July the 26th, 2018, Tokyo, Japan

ALIFE 2018

THE 2018 CONFERENCE ON ARTIFICIAL LIFE

Jul. 23-27, 2018
TOKYO


Marlbor Asllani    Timoteo Carletti


# Topological resilience in non-normal networked systems


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Namur Institute for Complex Systems

UNIVERSITÉ DE NAMUR

# Acknowledgements


## Main collaborators:

"Belgian" team:
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J. Petit (PhD)
A. Bellière, G. Planchon, R. Muolo (Master students)

Italian team:
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IAP VII/19 - DYSCO

## Warm up

Order from disorder is a leitmotif in Nature.

It emerges in a variety of wonderful self-organized shapes, colours and rhythms.

e.g. spots and stripes on the coat or the skin of animals, spatial distribution of vegetation, insect colonies, large complex ecosystems … and **life** itself.

Can we provide a simple and generic answer to such complex phenomenon?


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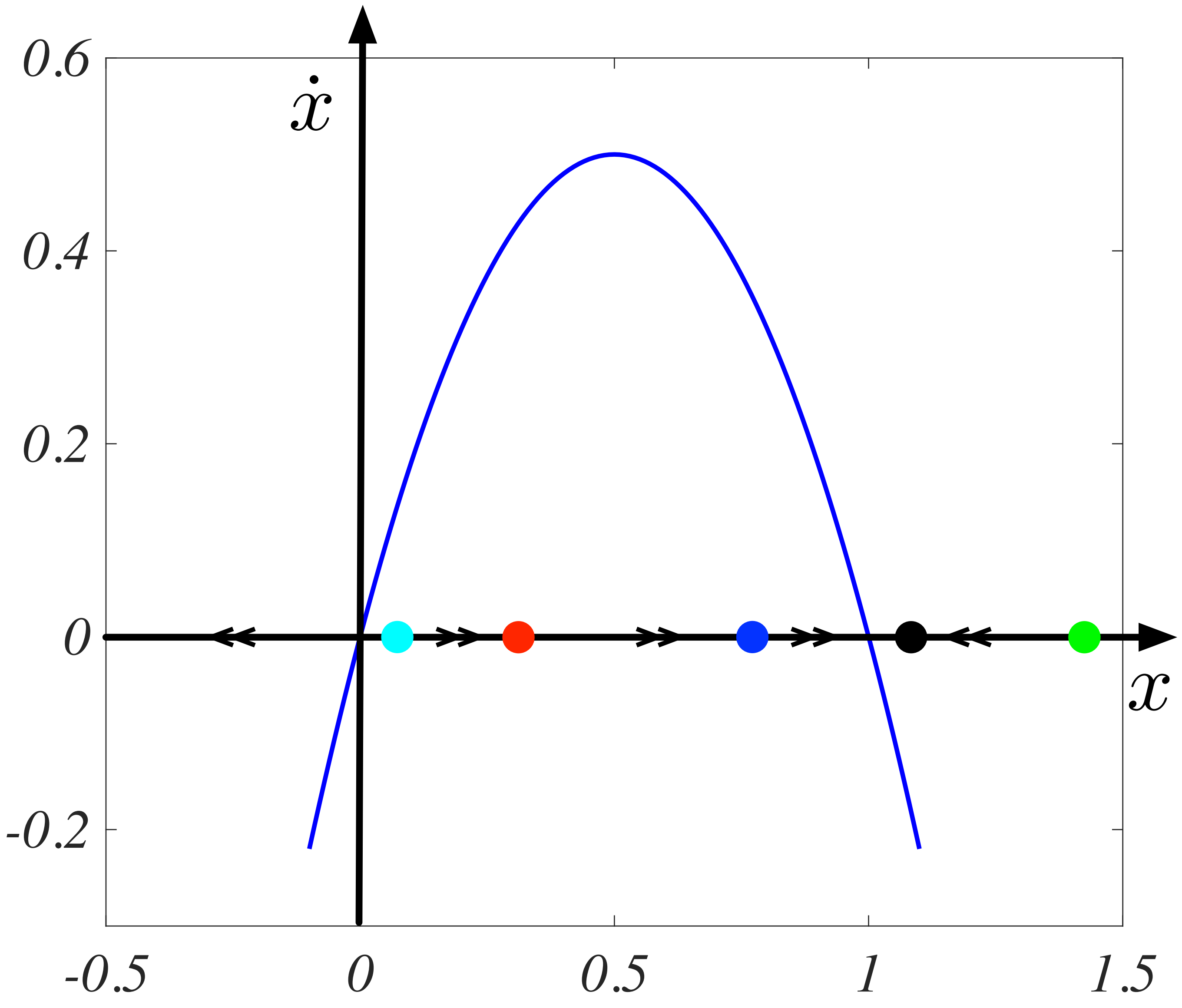
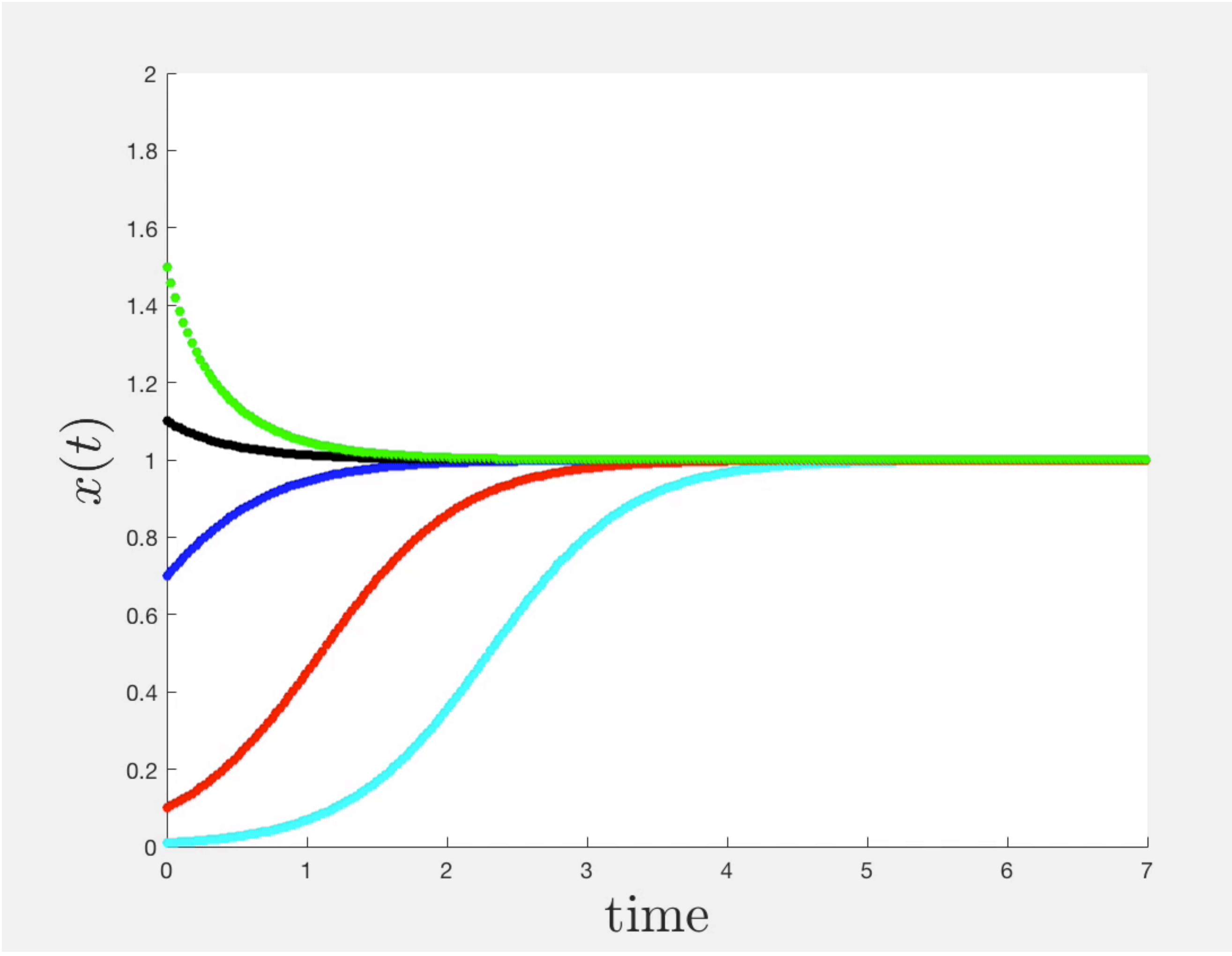
# Motivating example

Logistic model $\dot{x} = rx(1-x)$




$r = 2$

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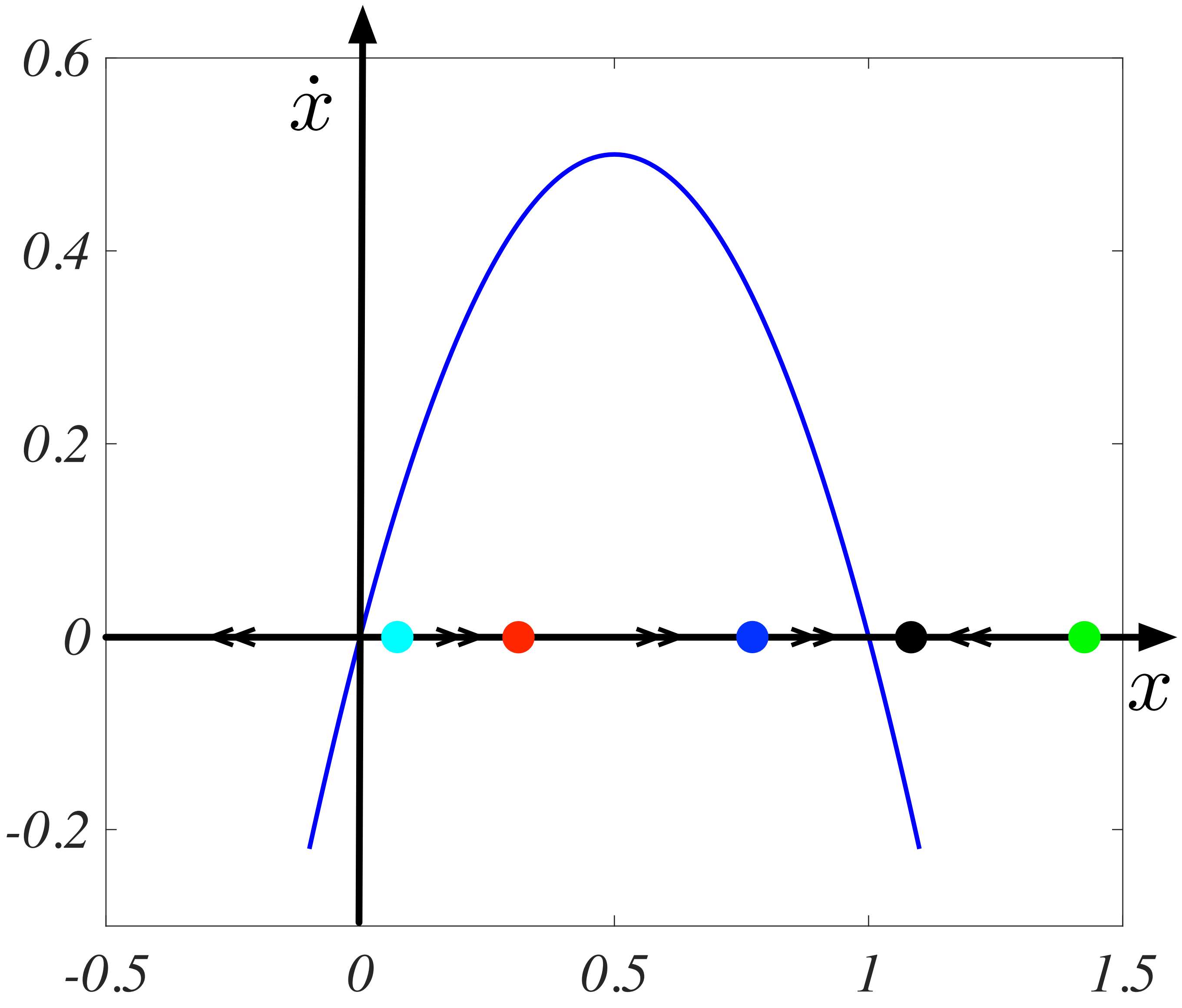
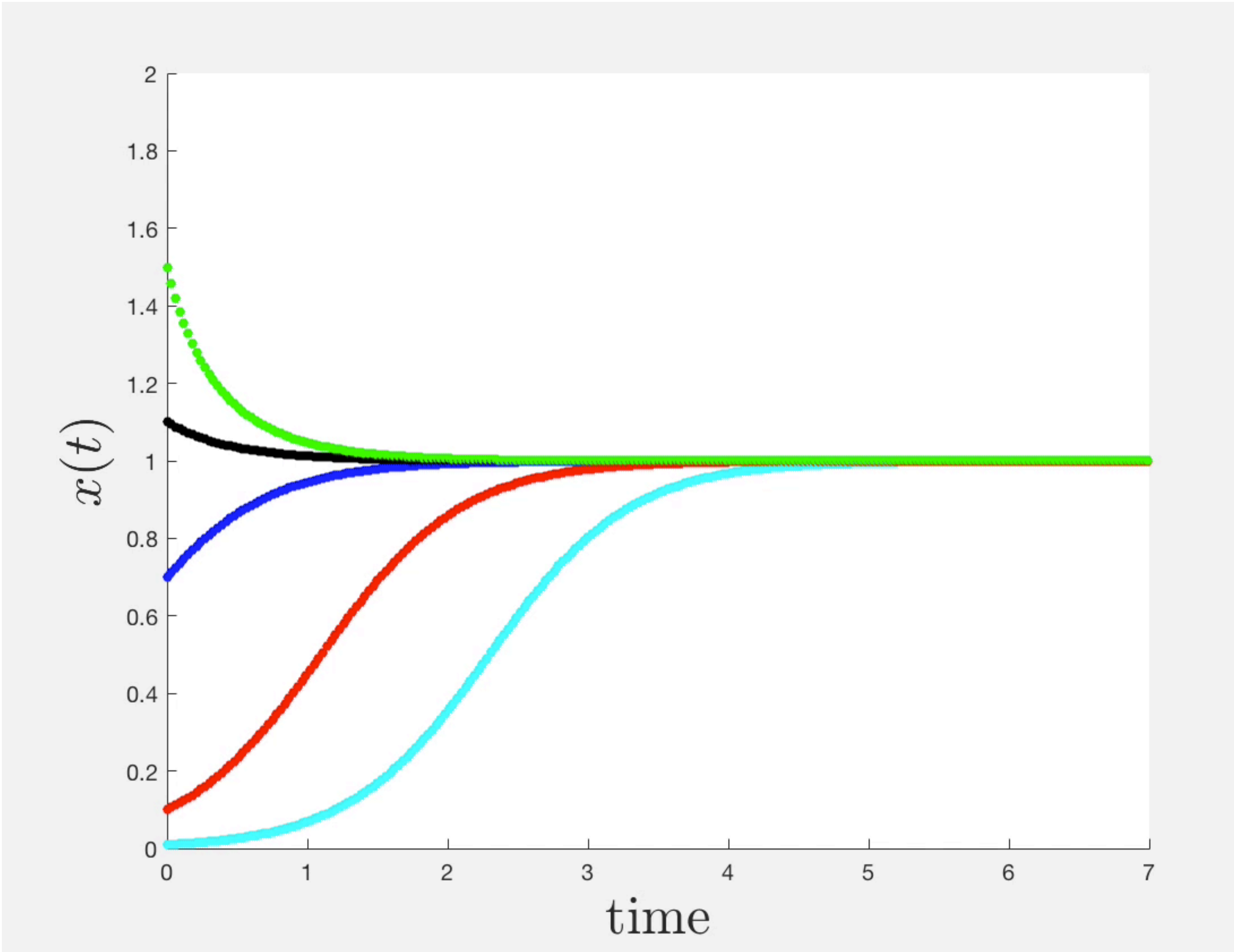
Logistic model $\dot{x} = rx(1-x)$

