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A numerical algorithm to find the equilibrium of a conservative system

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A numerical algorithm to find the equilibrium of a conservative system

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Why this algorithm?

• Aim: determine the exact location of a dynamical equilibrium (forced oscillations only) corresponding to a resonance



This algorithm has been used in some works yet:

- Problem: accurate (realistic) simulations give some parasitical (free) librations supposed to be damped
 - the initial conditions given by analytical studies are not accurate enough
- Solution: this algorithm use NAFF^[4] (Numerical Analysis of the Fundamental Frequencies) for the identification of the free and forced oscillations, the former being iteratively removed from the solution by carefully choosing the initial conditions.
- - Rotation problem ^[3,5,7]
 - Exoplanetary dynamics ^[1]
 - Dynamics of a probe around Vesta ^[2]

• ...

Here^[6] we provide the convergence proof and give the quadratic convergence in the Hamiltonian case.

	The .	An example					
• Step 1, n=0	Step 0 Initialisation	• Choose an initial condition \vec{x}_0 close to the equilibrium (e.g. using an analytical solution) and set n=0	Earth-Moon Spin-Orbit resonance: $H(y, \sigma, L, \lambda) = \frac{y^2}{2}$ The angular f momentum				
	Step 1 Integration	• Obtain the evolution of \vec{x} with respect to the time: $\vec{x}(t)$	$-\epsilon \left[\alpha_1 \cos(2\sigma + \lambda) + \alpha_2 \cos(2\sigma) & \sigma \right] $ $+\alpha_3 \cos(2\sigma - \lambda) $ $The integration is a constant of the integral of$				
• Step 1, n=1	Step 2	• Express the variable \vec{x} of the problem by a quasi-periodic decomposition	$+\alpha_4 \cos(2\sigma - 2\lambda) \qquad \qquad$				

Image: state stat	Identification Step 3 New Initial	 (e.g. using NAFF^[4]) Isolate the free (to remove) and the forced (to keep) oscillations Remove the free oscillations from the initial conditions <i>x</i>_n to obtain the new initial conditions <i>x</i>_{n+1} 	• Step 0: the averaged Hamiltonian $\bar{H}(y, \sigma, L, -) = \frac{(y-1)^2}{2} - \epsilon \alpha_2 \cos(2\sigma) + L$ gives the first initial condition: $y_0 = 0$, $\sigma_0 = 0$ • Step 1 (numerical integration) + Step 2 (NAFE) Frequency decomposition of v(t)
Step I, n=Z Image: step I and the step I	Conditions	$x_{n+1} = x_n - \sum_{i \in \text{Free terms}} \text{Ampl}_i \cos(\text{Phase}_i)$ • Increase n: n=n+1 • Go to Step 1 until $\ \vec{x}_{n+1} - \vec{x}_n\ < \varepsilon_{IC}$ or until $\text{Ampl}_i < \varepsilon_{\text{Ampl}} \forall i$	$\frac{N}{1} \frac{\text{Ampl.}}{1.000000} \frac{\text{Freq.}}{0.00000} \frac{\text{Phase (rad)}}{2.462 \times 10^{-14}} \frac{\text{Id.}}{1.000000}$ $\frac{2}{2.462 \times 10^{-14}} \frac{\text{Id.}}{2.462 \times 10^{-14}}$ $\frac{3}{2} \frac{3.010500 \times 10^{-5}}{10^{-5}} \frac{2.61686 \times 10^{-2}}{2.00000} \frac{3.141588}{3.141587} \frac{\lambda}{2\lambda}$ $\frac{4}{5} \frac{4.392058 \times 10^{-6}}{3.328440 \times 10^{-7}} \frac{2.00000}{3.00000} \frac{3.141587}{3.141587} \frac{3\lambda}{3\lambda}$ $\frac{1}{1000} \frac{1}{1000} \frac$
Convergen Let $\dot{\vec{X}} = f(\vec{X}) + g(\vec{X}, \vec{X})$	t) with the solution	$y_1 = y_0 - 8.01050010^{-5} \cos(0.000008) = 0.999920$ $\sigma_1 = \sigma_0 - 3.06110710^{-3} \cos(1.570796) = 3.10699110^{-10}$ • Some iterations of the algorithm	
$\phi(t;\vec{X}) = \sum_{m \in \mathbb{Z}}$	$\phi_{0m}(\vec{X})e^{\imath\nu mt} + \sum_{l\neq 0}^{l}$	$\sum_{0,m\in\mathbb{Z}}\phi_{lm}(\vec{X})e^{i(\omega l+\nu m)t} := S(t;\vec{X}) + L(t;\vec{X})$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

and the fixed point \vec{X}_{∞} such as $\phi(t; \vec{X}_{\infty}) = S(t; \vec{X}_{\infty})$. Assuming an Hamiltonian framework and an initial condition \vec{X}_0 such as $|\vec{X}_0 - \vec{X}_\infty| < 1$. Then, the algorithm gives a sequence $(\vec{X}_n)_n$ where $\vec{X}_n \longrightarrow \vec{X}_\infty$ and the convergence rate is quadratic $|\vec{X}_{n+1} - \vec{X}_{\infty}| \propto |\vec{X}_n - \vec{X}_{\infty}|^2$ Idea of proof (one dim. to simplify)

We have to prove that x_{∞} is an attractor:

$$|f'(x_{\infty})| < 1 \iff \lim_{x \to x_{\infty}} \frac{\partial_x S(0;x) / \partial_x L(0;f(x))}{\partial_x S(0;f(x)) / \partial_x L(0;f(x)) + 1} = 0$$

Using the d'Alembert rule we can state that (x close to x_{∞})

 $S(0;x) \sim x_{\infty} + a|x - x_{\infty}| + \dots$ and $L(0;x) \sim x_{\infty} + b\sqrt{|x - x_{\infty}| + \dots}$

Then, $\partial_x S(0;x) / \partial_x L(0;f(x)) \xrightarrow[x \to x_{\infty}]{} 0$ and the convergence rate is quadratic.

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	T	σ	3.106	991 0	$59 \times$	10^{-10}	2.	969 !	532 ×	(10^{-8})	³ 2	2.6168	$370 \times$	10^{-2}	
	<u> </u>	У	0.999	919 90)4		1.	264	196 ×	(10^{-1})	¹⁵ 2	2.6168	$360 \times$	10^{-2}	
	Z	σ	3.490	283 08	$35 \times$	10^{-15}	4.	830 9	976 ×	(10^{-1})	^{L4} 2	2.6168	$360 \times$	10^{-2}	
~	 Final results w.r.t first integration 														
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	Re	efere	nces :												

[1] Couetdic J. et al (2010), Astronomy and Astrophysics, 519 [2] Delsate N. (2011), Planetary and Space Science, 59 [3] Dufey J. et al. (2009), Icarus, 203 [4] Laskar J. (1993), Celestial Mechanics and Dynamical Astronomy, 56 [5] Noyelles B. (2009), Icarus, 202 [6] Noyelles et al., arXiv:1101.2138 [7] Robutel P. et al. (2011), Icarus, 211