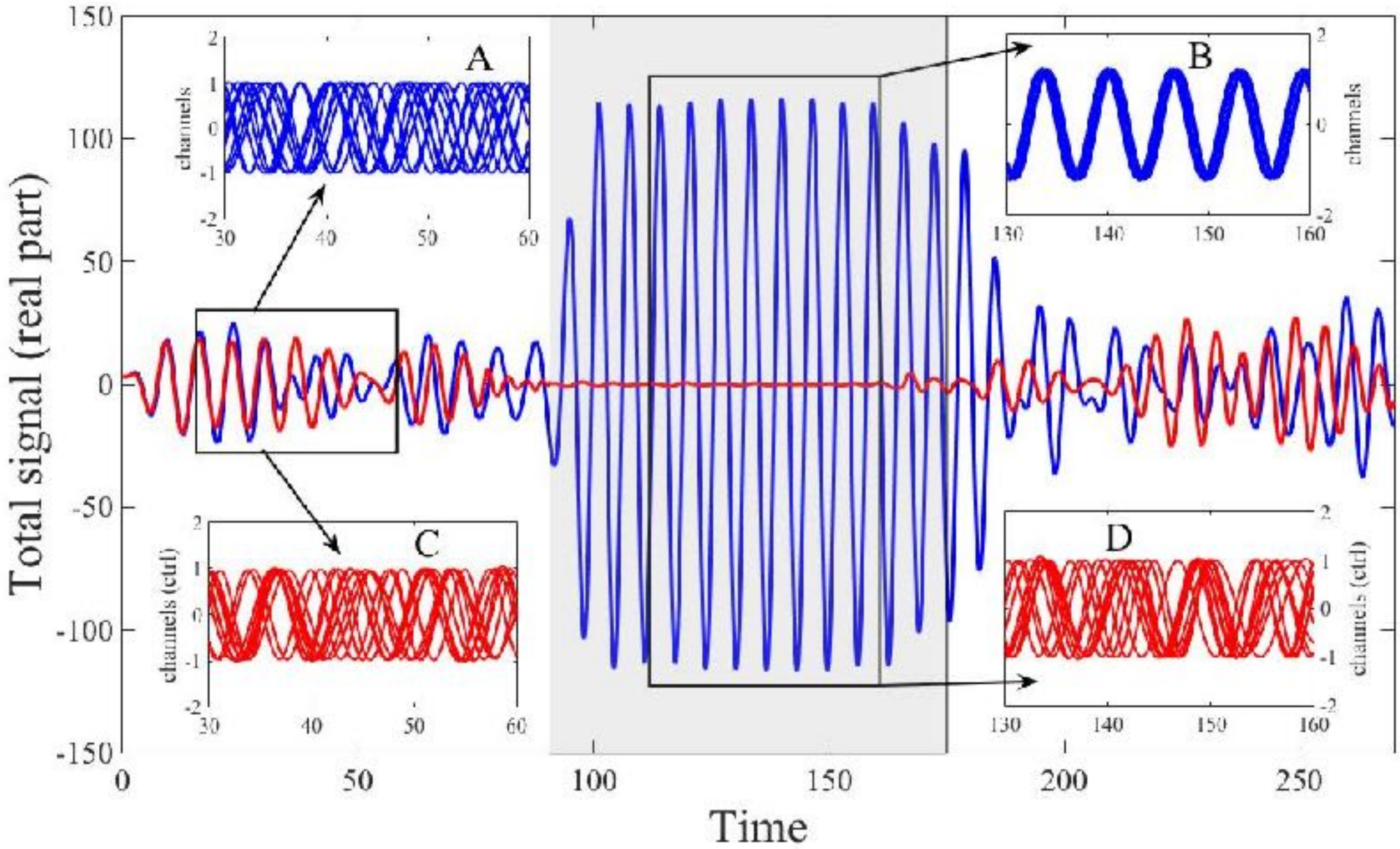


K



From the Kuramoto back to Stuart-Landau

$$\dot{z}_k = (1 + \iota\omega_k - |z_k|^2)z_k + Z_k + Z_k^{ctrl}$$



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On the number of controllers and their strength