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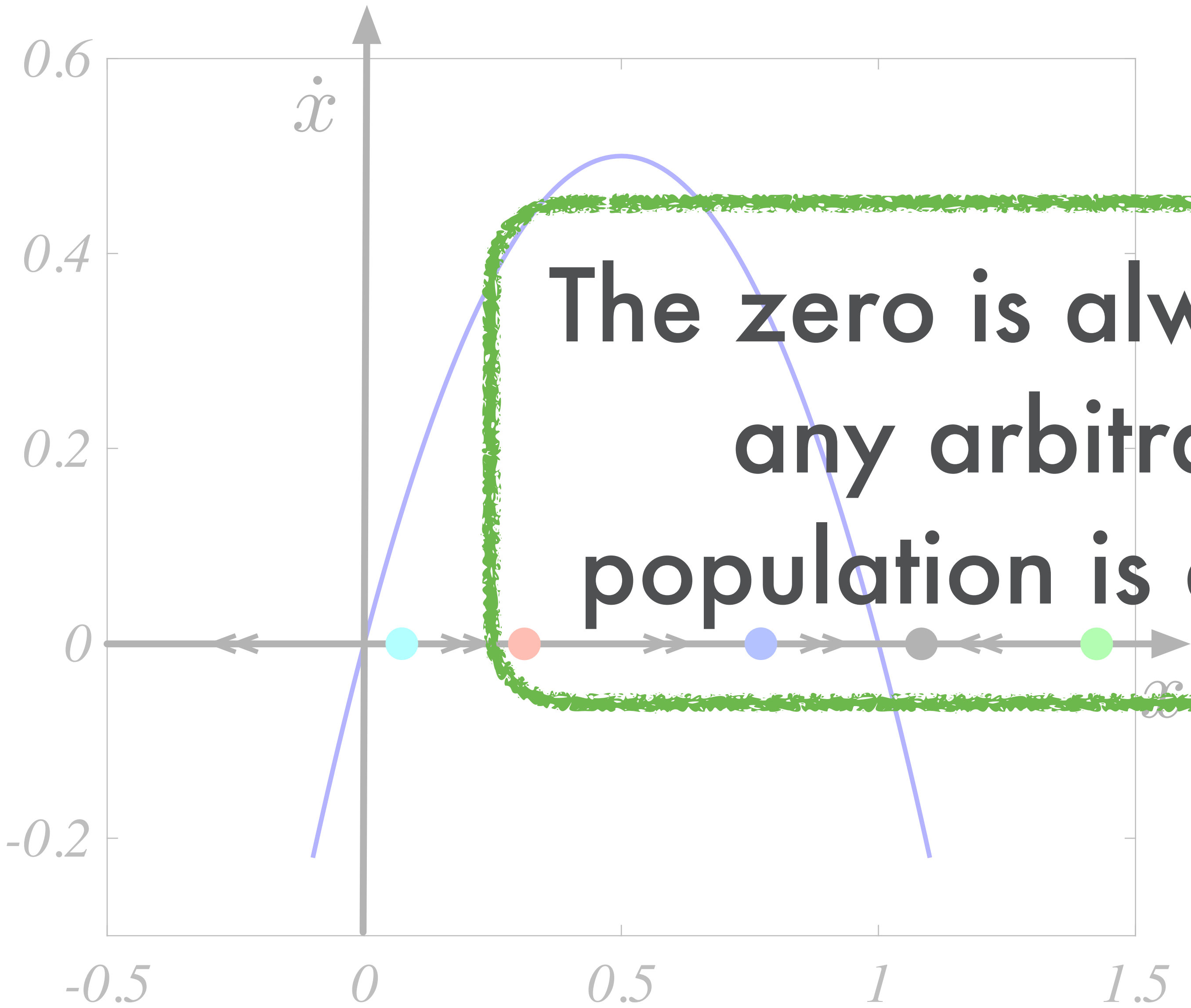


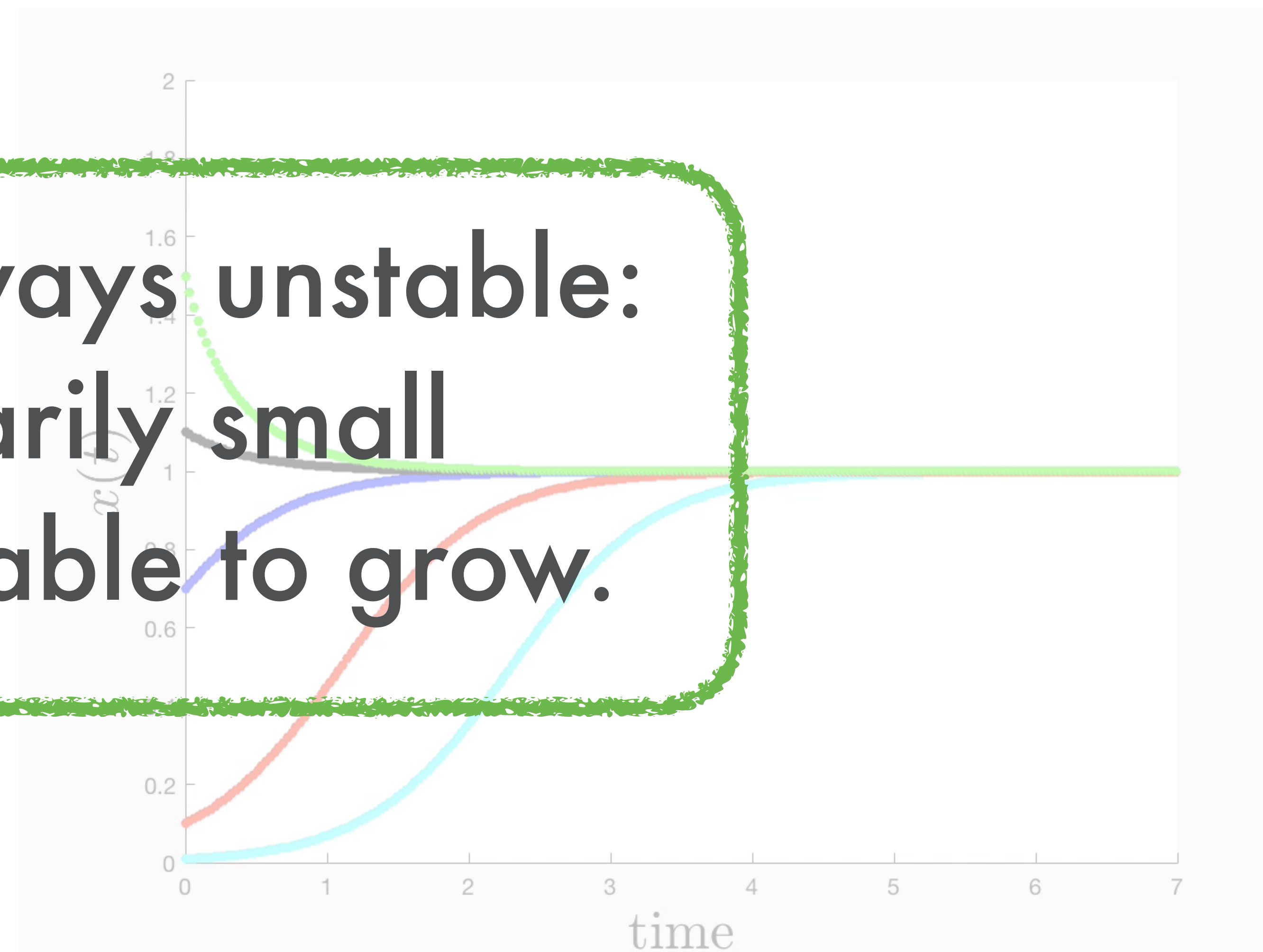


The zero is always unstable: any arbitrarily small population is able to grow.

$r = 2$

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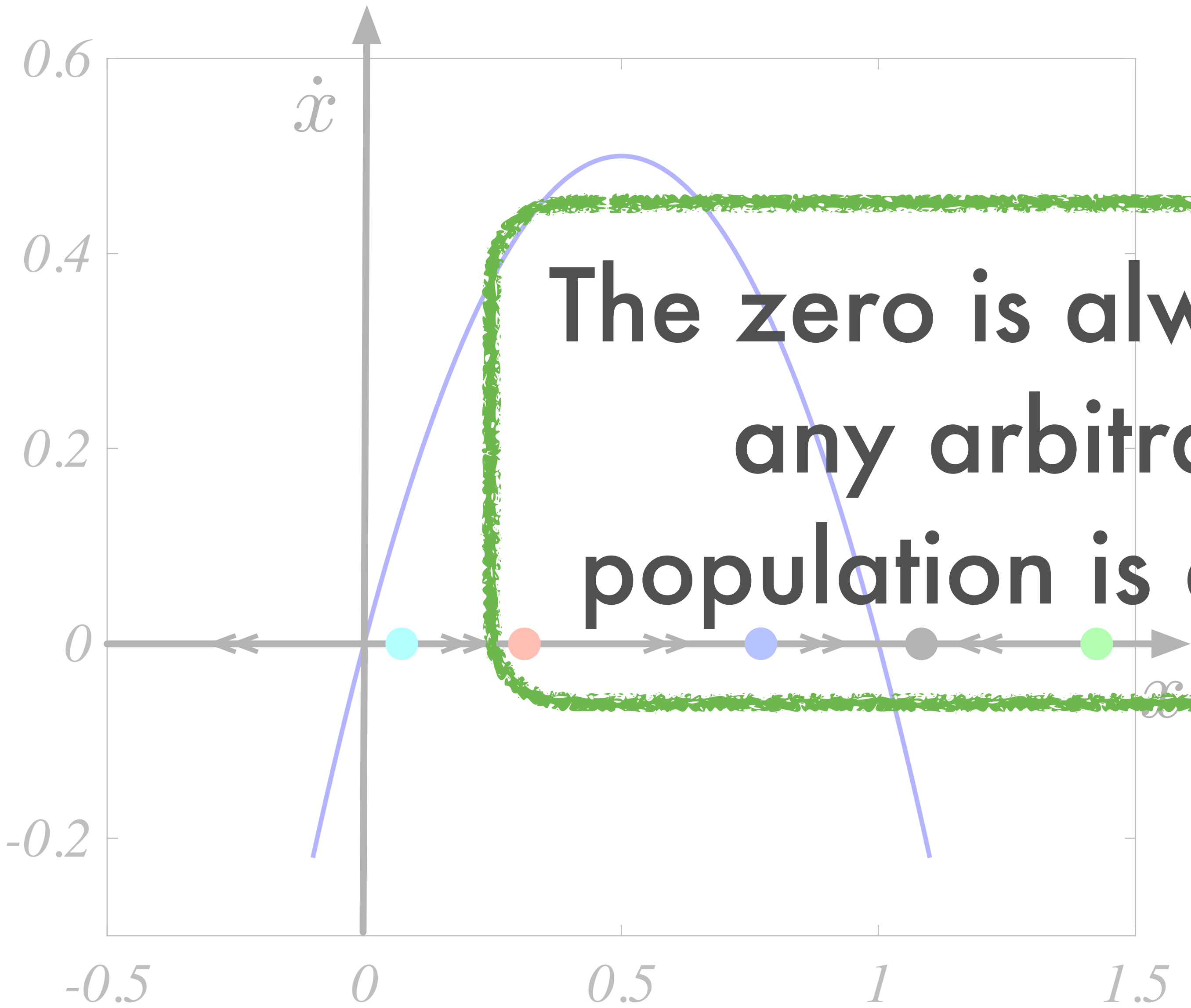


