Barcelona | November 6 – 8, 2017

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Hopping in the crowd to unveil network topology

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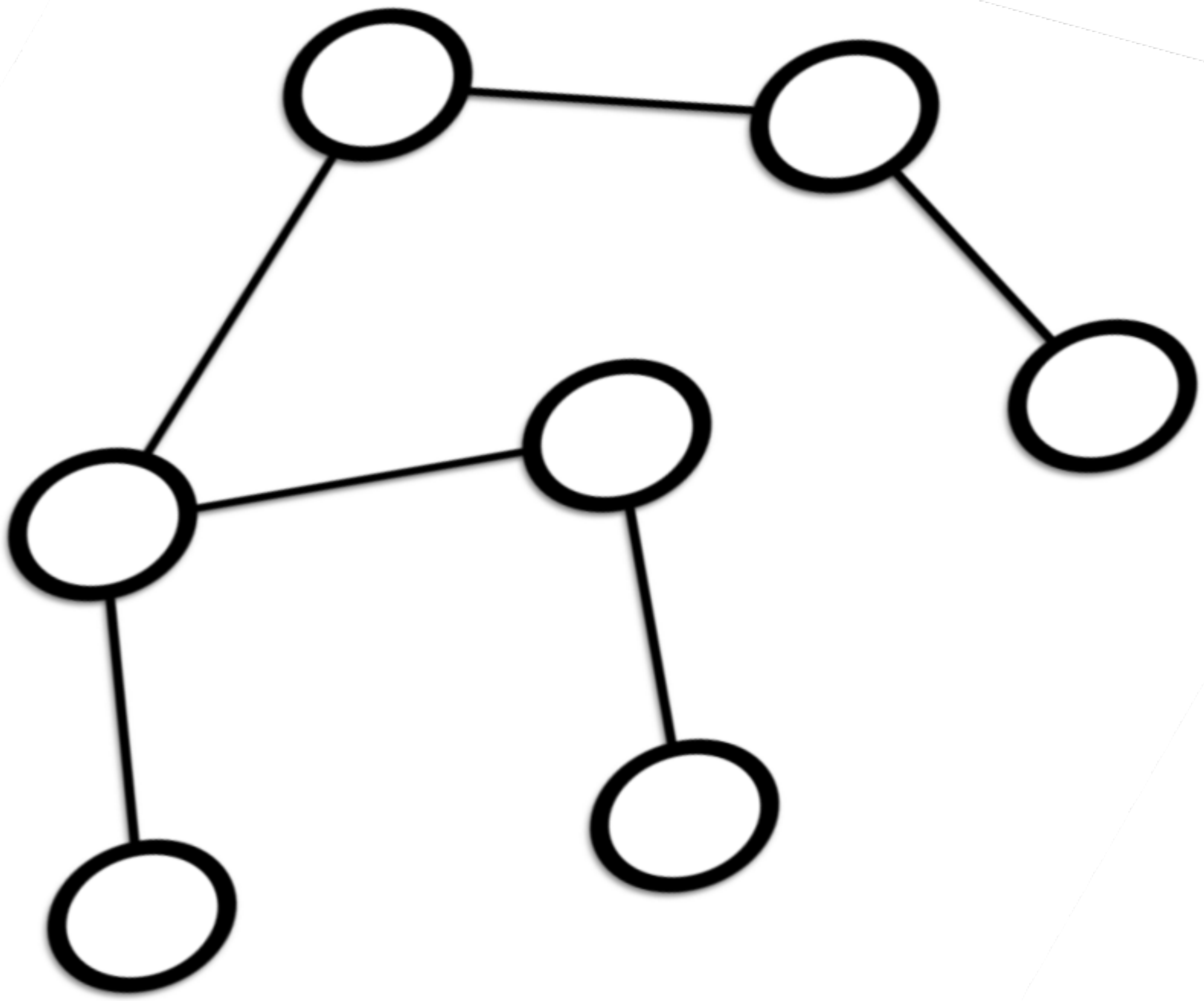
(b) UNIVERSITAS STUDIORUM FLORENTINA
(c) UNIVERSITE D'ORLEANS

