Lyon | November 29th – December 1st, 2017

Timoteo Carletti [a]
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Hopping in the crowd to unveil network topology

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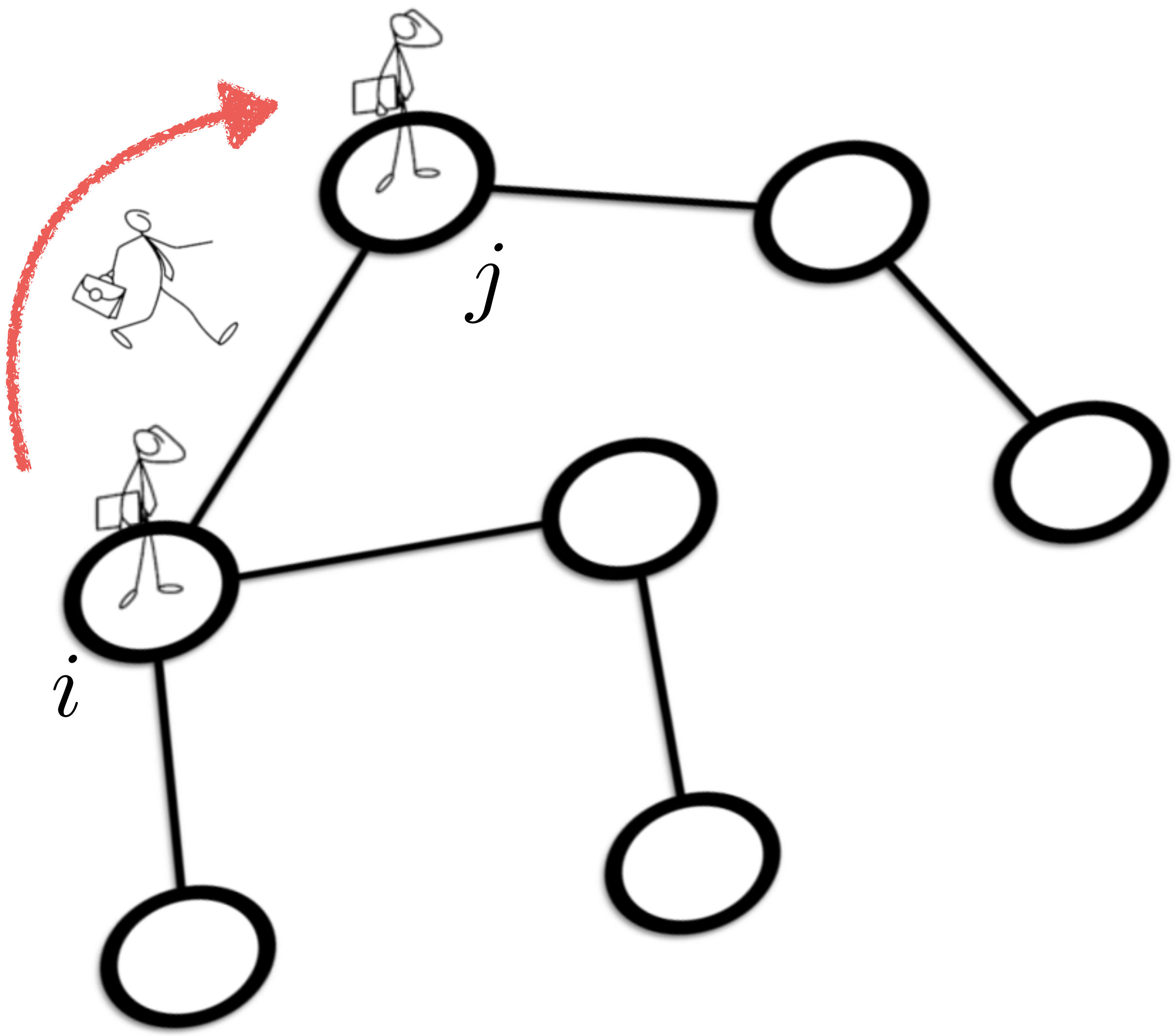
(b)
(c) UNIVERSITE D'ORLEANS