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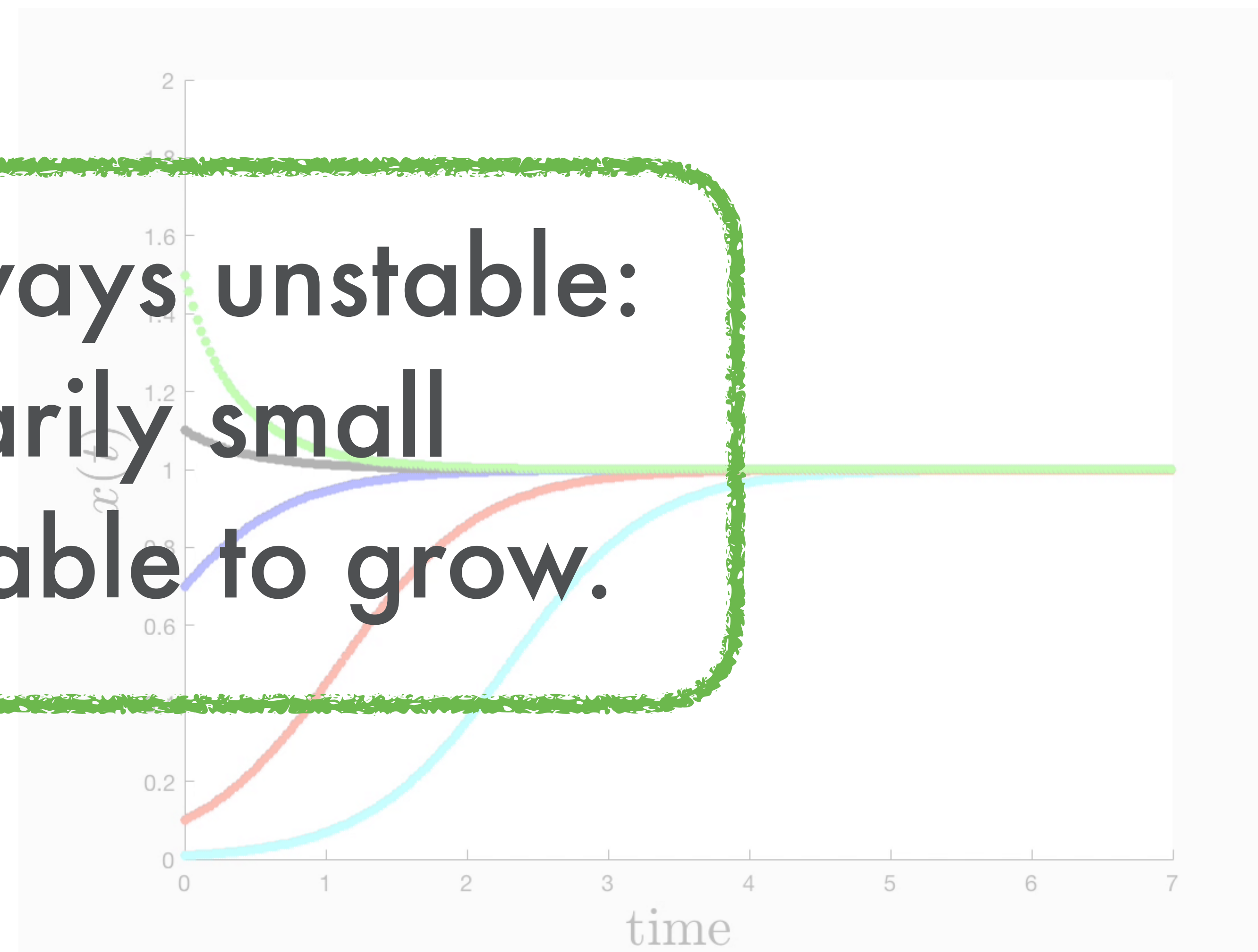
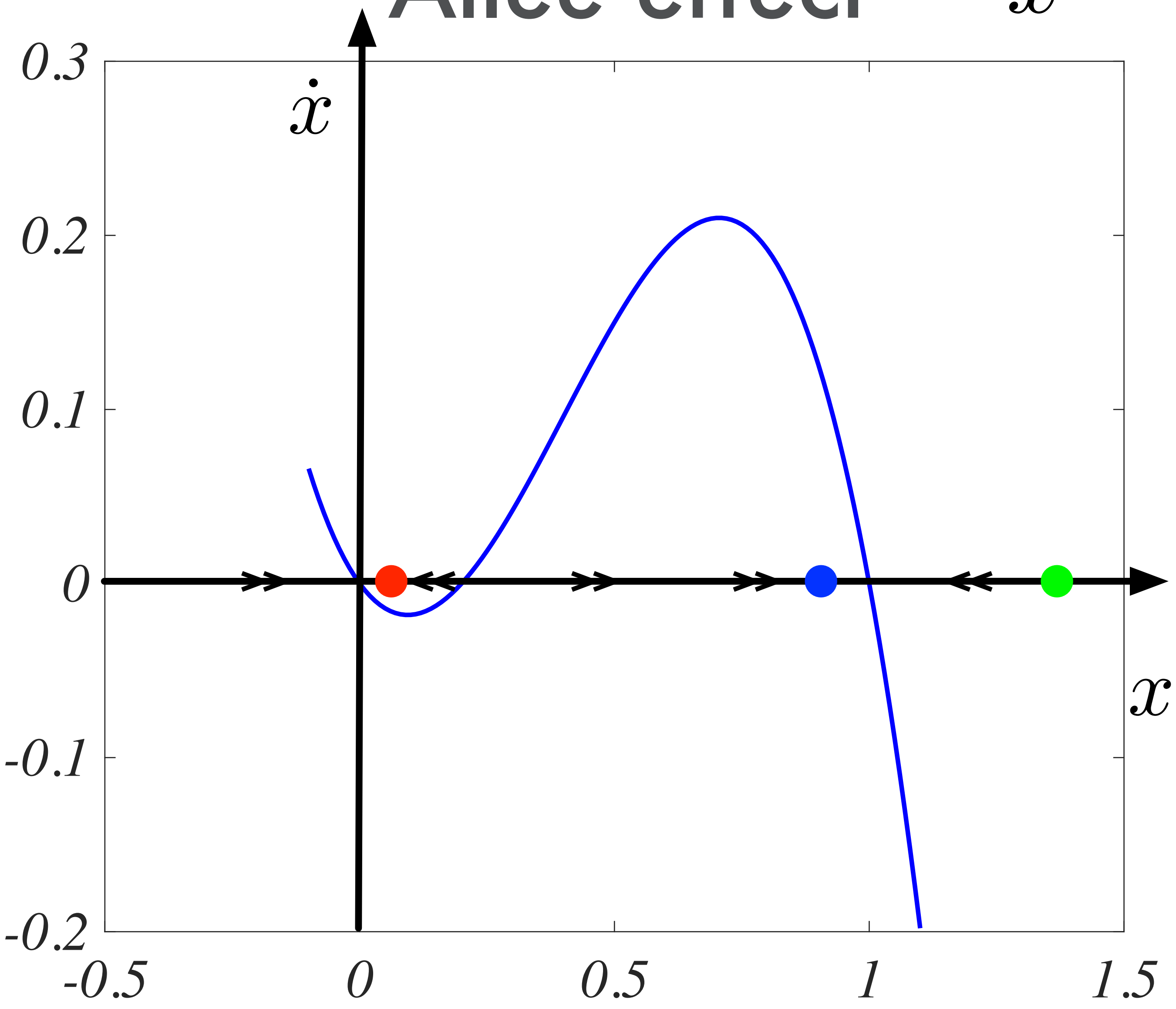


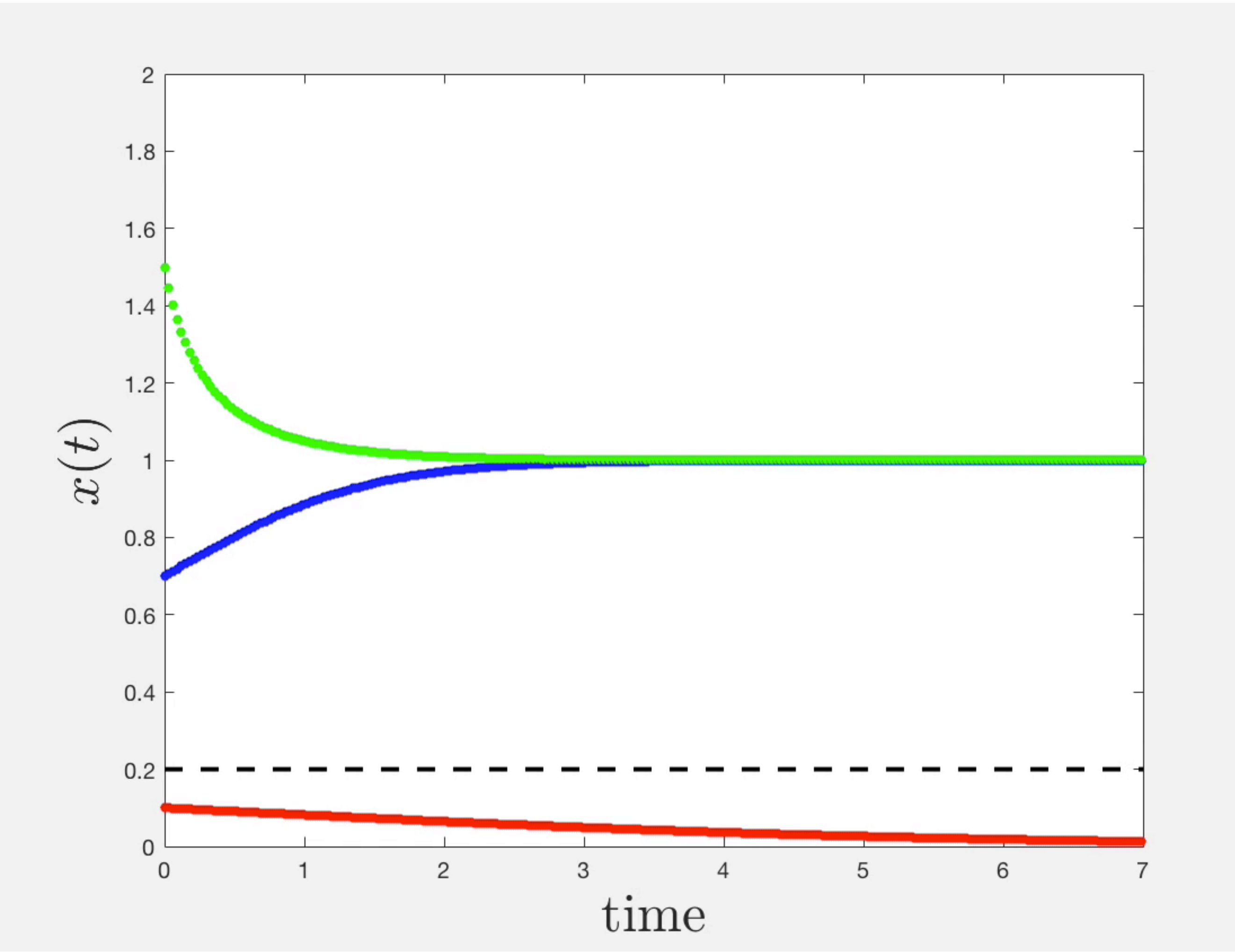

**The zero is always unstable: any arbitrarily small population is able to grow.**

# Motivating example

However there is empirical evidence that the size does matter.
(cooperation for hunting, raising of cubs, and defense)