Random walk on networks



Number of nodes: Ω

Adjacency matrix: $A_{ij} = A_{ji} = 1$

Nodes degree: $k_i = \sum_j A_{ij}$

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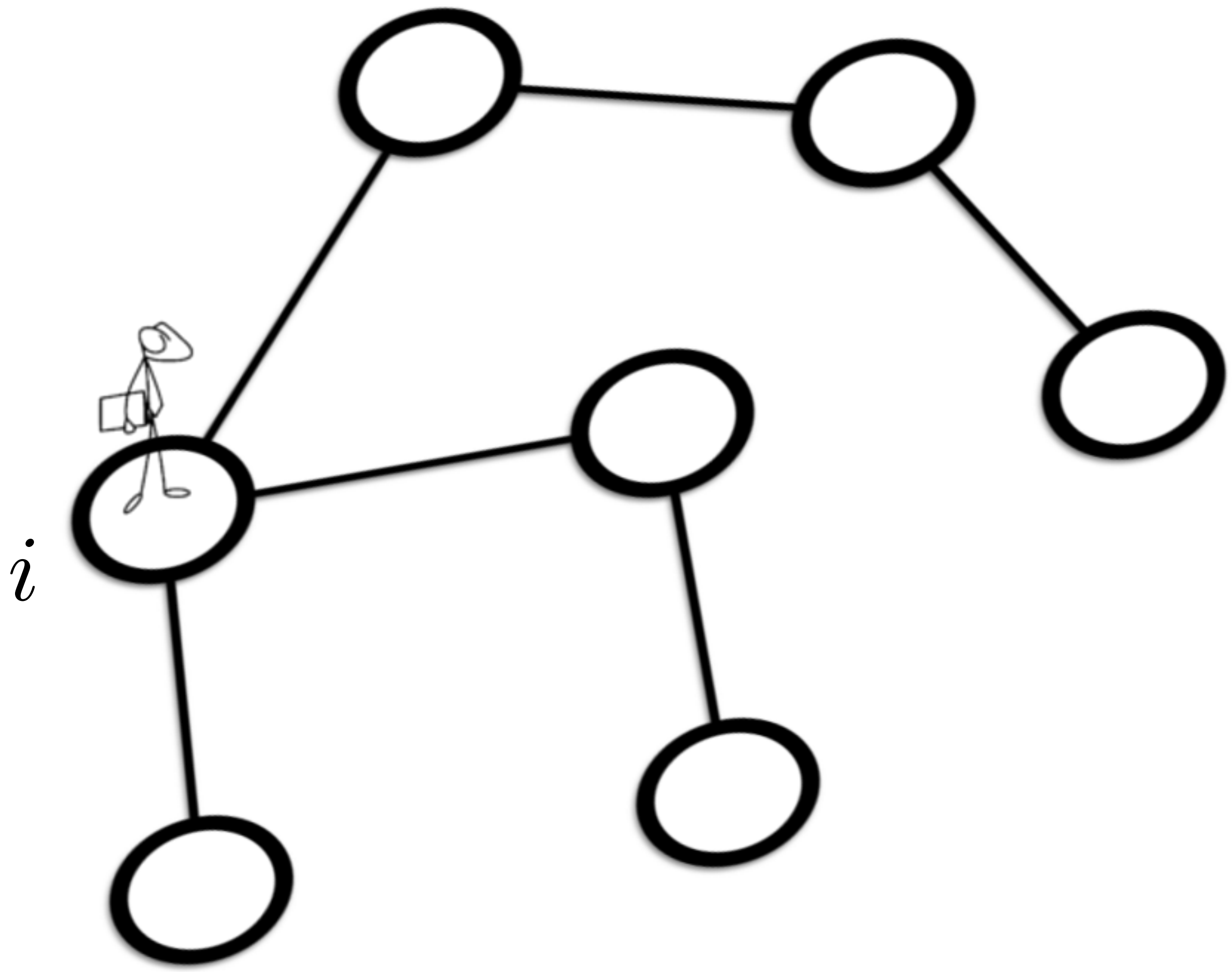

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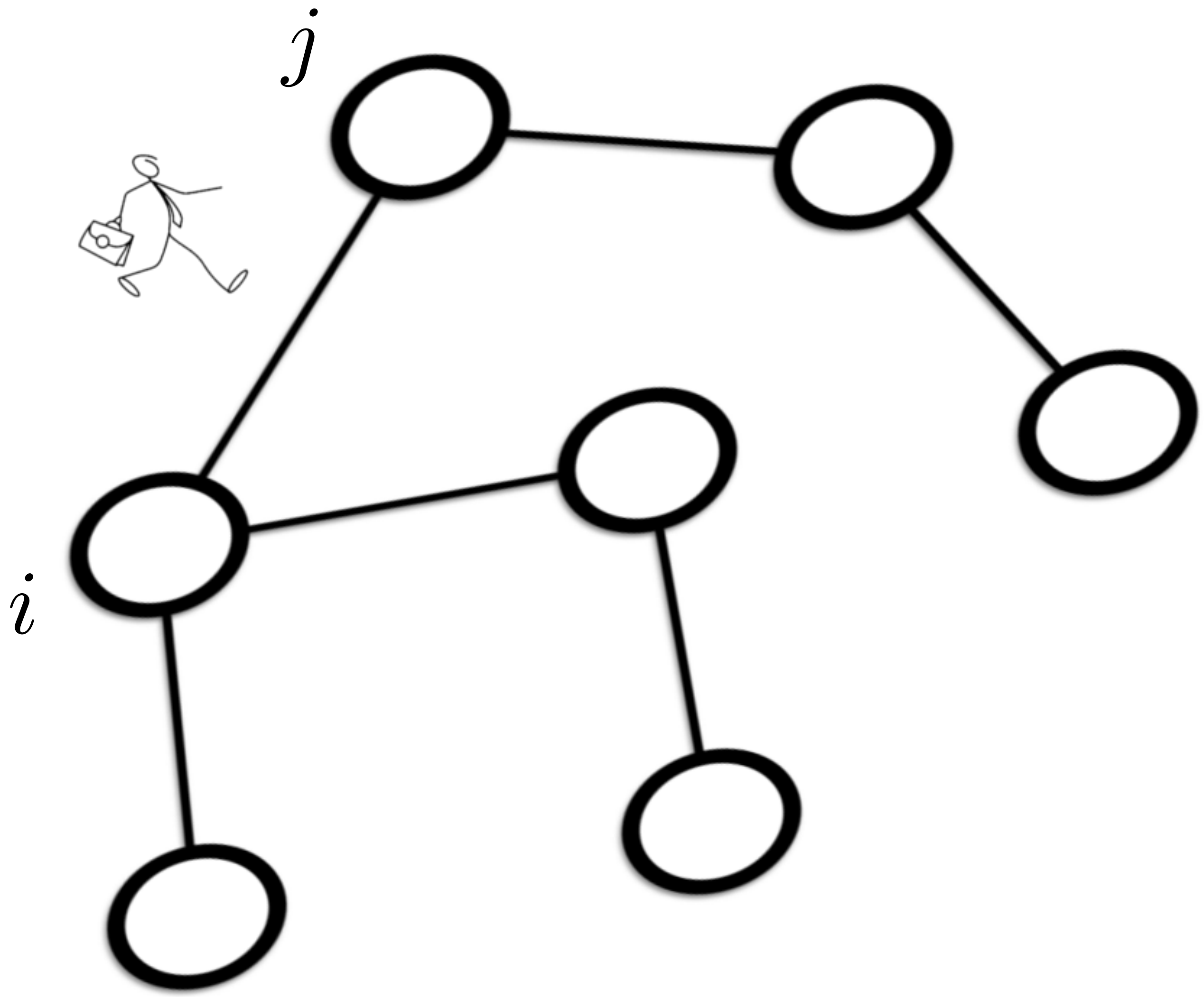