Random walk on networks



Number of nodes: Ω

Adjacency matrix: $A_{ij} = A_{ji} = 1$

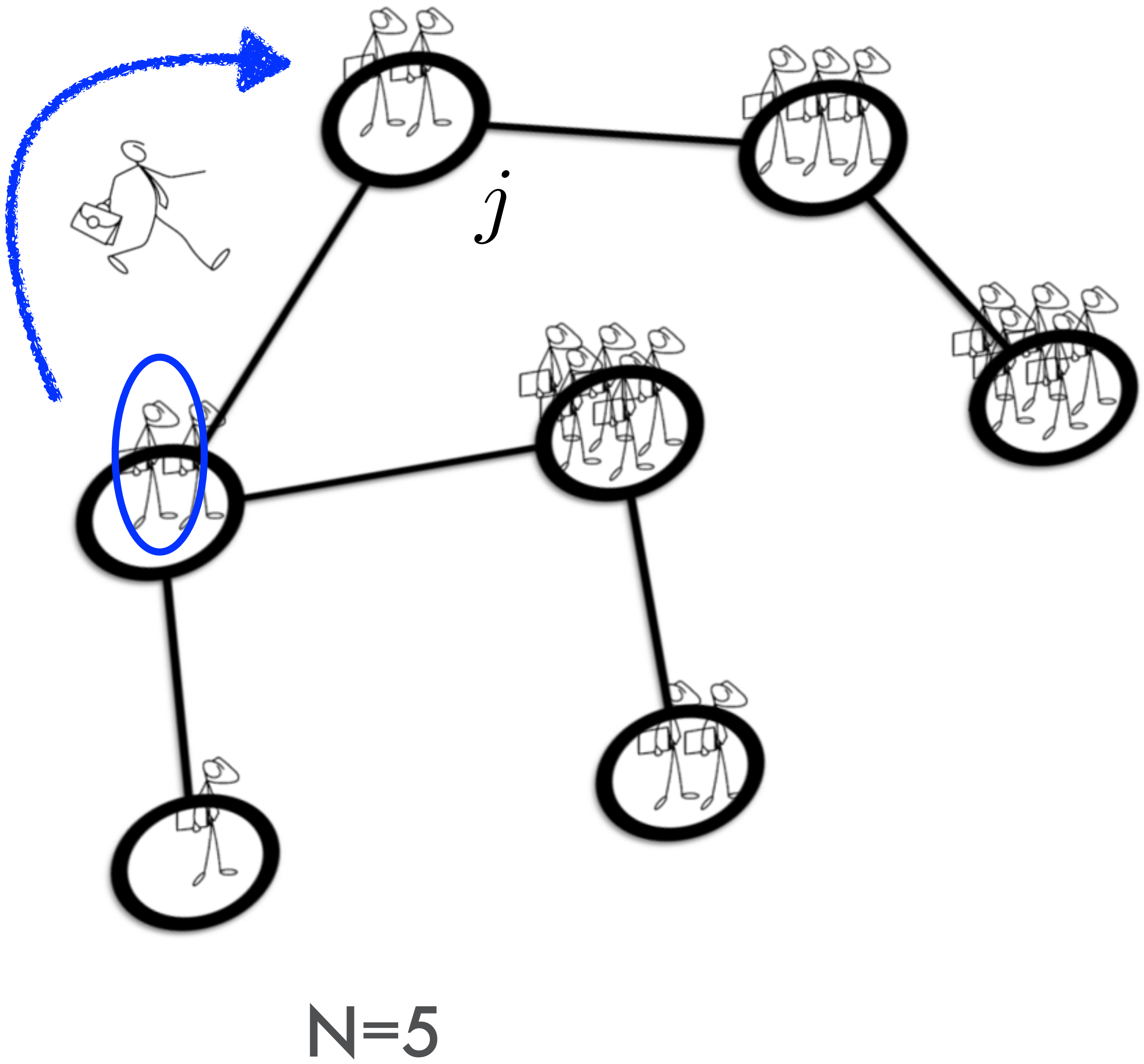
Nodes degree: $k_i = \sum_j A_{ij}$

Transition probability form i to j : $T(j|i) = \frac{A_{ij}}{k_i}$

Probability to find the walker on node i at time t: $P_i(t)$

$P_i(t) \sim k_i$ (asymptotically)