Allee effect $\dot{x} = rx(1-x)(x-a)$




$r = 2$ $a = 0.2$

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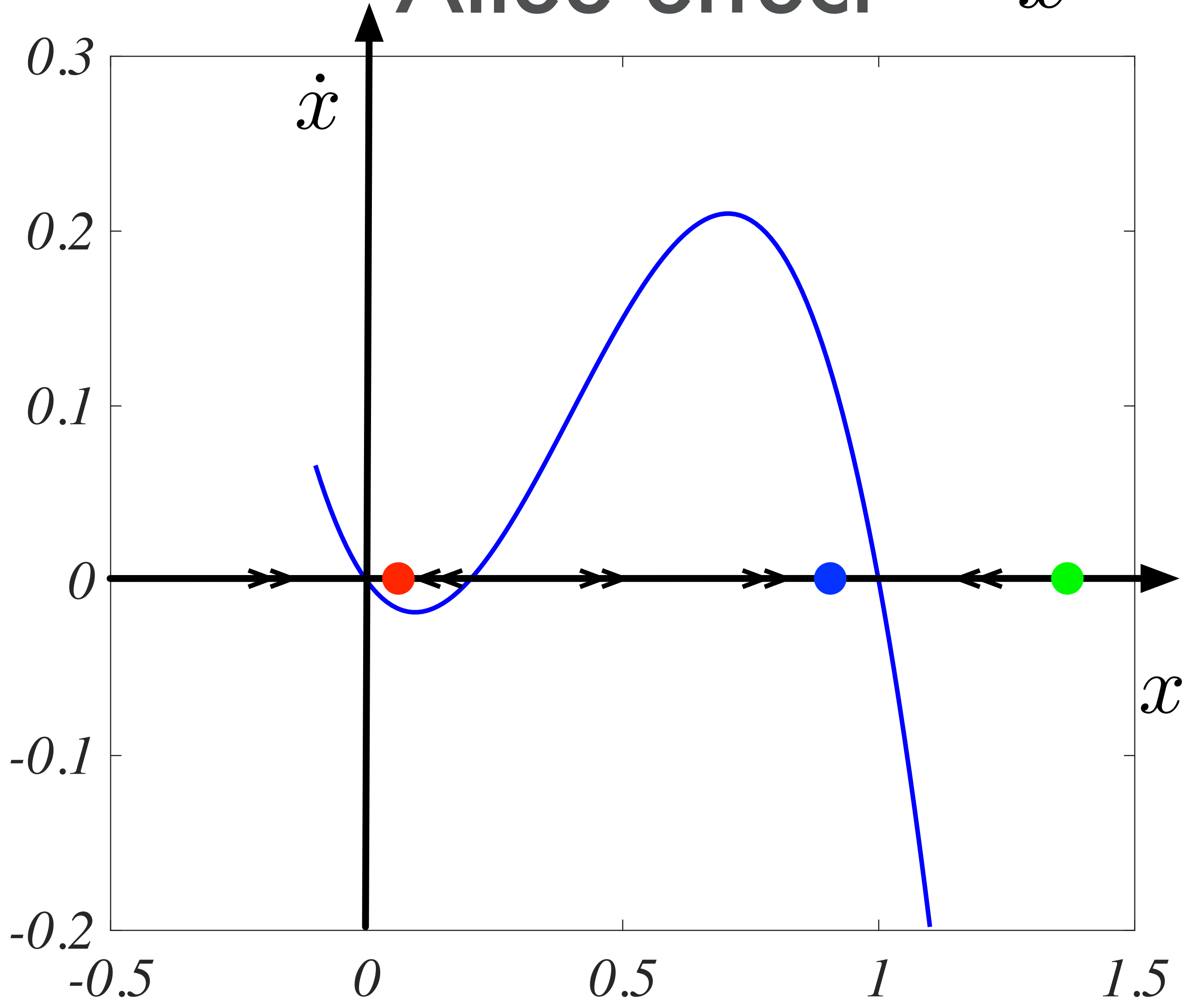
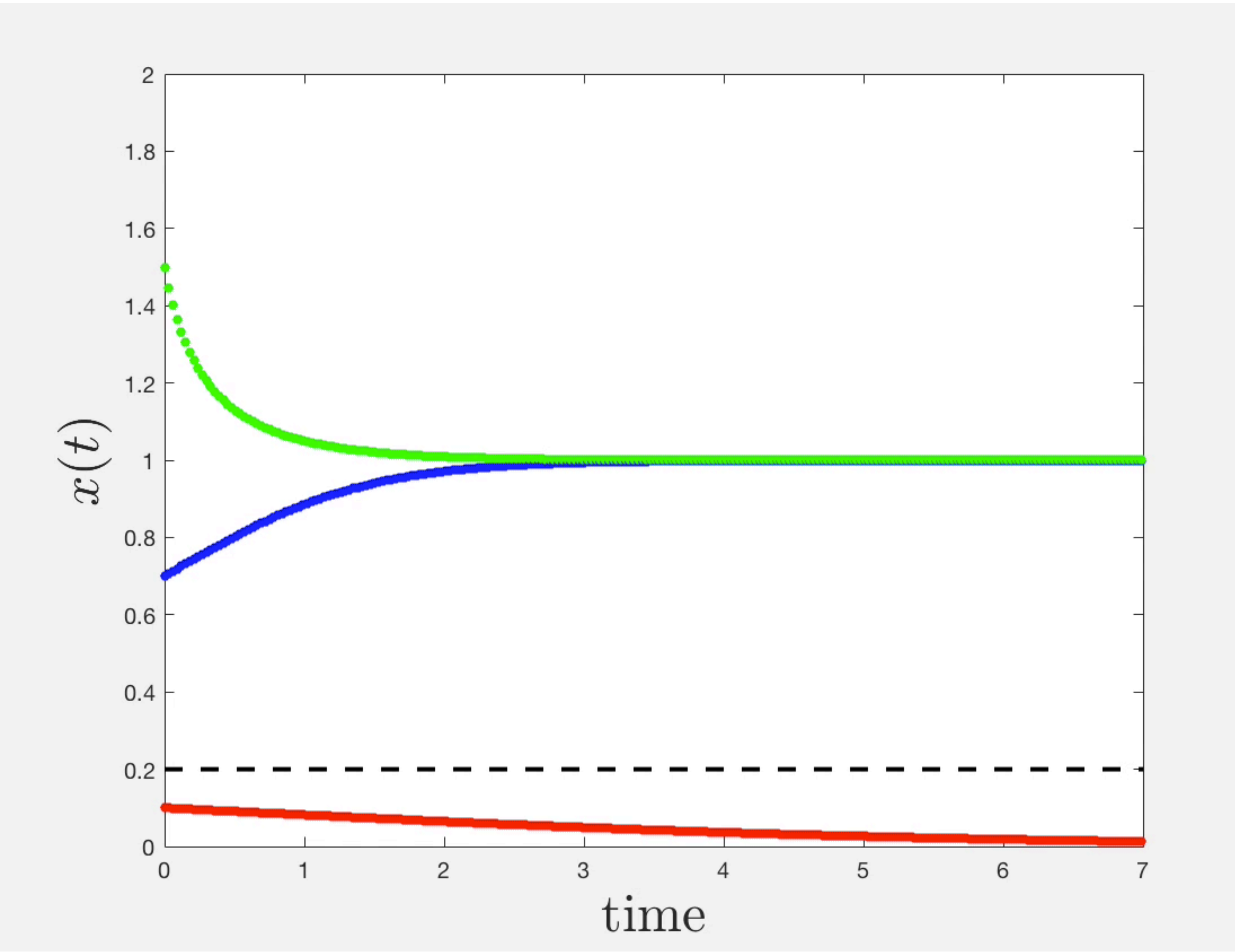
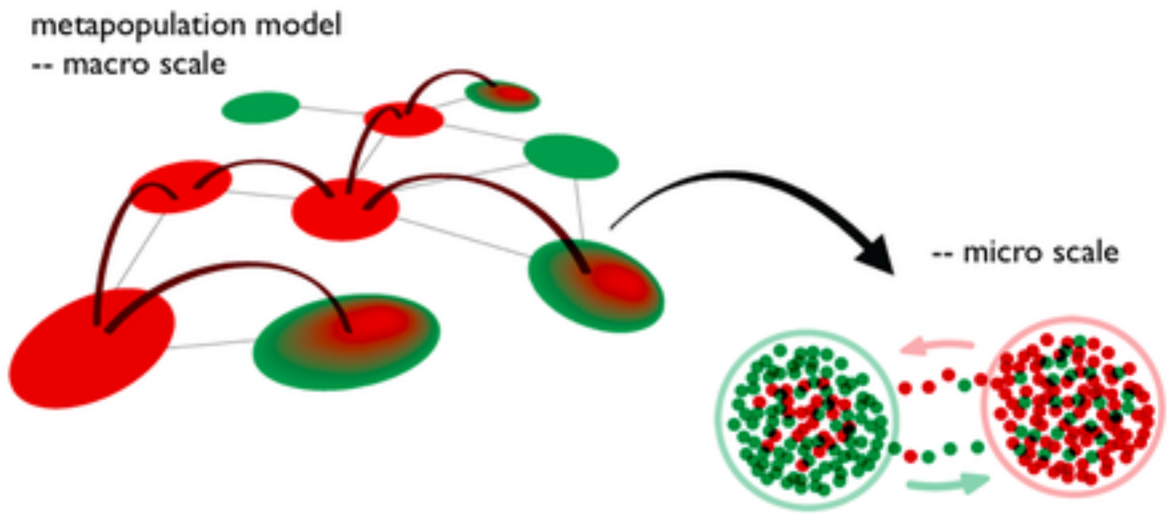



$r = 2 \qquad a = 0.2$

# Motivating example

Assume species to live in a networked environment (niches).




Metapopulation models
e.g. in the framework of ecology:

May R., Will a large complex system be stable? Nature, 238, pp. 413, (1972)

Species interact inside each node and diffuse among nodes across available edges in both directions: symmetric network.

# Motivating example

## Allee effect with diffusion

$$\dot{x}_i = r x_i (1 - x_i)(x_i - a) + D \sum_{j=1}^{M} L_{ij} x_j$$

**Laplacian matrix**

$$L_{ij} = A_{ij} - k_i \delta_{ij}$$

**Node degree**

$$k_i = \sum_{j=1}^{M} A_{ij}$$

**Adjacency matrix**

$$A_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$

# Motivating example

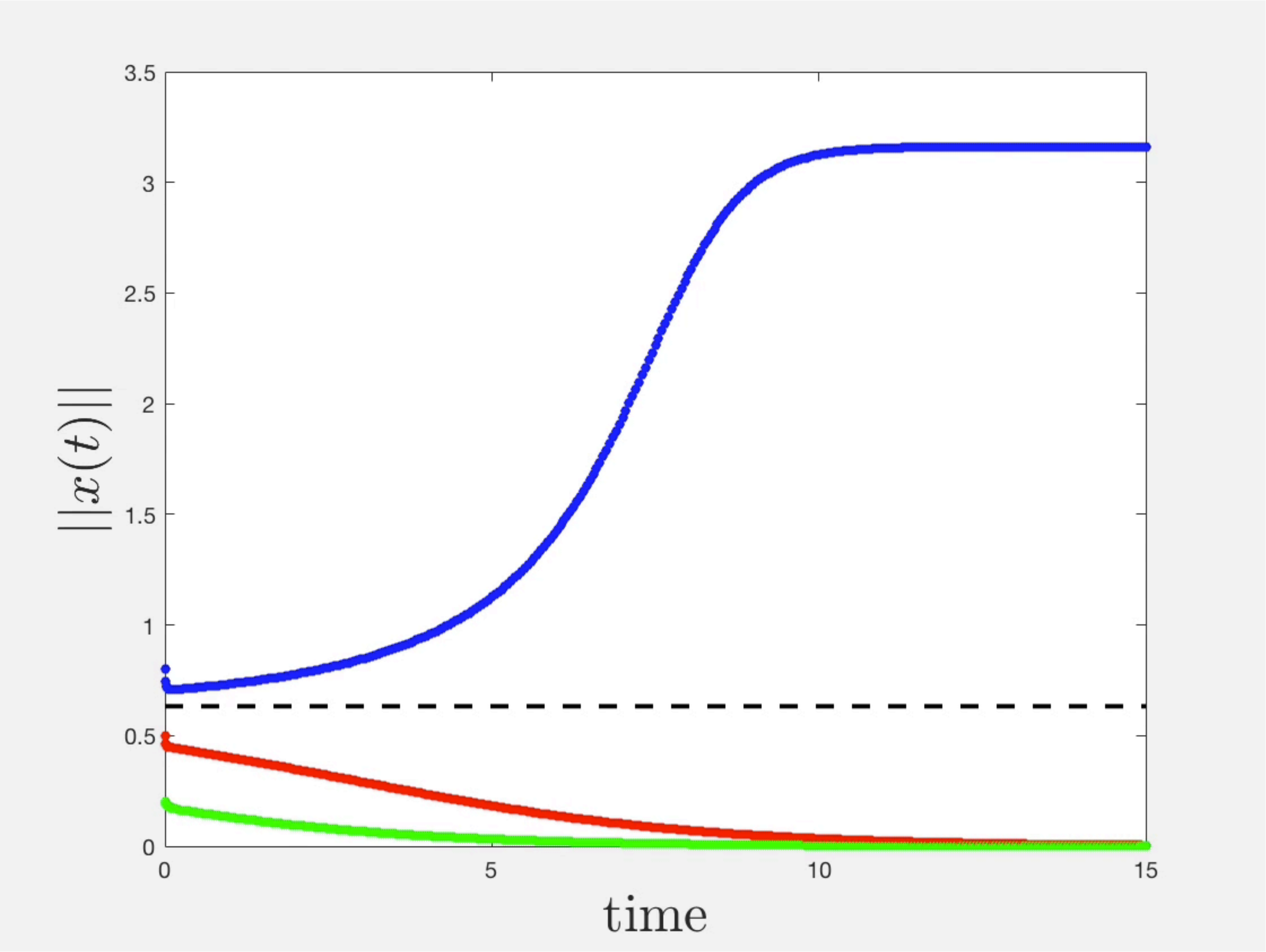
Assign $\forall i = 1, \ldots, M \quad x_i(0) \geq 0$