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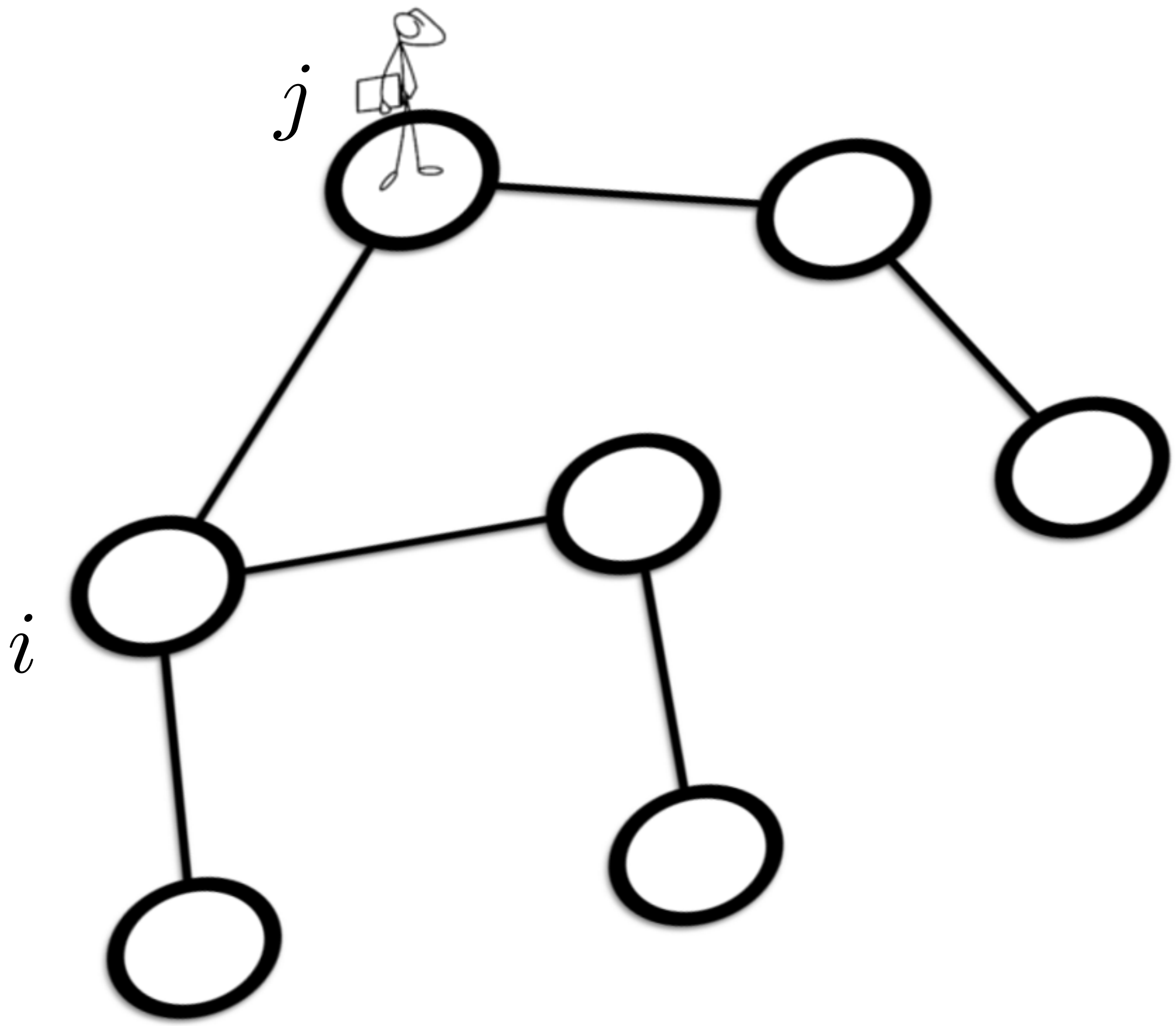

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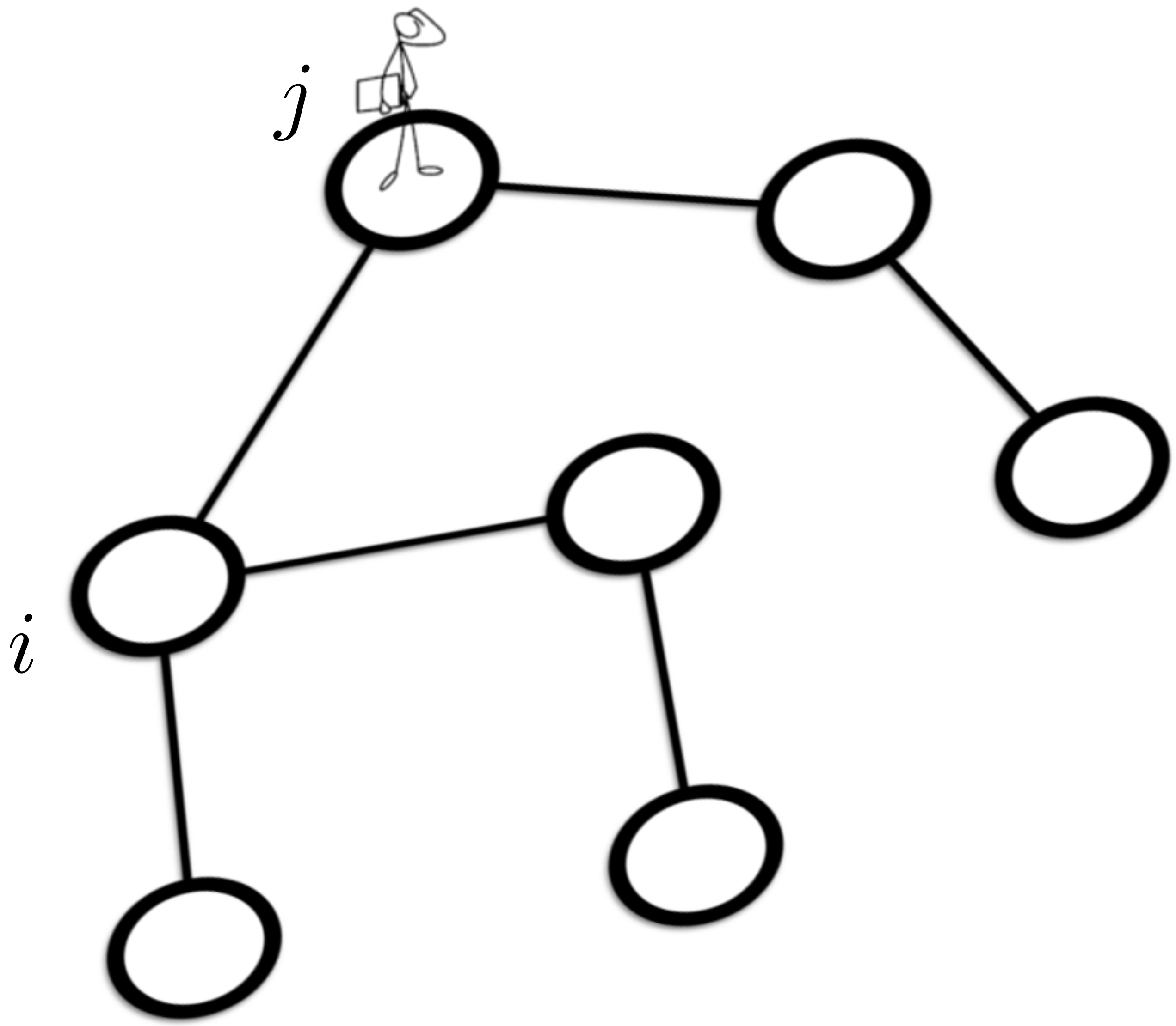