Crowding Random walk on networks

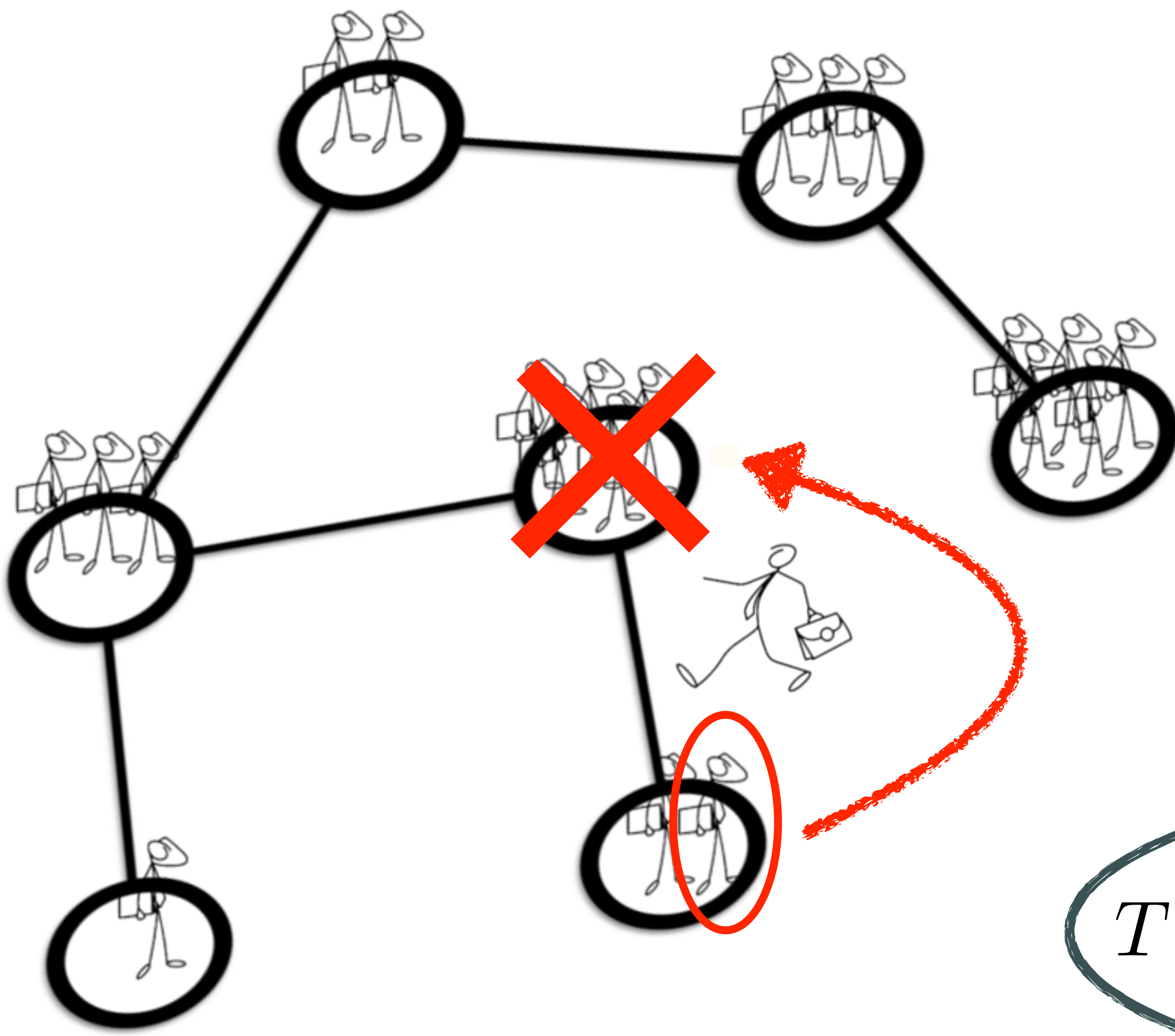


Node capacity: N

Number of walkers: $\beta\Omega$

Number walkers at node i: n_i

Crowding Random walk on networks



Node capacity: N

Number of walkers: $\beta\Omega$

Number walkers at node i: n_i

Transition probability form i to j :

$$T(n_i - 1, n_j + 1 | n_i, n_j) = \frac{A_{ji}}{k_i} \frac{n_i}{N} \frac{(N - n_j)}{N}$$

N=5

$P(n_1, \dots, n_\Omega, t)$ Probability to find the system in the state $(n_1, \dots, n_\Omega)$ at time t

A new non-linear transport operator

$$\langle n_i \rangle \equiv \sum_{\mathbf{n}} n_i P(n_1, \ldots, n_\Omega, t)$$

$$\rho_i = \lim_{N \to \infty} \langle n_i \rangle / N$$

$$\boxed{\frac{\partial}{\partial t} \rho_i = \sum_{j=1}^{\Omega} \Delta_{ij} \left[\rho_j \left(1 - \rho_i\right) - \frac{k_j}{k_i} \rho_i \left(1 - \rho_j\right) \right]}$$

$$\boxed{\rho_i^\infty = \frac{a k_i}{1 + a k_i} \quad \text{(asymptotically)}}$$

$$\Delta_{ij} = A_{ij} / k_j - \delta_{ij}$$

$$\beta = \sum_{i=1}^{\Omega} \rho_i(t)$$

The total "mass" is preserved

Application: reconstruction of p(k)