System "mass"

$$||x(t)||^2 = \sum_{i=1}^{M} [x_i(t)]^2$$

- $||x(0)|| = 0.2$ and $x_i(0) << a$
- $||x(0)|| = 0.5$ and $x_i(0) \lesssim a$
- $||x(0)|| = 0.8$ and $\exists j : x_j(0) \geq a$

Symmetric network



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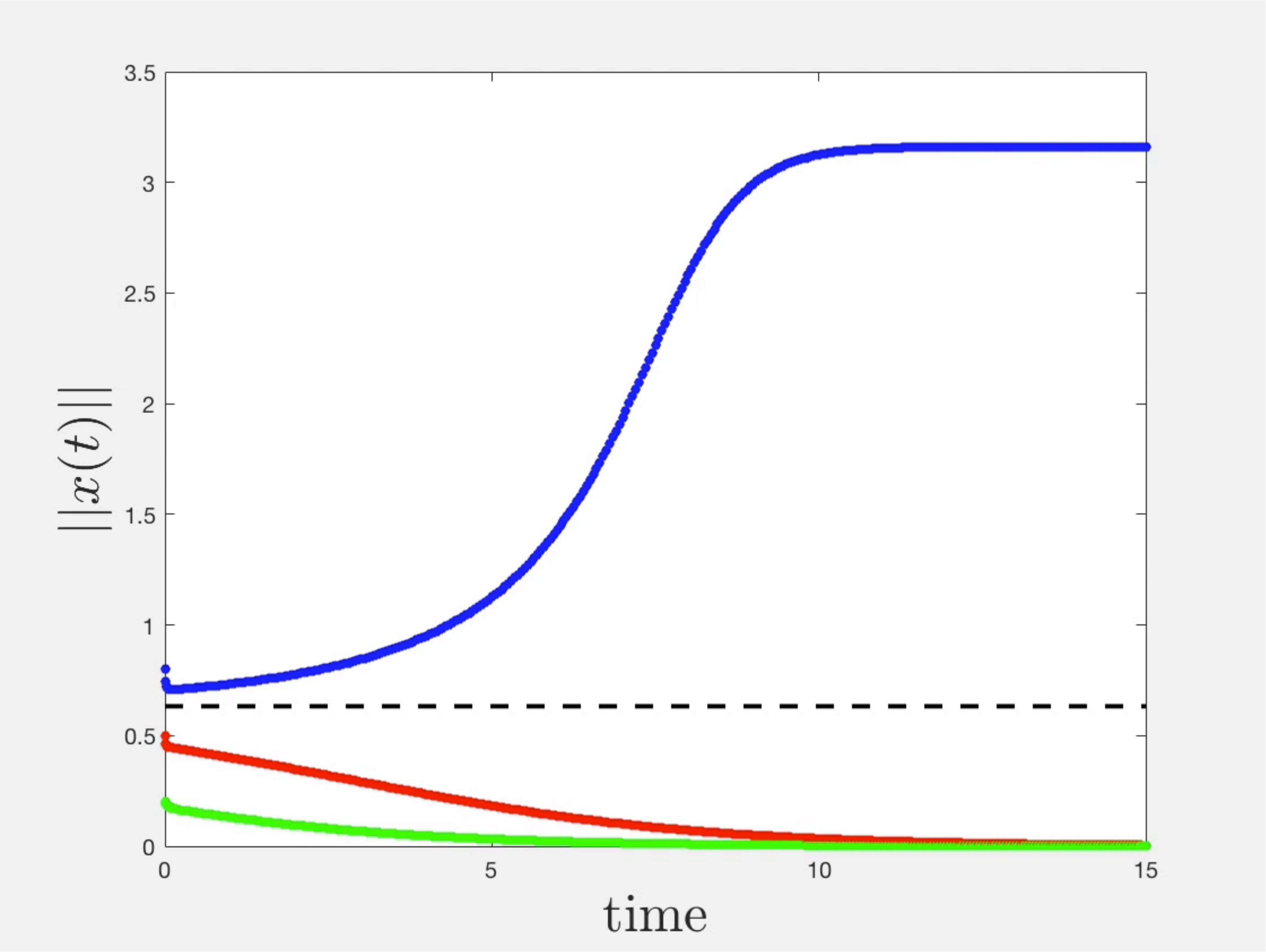


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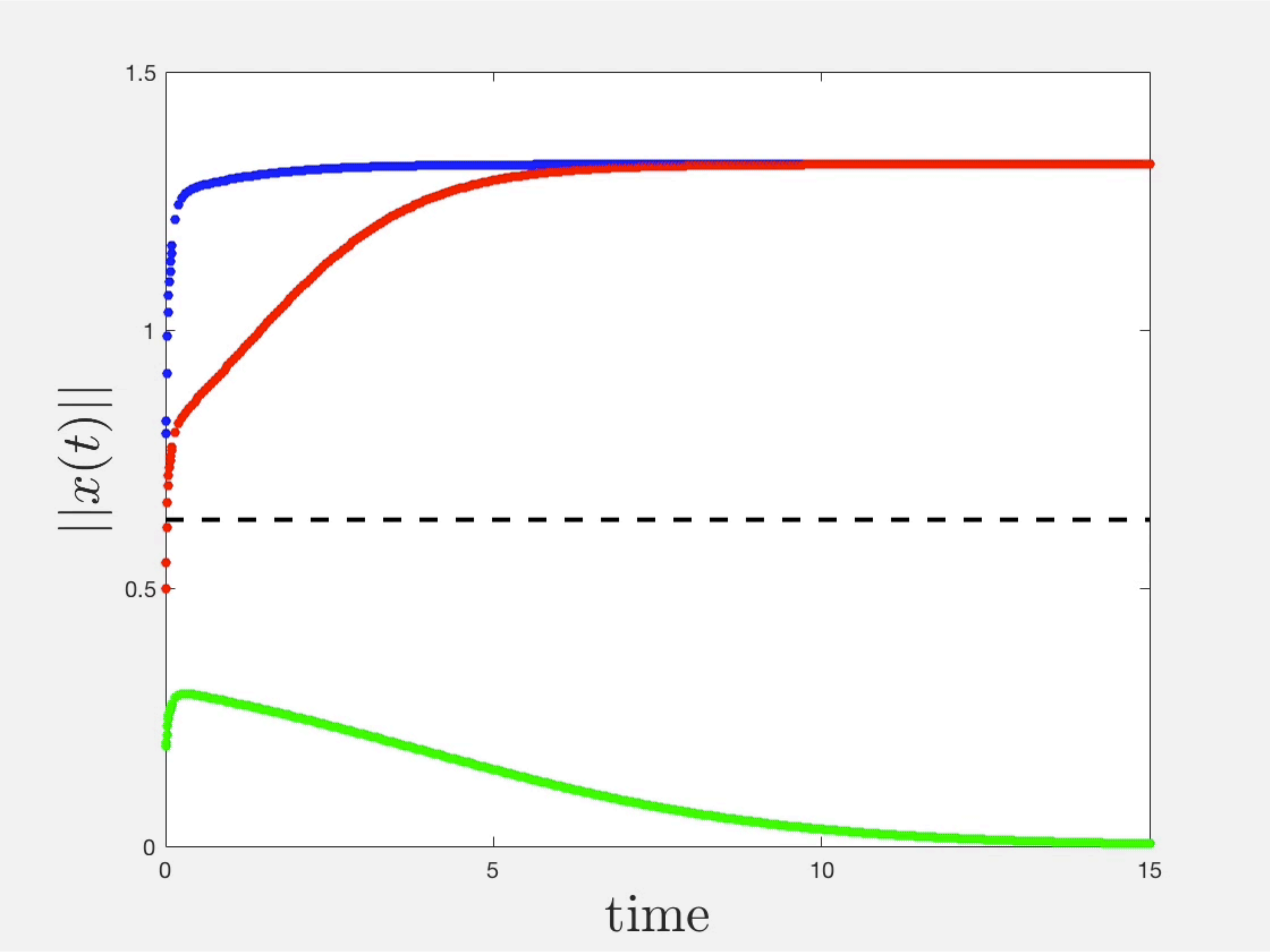
**non-normal network**

(directed links)



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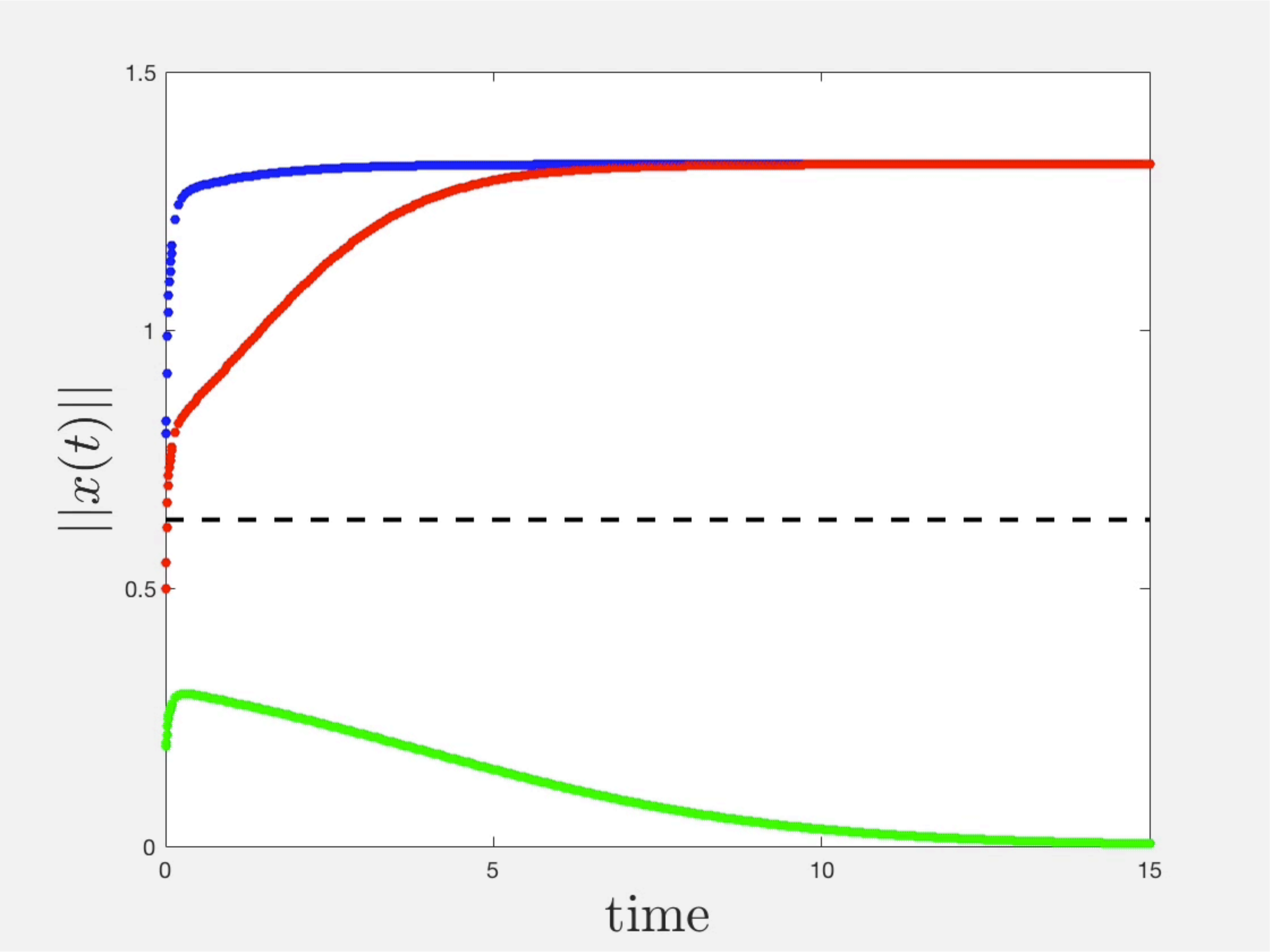
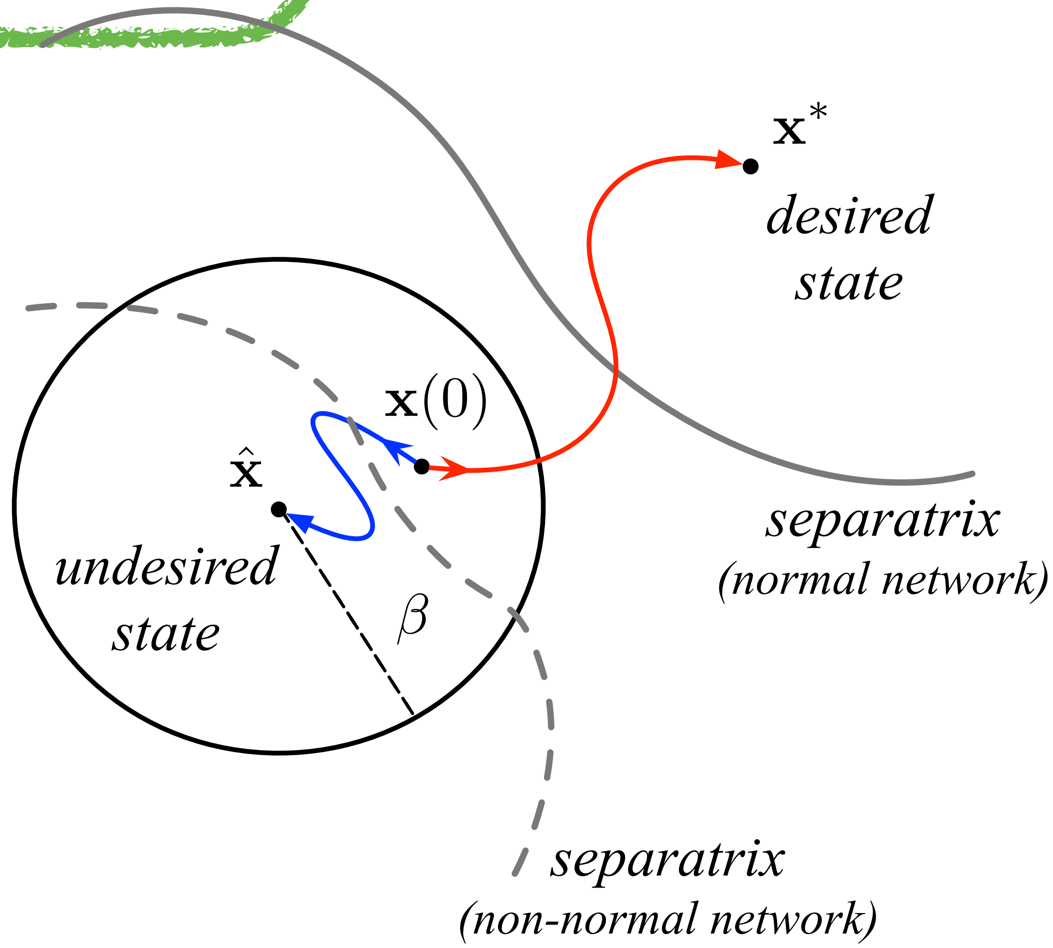