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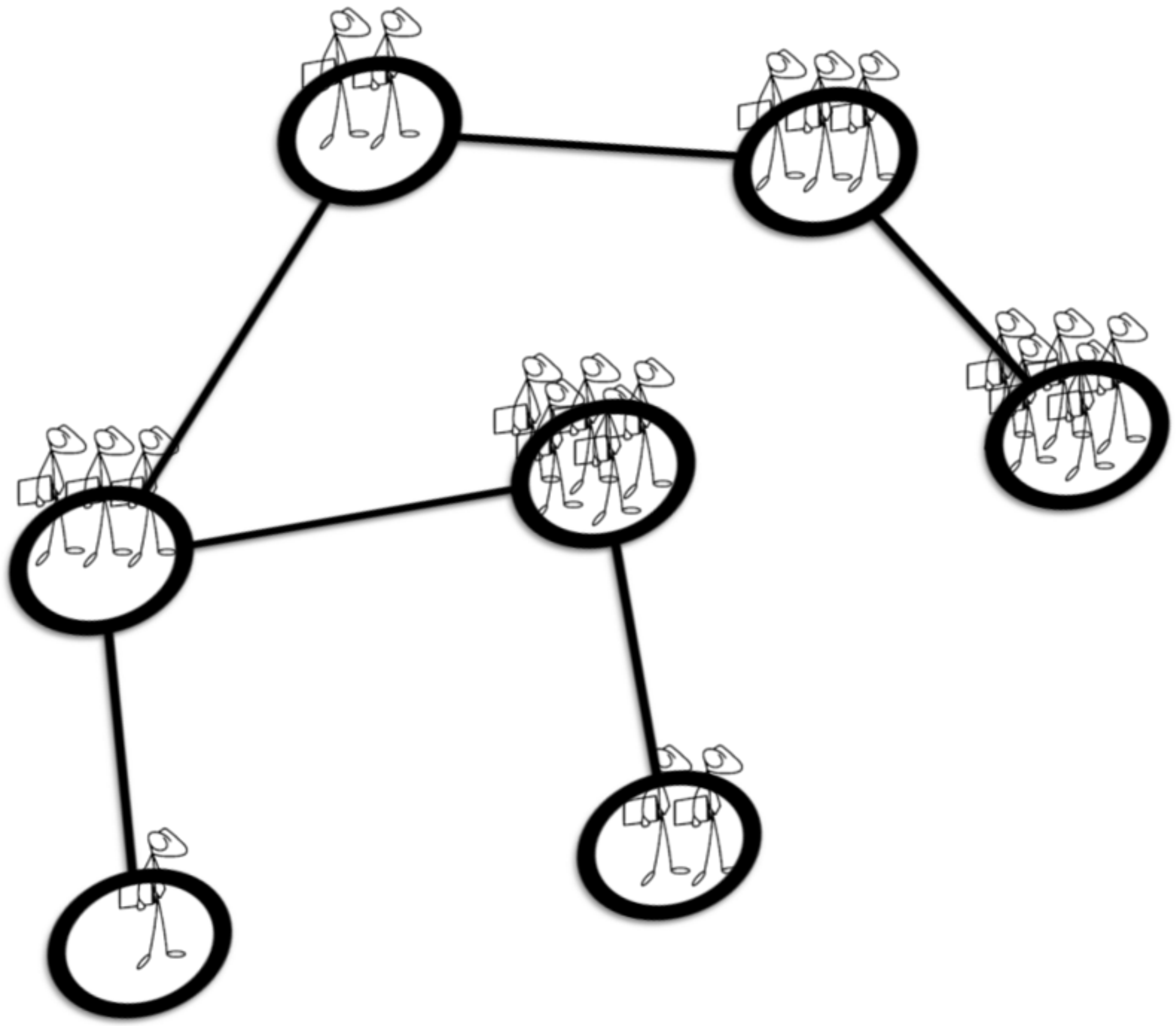
Probability to find the walker on node i at time t: $P_i(t)$

$P_i(t) \sim k_i$ (asymptotically)