- ✓ The "mass" conservation provides

$$\beta = \sum_k p(k) \frac{a(\beta)k}{1+a(\beta)k}$$

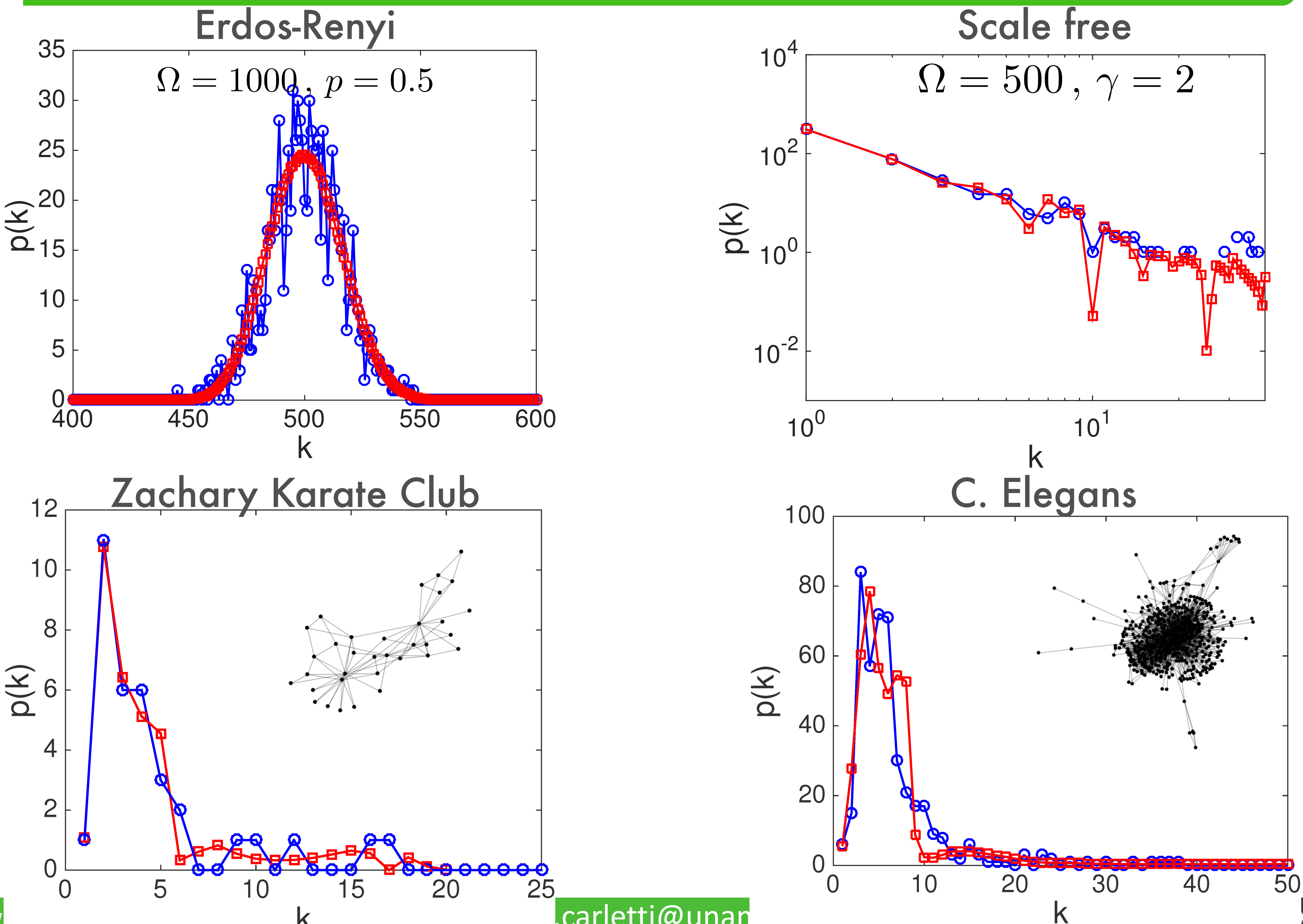
- ✓ Let β_1 walkers to diffuse in the crowded network
- ✓ From a single node measure ρ_1^∞ and k_1
- ✓ Compute $a_1 := a(\beta_1) = \frac{\rho_1^\infty}{1-\rho_1^\infty}\frac{1}{k_1}$

"experiment"

- ✓ Repeat s times the experiment and compute $a_s = a(\beta_s)$
- ✓ Solve

$$\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_s \end{pmatrix} = \mathbf{F} \begin{pmatrix} p(1) \\ \vdots \\ p(s) \end{pmatrix} \qquad F_{lr} = \frac{ra_l}{1+ra_l}$$

Application: reconstruction of p(k)



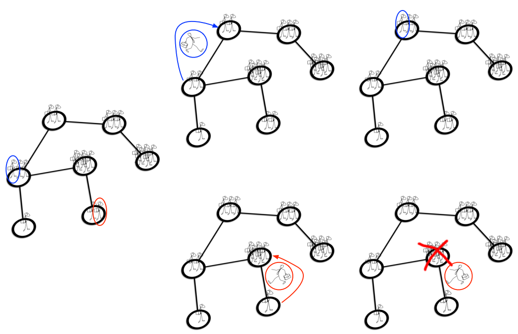
Thank you for your attention

The poster

Hopping in the crowd to unveil network topology

Timoteo Carletti [a], Malbor Asllani [a], Francesca Di Patti [b], Duccio Fanelli [b] and Francesco Piazza [c]

Random walk on a crowded network.
Given an undirected network made by Ω nodes, let $A_{ij}=A_{ji}$ be its adjacency matrix and $k_i=\sum A_{ij}$ the nodes degree. Assume each node can host at most N agents and let n_i be the number of agents in node i.