# Motivating example

Network non-normality reduces the stability of the origin: only a very small population is not able to grow.

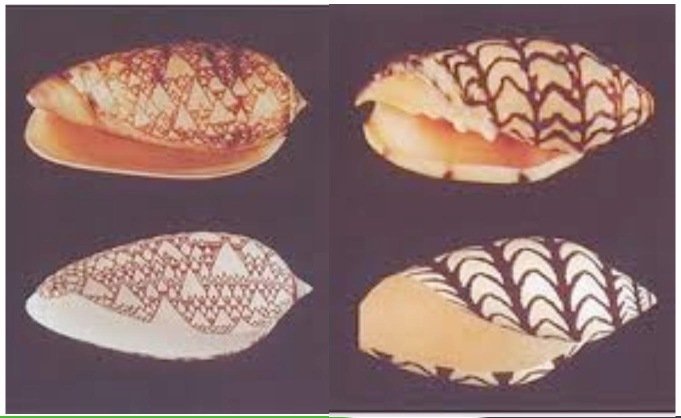


## First conclusions

Network non-normality can strongly modify the system behavior.

The result is general: it holds for any number of involved species/agents reacting in the nodes and diffusing across links.

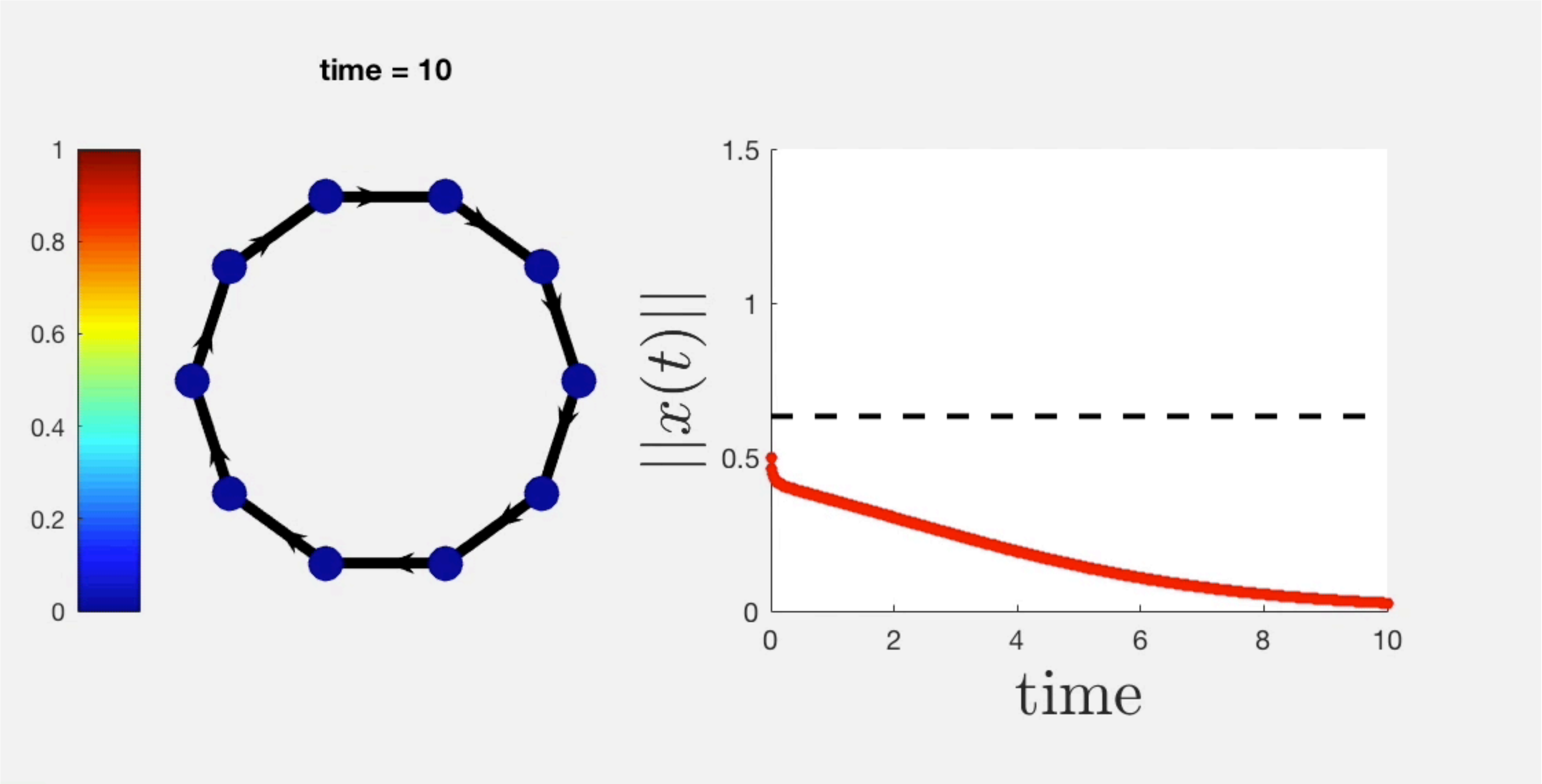
This mechanism generalizes the Turing instability. Systems with a single species cannot have Turing patterns.



# Non-normal networks

Definition (mathematical). A network is non-normal, if its adjacency matrix $A_{ij}$ does satisfy $AA^T \neq A^T A$ .

Definition (operational). It must contain a large Direct Acyclic Graph (DAG).



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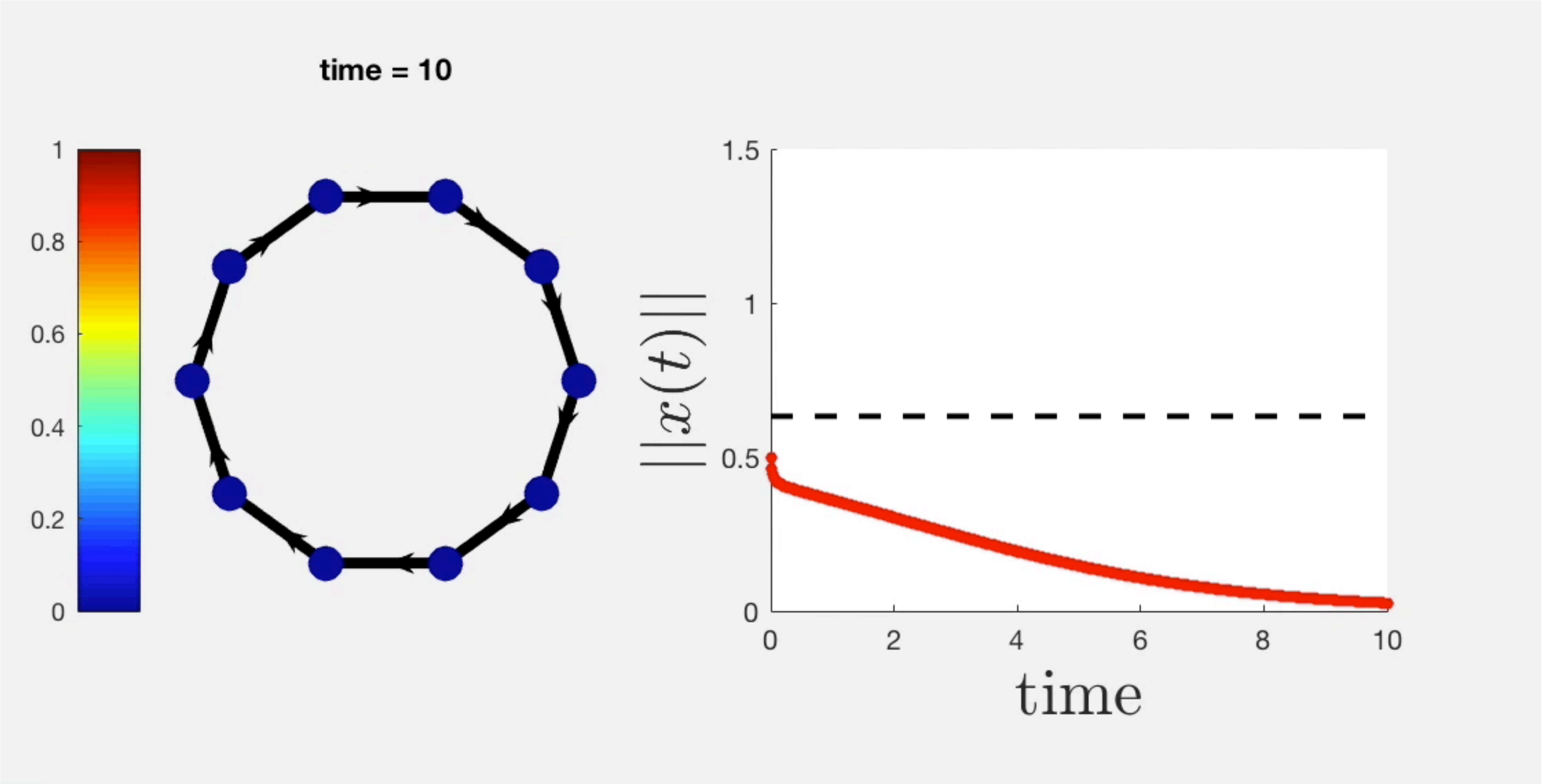