Crowding Random walk on networks



Node capacity: N

Number of walkers: $\beta\Omega$

Number walkers at node i: n_i

N=5

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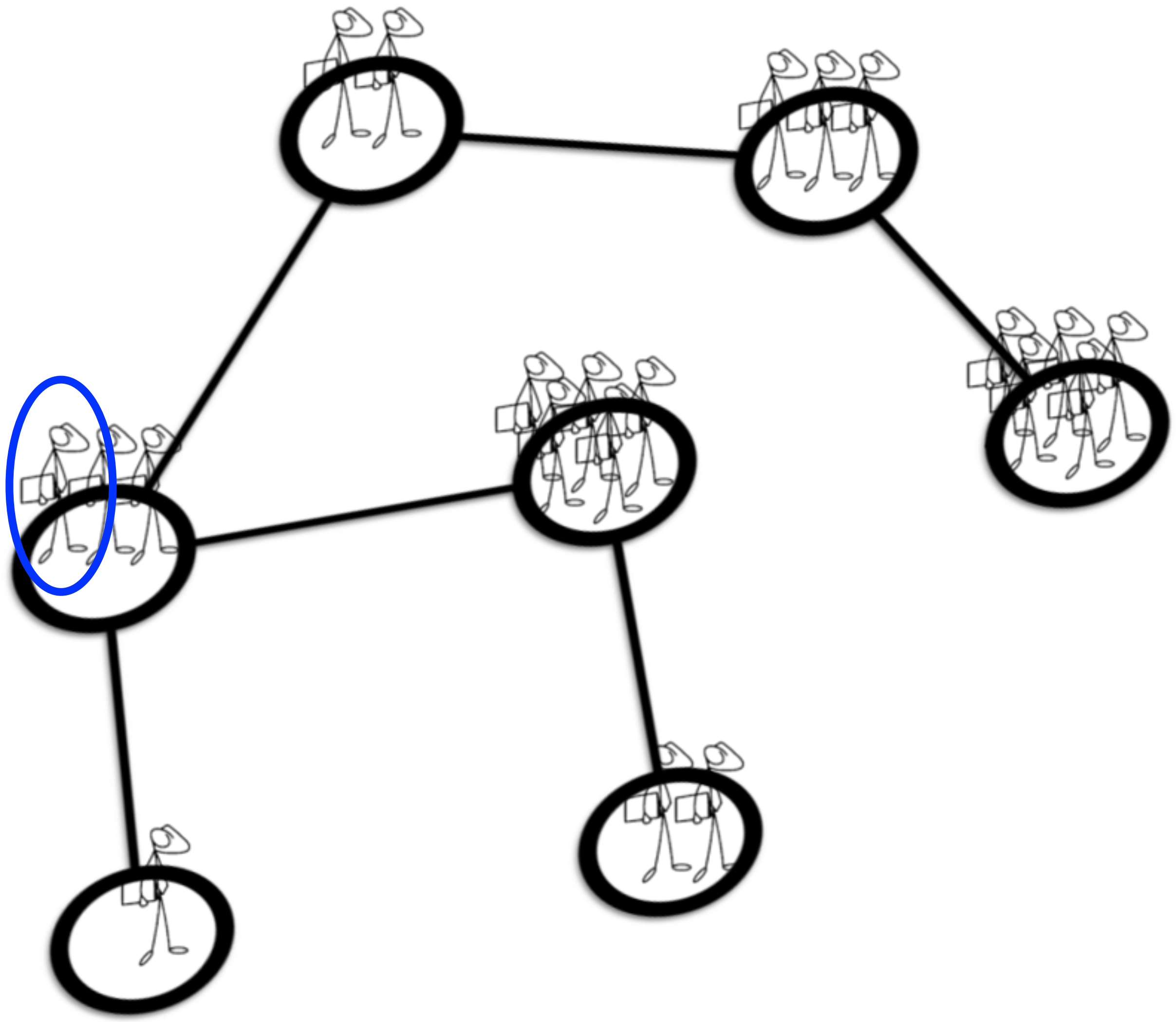

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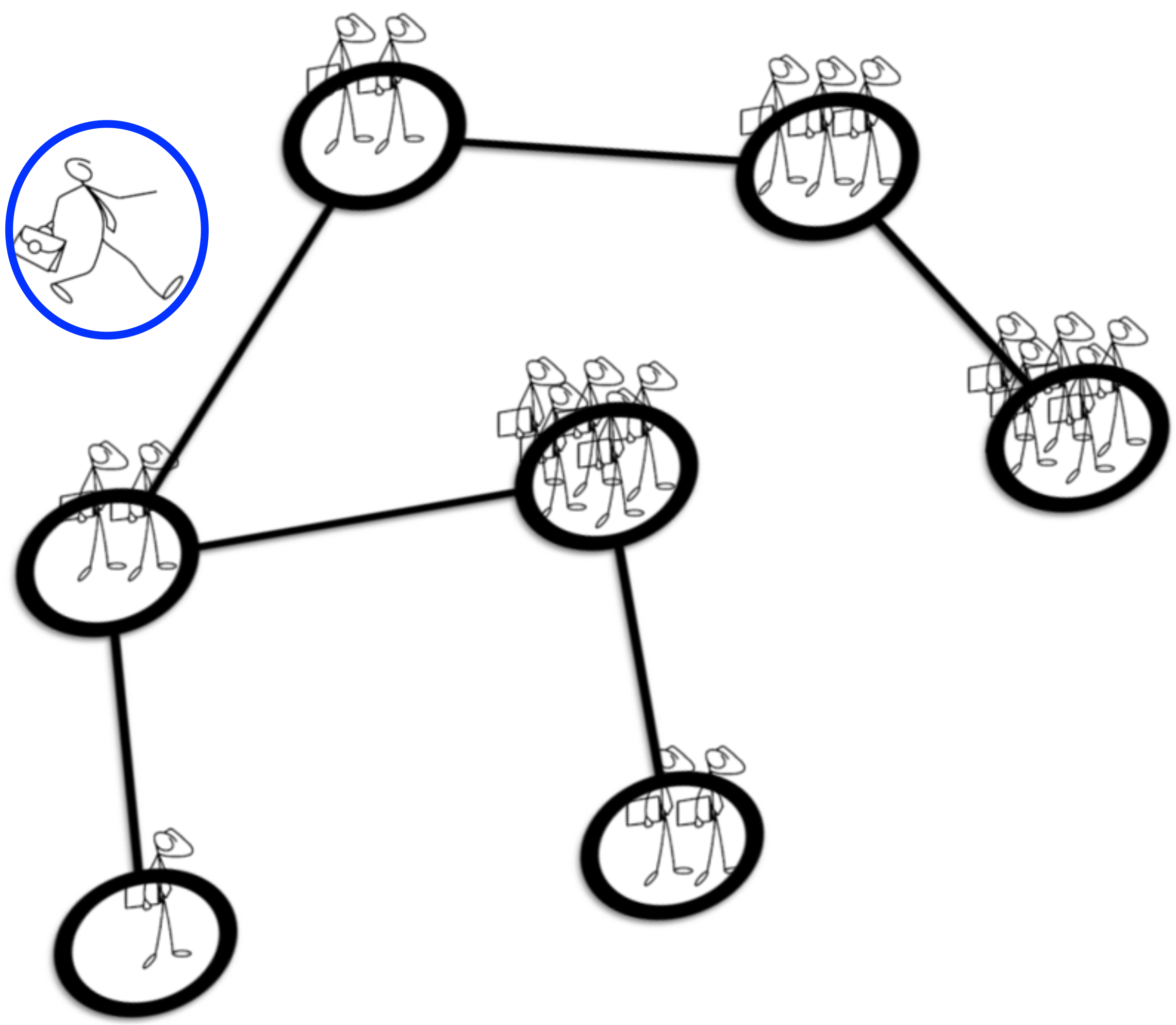