Walkers hop from node to node (using existing links and if space at destination is available) according to the transition probability:

$$T(n_i-1,n_j+1|n_i,n_j)=\frac{A_{ij}}{k_i}\frac{n_i}{N}\frac{(N-n_j)}{N}$$

A new non-linear transport operator.
Let $P(n_1,\dots,n_\Omega,t)$ be the probability to find the system in state $(n_1,\dots,n_\Omega)$ at time t. Let denote the average density of node i : $\rho_i=\lim_{N\to\infty}\langle n_i\rangle/N$
Then

$$\frac{\partial}{\partial t}\rho_i=\sum_{j=1}^{\Omega}\Delta_{ij}\left[\rho_j(1-\rho_i)-\frac{k_j}{k_i}\rho_i(1-\rho_j)\right]$$

where $\Delta_{ij}=A_{ij}/k_j-\delta_{ij}$ is the Random-walk Laplacian.

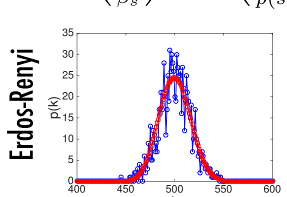
a) $\rho_i^\infty=\frac{ak_i}{1+ak_i}$ asymptotically

b) The "total mass" is preserved $\beta=\sum_{i=1}^{\Omega}\rho_i(t)$

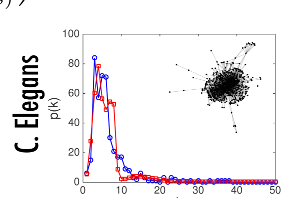
Network reconstruction.
The mass conservation implies

$$\beta=\sum_k p(k)\frac{a(\beta)k}{1+a(\beta)k}$$

i) Let β_1 walkers to diffuse in a crowded network ;
ii) From a single node, measure ρ_1^∞ and k_1
iii) Then compute $a_1:=a(\beta_1)=\frac{\rho_1^\infty}{1-\rho_1^\infty}\frac{1}{k_1}$
iv) Repeat s times and get $a_s=a(\beta_s)$
v) Solve $\begin{pmatrix}\beta_1\\ \vdots\\ \beta_s\end{pmatrix}=\mathbf{F}\begin{pmatrix}p(1)\\ \vdots\\ p(s)\end{pmatrix}$ $\quad F_{lr}=\frac{ra_l}{1+ra_l}$



$\Omega=1000$, $p=0.5$



$\Omega=453.76$

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The preprint: Arxiv:1711.06733

arXiv.org > cond-mat > arXiv:1711.06733

Condensed Matter > Statistical Mechanics

Hopping in the crowd to unveil network topology

Malbor Asllani, Timoteo Carletti, Francesca Di Patti, Duccio Fanelli, Francesco Piazza

(Submitted on 17 Nov 2017)

We introduce a nonlinear operator to model diffusion on a complex undirected network under crowded conditions. We show that the asymptotic distribution of diffusing agents is a nonlinear function of the nodes' degree and saturates to a constant value for sufficiently large connectivities, at variance with standard diffusion in the absence of excluded-volume effects. Building on this observation, we define and solve an inverse problem, aimed at reconstructing the a priori unknown connectivity distribution. The method gathers all the necessary information by repeating a limited number of independent measurements of the asymptotic density at a single node that can be chosen randomly. The technique is successfully tested against both synthetic and real data, and shown to estimate with great accuracy also the total number of nodes.

Subjects: Statistical Mechanics (cond-mat.stat-mech); Physics and Society (physics.soc-ph)
Cite as: arXiv:1711.06733 [cond-mat.stat-mech]
(or arXiv:1711.06733v1 [cond-mat.stat-mech] for this version)

Submission history
From: Timoteo Carletti [view email]
[v1] Fri, 17 Nov 2017 21:37:18 GMT (3222kb,D)

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