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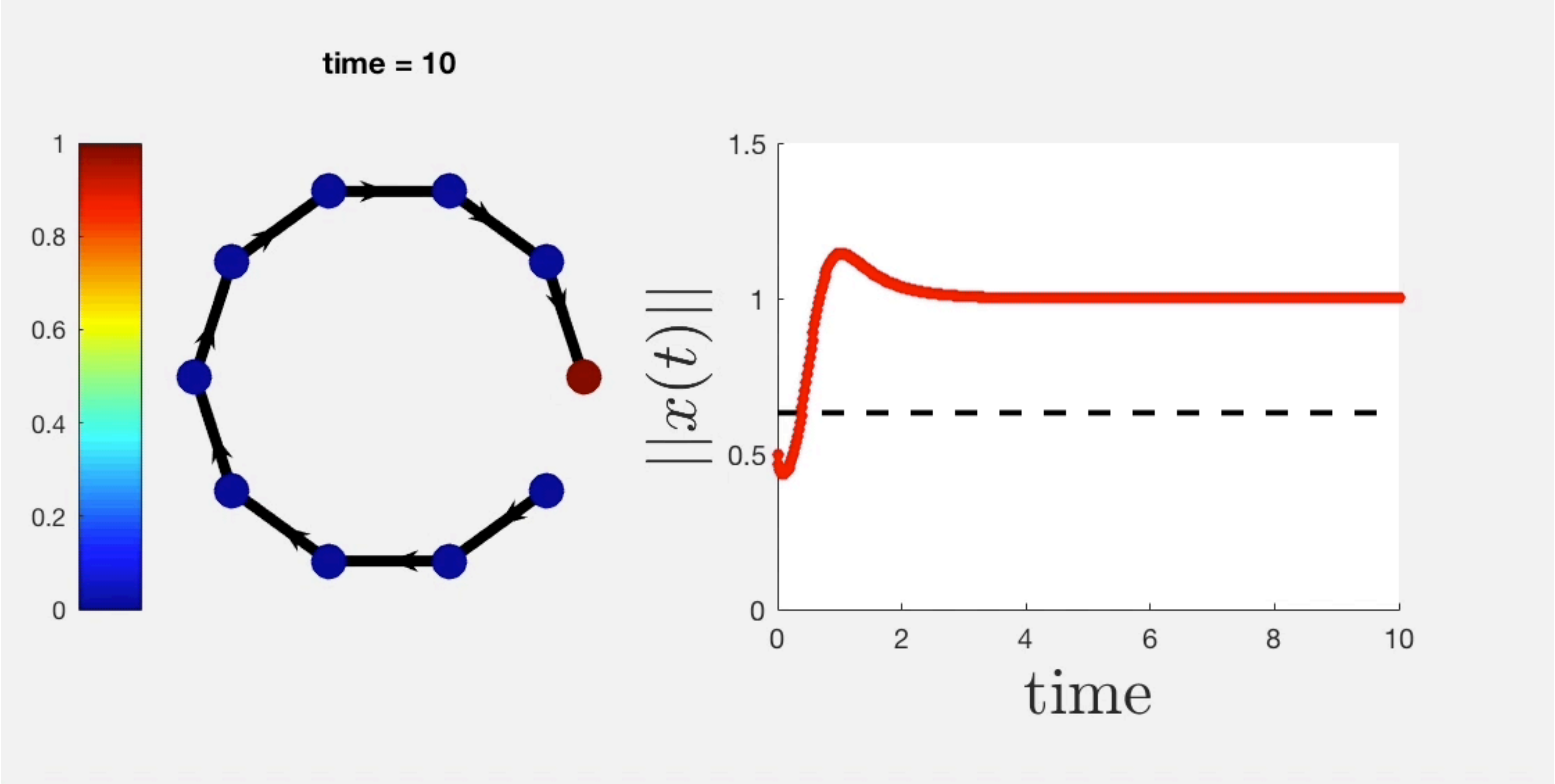

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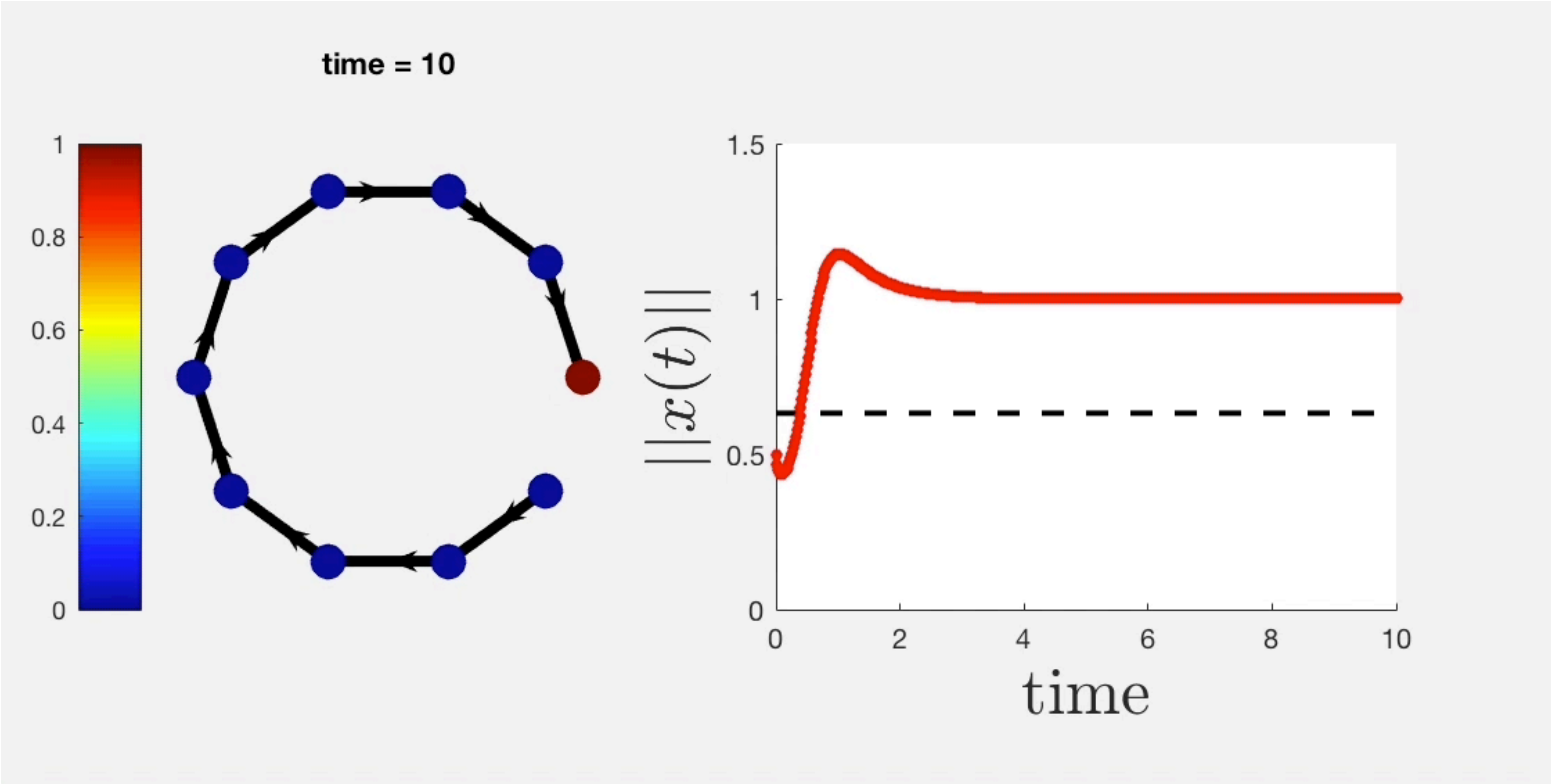

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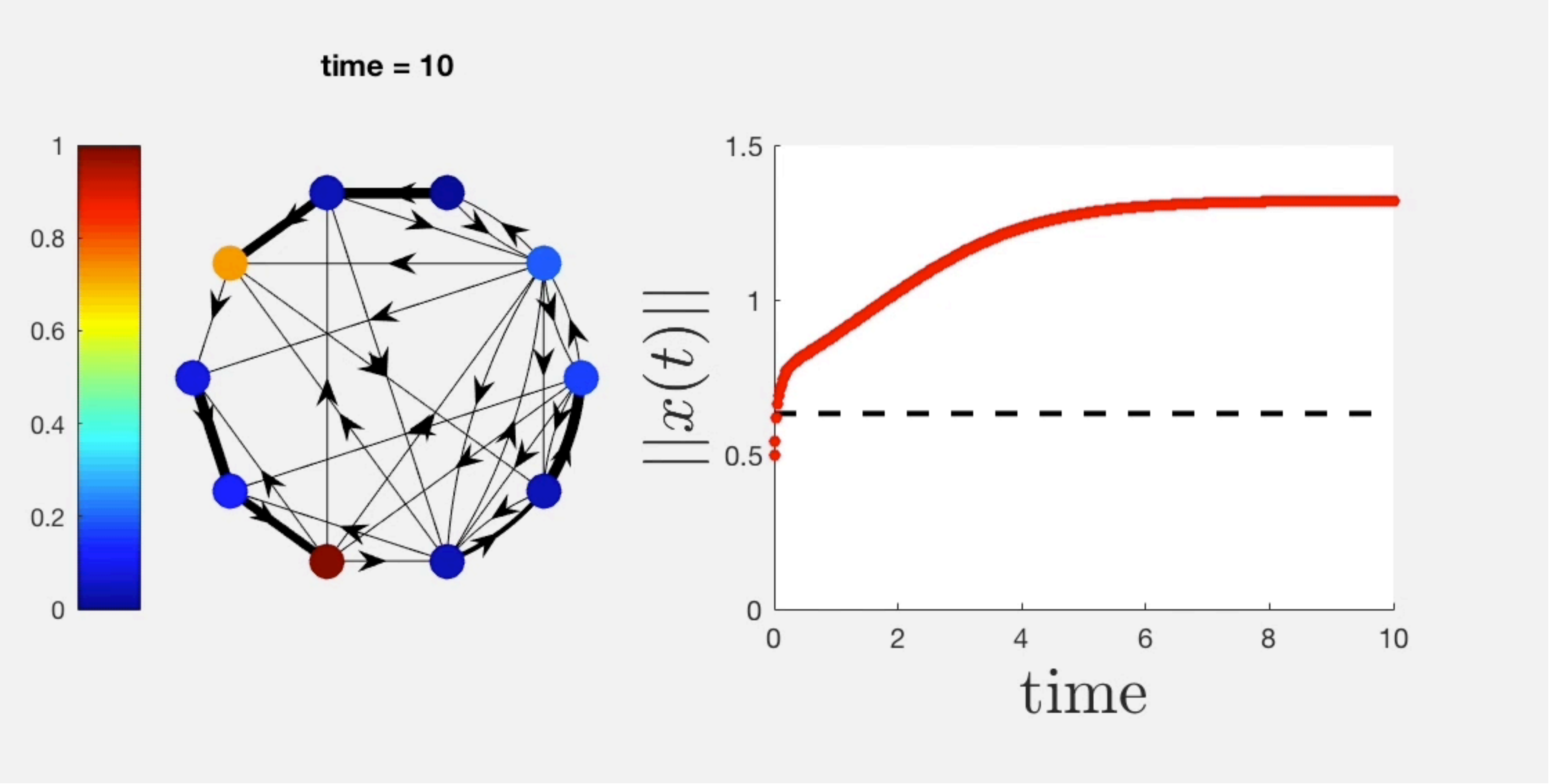

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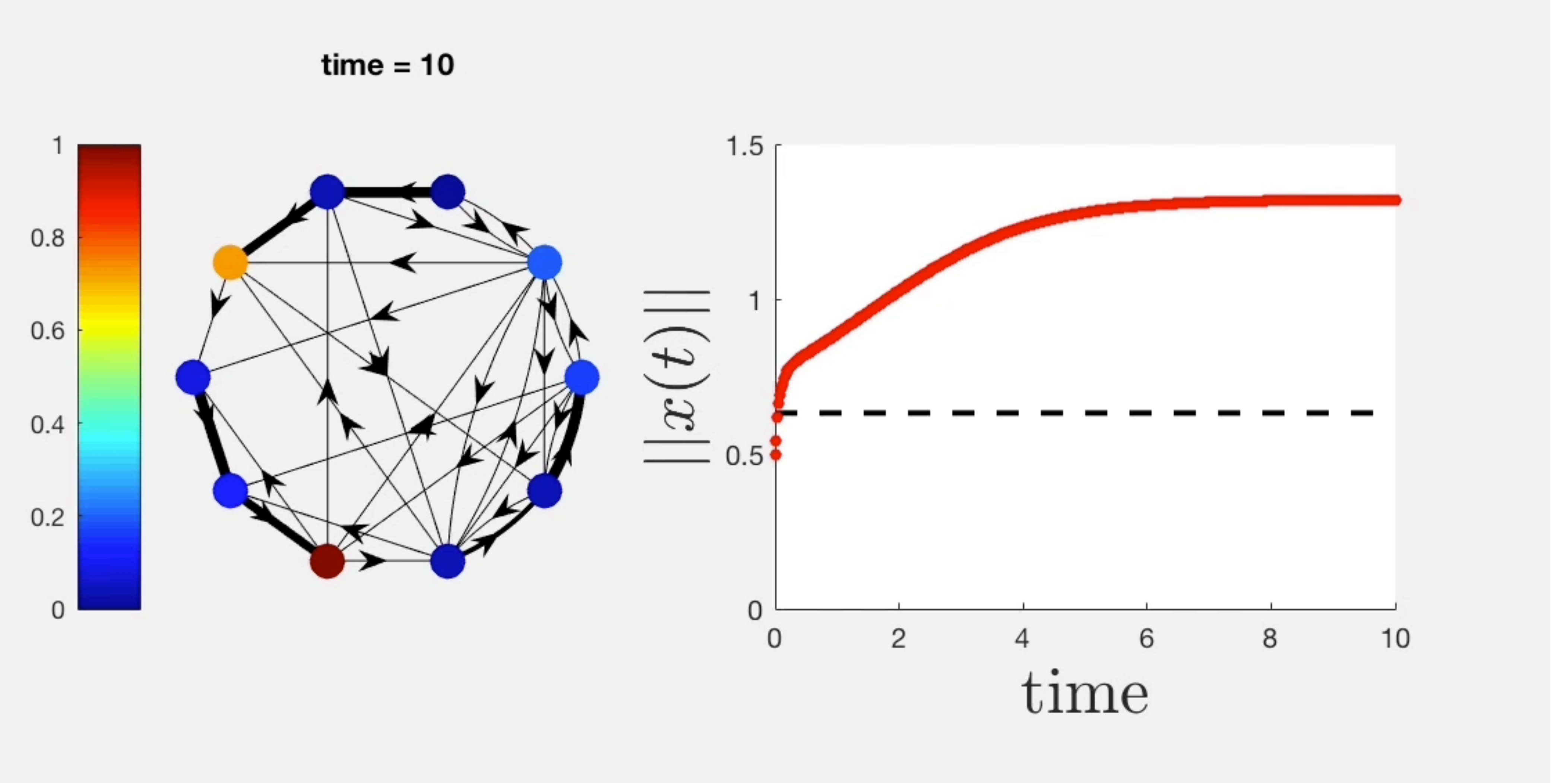