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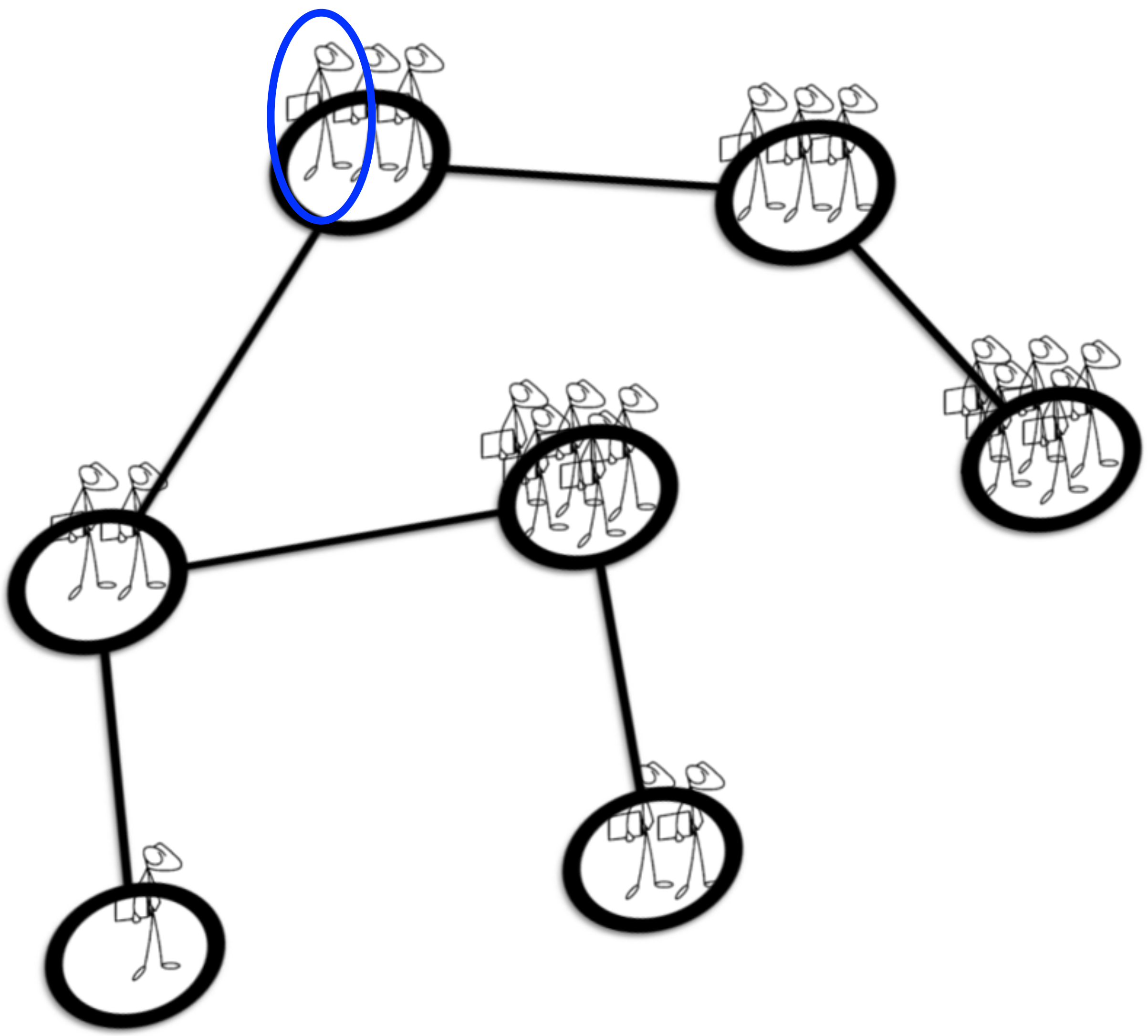

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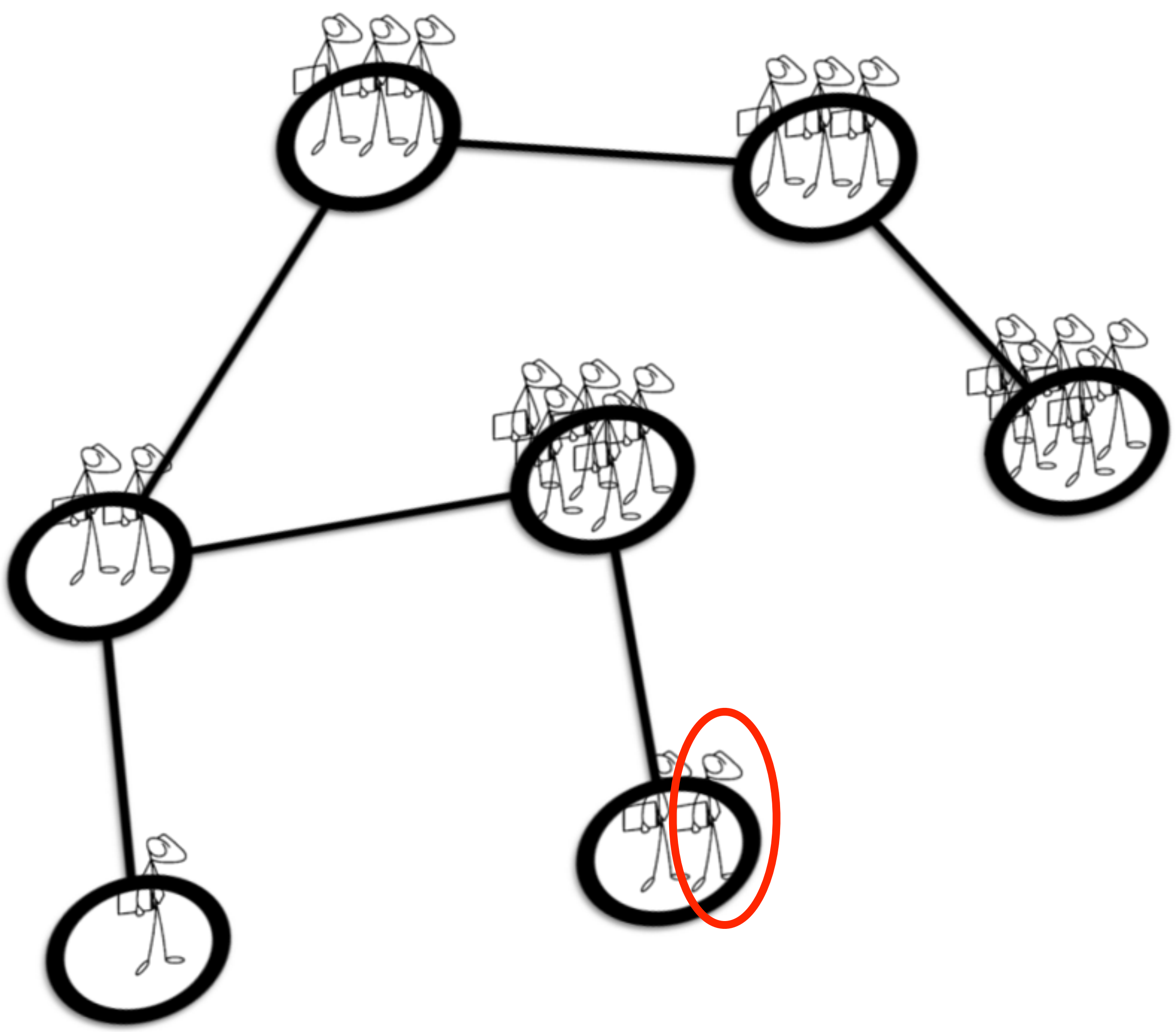

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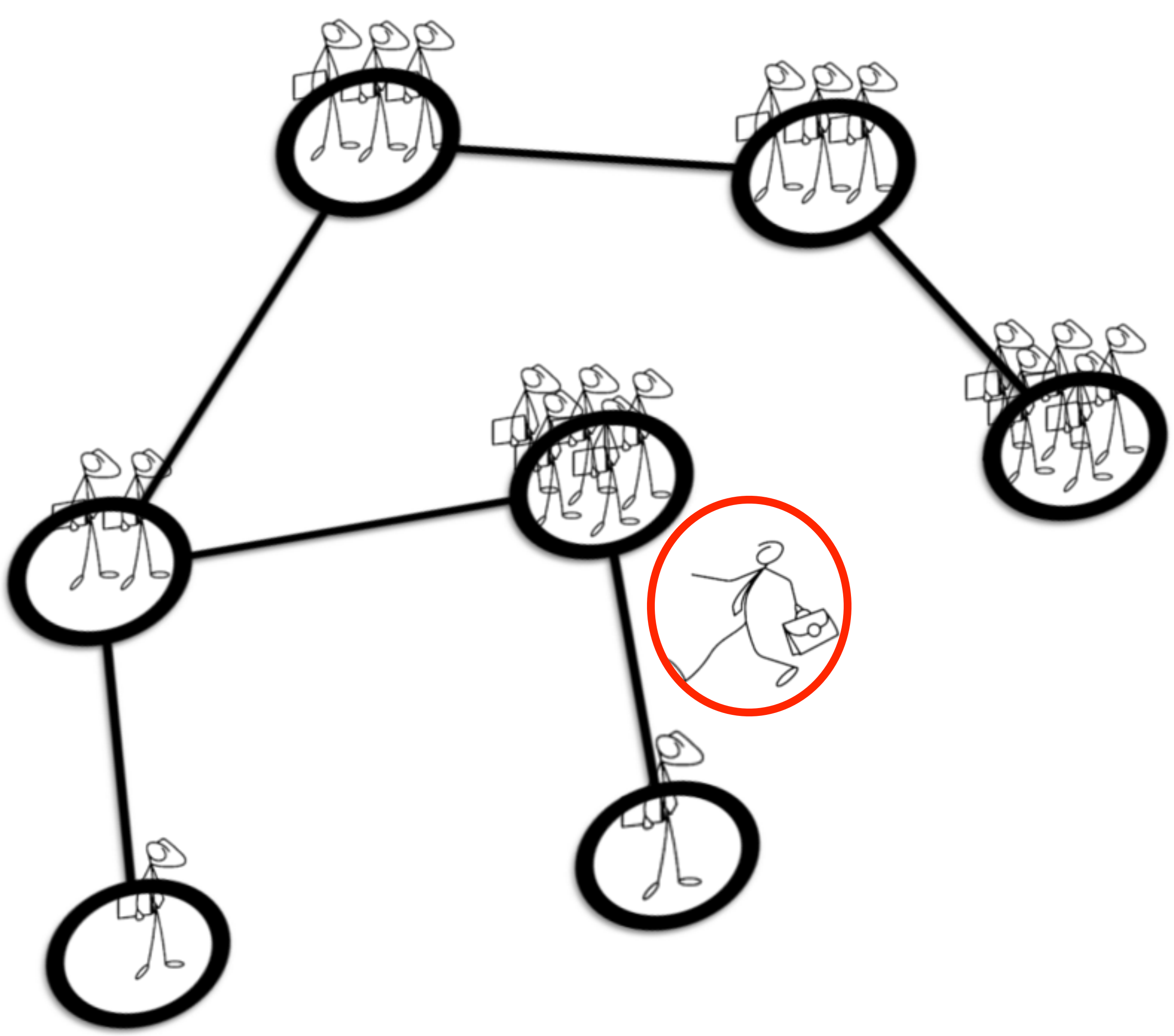

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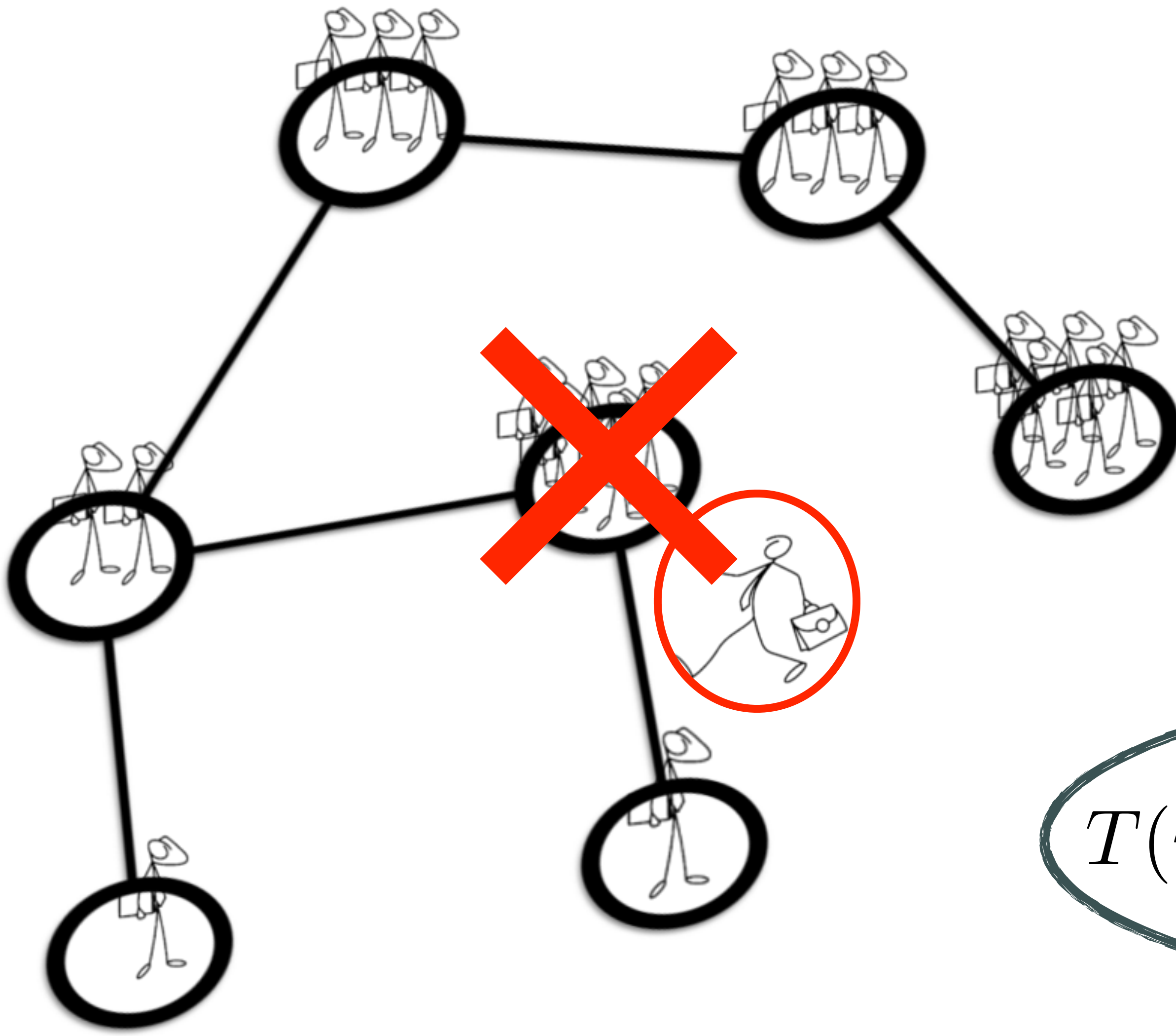

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Transition probability form i to j :

$$T(n_i - 1, n_j + 1 | n_i, n_j) = \frac{A_{ji}}{k_i} \frac{n_i}{N} \frac{(N - n_j)}{N}$$

$P(n_1, \dots, n_\Omega, t)$ Probability to find the system in the state $(n_1, \dots, n_\Omega)$ at time t

A new non-linear transport operator

$$\langle n_i \rangle \equiv \sum_{\mathbf{n}} n_i P(n_1, \dots, n_\Omega, t)$$

$$\rho_i = \lim_{N\to\infty} \langle n_i \rangle / N$$

$$\boxed{\frac{\partial}{\partial t}\rho_i = \sum_{j=1}^{\Omega} \Delta_{ij} \left[\rho_j \left(1 - \rho_i\right) - \frac{k_j}{k_i}\rho_i \left(1 - \rho_j\right)\right]}$$

$$\Delta_{ij} = A_{ij}/k_j - \delta_{ij}$$

$$\beta = \sum_{i=1}^{\Omega} \rho_i(t)$$

The total "mass" is preserved

$$\boxed{\rho_i^\infty = \frac{a k_i}{1 + a k_i} \quad \text{(asymptotically)}}$$

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Application: reconstruction of p(k)

- ✓ The "mass" conservation provides

$$\beta = \sum_k p(k) \frac{a(\beta)k}{1 + a(\beta)k}$$

- ✓ Let β_1 walkers to diffuse in the crowded network
- ✓ From a <u>single node</u> measure ρ_1^∞ and k_1
- ✓ Compute $a_1 := a(\beta_1) = \frac{\rho_1^\infty}{1-\rho_1^\infty}\frac{1}{k_1}$

"experiment"

- ✓ Repeat s times the experiment and compute $a_s = a(\beta_s)$
- ✓ Solve

$$\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_s \end{pmatrix} = \mathbf{F} \begin{pmatrix} p(1) \\ \vdots \\ p(s) \end{pmatrix} \qquad F_{lr} = \frac{ra_l}{1 + ra_l}$$

Application: reconstruction of p(k)