# Non-normal networks: DAG backbone

**Adjacency matrix directed network**

$$A_{ij} = \begin{cases} 1 & \text{if there exists } j \to i \\ 0 & \text{otherwise} \end{cases}$$

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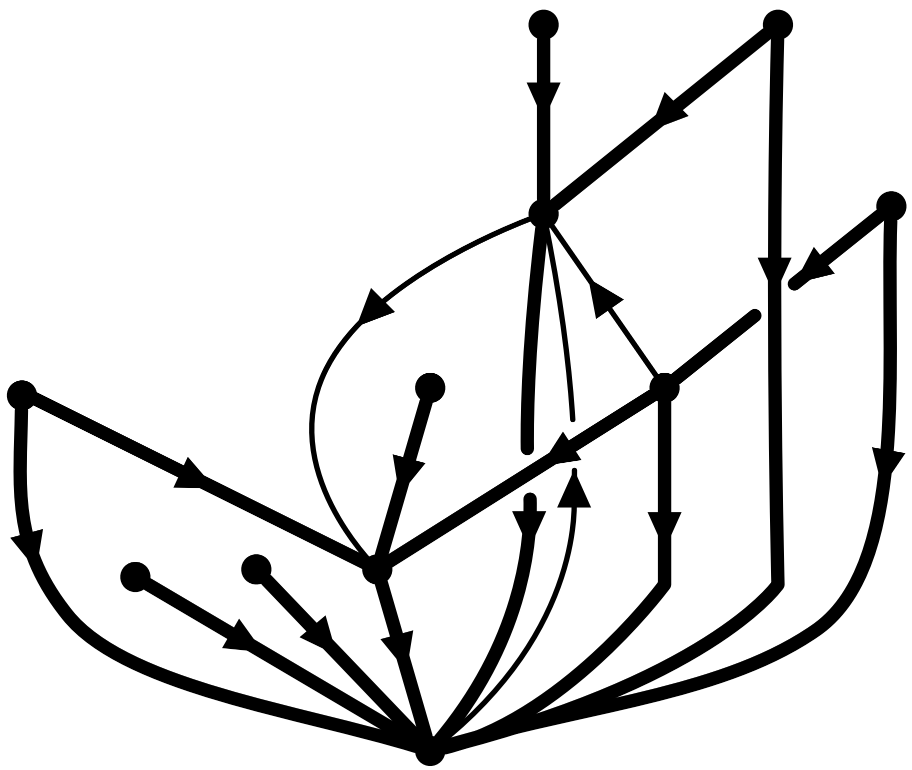

$$A = \begin{pmatrix} 0 & & & \text{0's \& 1's} \\ & 0 & & \\ & & \ddots & \\ \text{0's \& 1's} & & & 0 \end{pmatrix}$$

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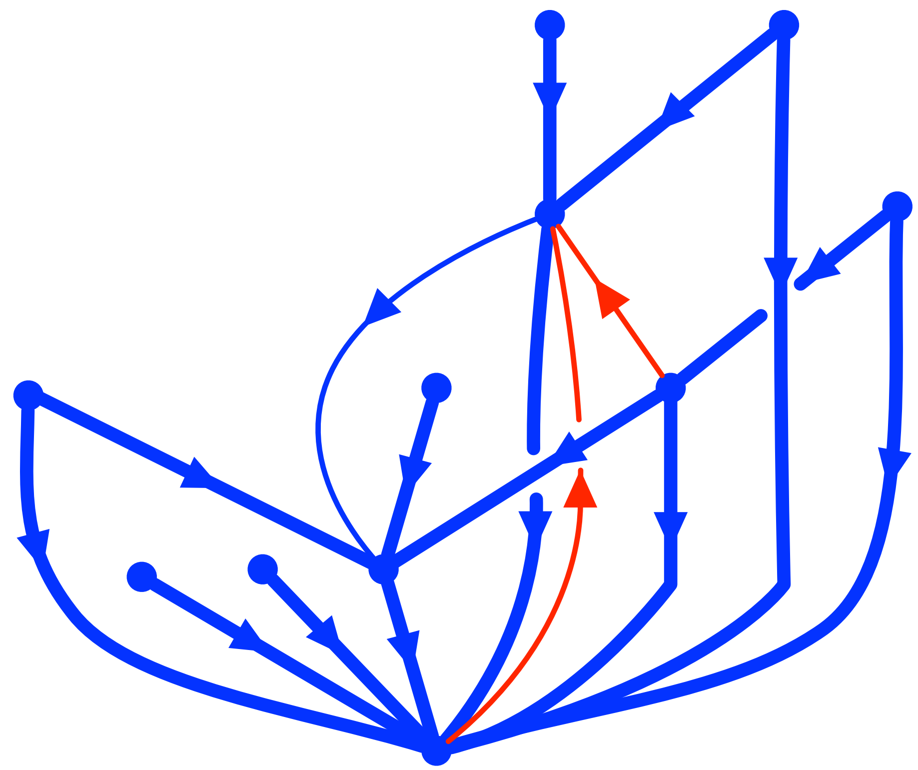

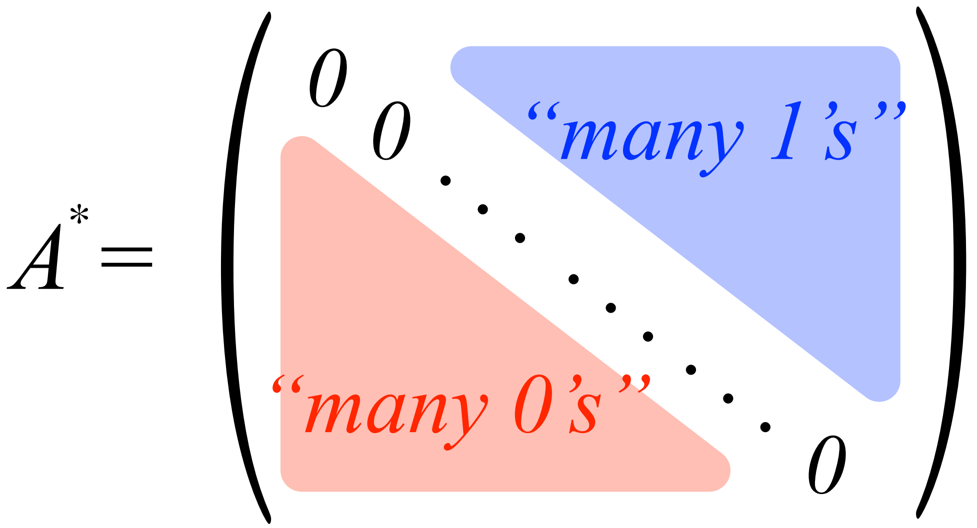


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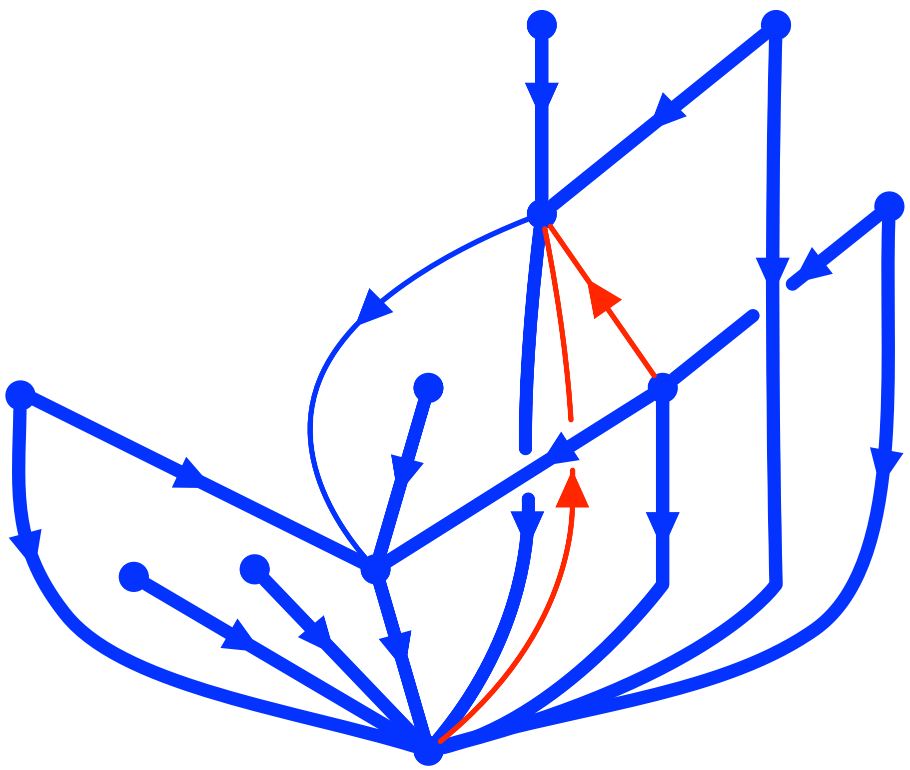

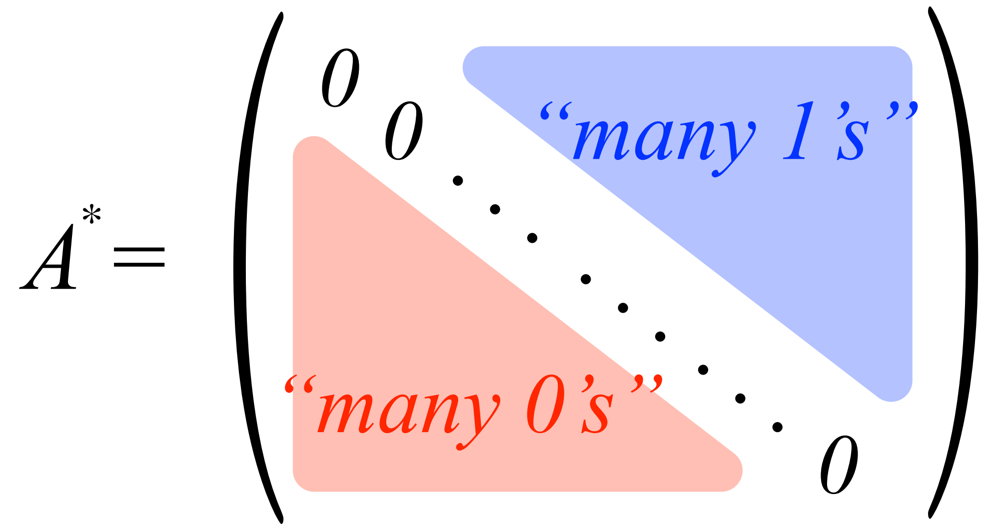


A (simple) measure of non-normality

$$\Delta := |K^< - K^>|/K$$

Adjacency matrix directed network

$$A_{ij} = \begin{cases} 1 & \text{if there exists } j \to i \\ 0 & \text{otherwise} \end{cases}$$

$$K^< = \sum_{i<j} A^*_{ij} \qquad K^> = \sum_{i>j} A^*_{ij}$$

$$K = K^< + K^>$$

# Non-normal networks are "the rule"

| Network name | nodes | links | $\omega$ | $\omega - \alpha$ | $\alpha_\epsilon$ | $\Delta$ | $\hat{d}_F$ |
|---|---|---|---|---|---|---|---|
| **Foodwebs** | | | | | | | |
| Cypress wetlands South Florida (wet) | 128 | 2016 | 296.71 | 132.11 | 167.46 | 0.83 | 1.00 |
| Cypress wetlands South Florida (dry) | 128 | 2137 | 217.60 | 152.50 | 82.20 | 0.89 | 1.00 |
| Little Rock Lake (Wisconsin, US) | 183 | 2494 | 21.69 | 14.69 | 10.02 | 0.95 | 0.93 |
| **Biological** | | | | | | | |
| Transcriptional regulation network (*E. coli*) | 423 | 578 | 5.11 | 4.11 | 2.52 | 0.81 | 0.93 |
| Metabolic network (*C. Elegans*) | 453 | 4596 | 13.44 | 12.44 | 6.89 | 0.98 | 1.00 |
| Pairwise proteins interaction (*Homo sapiens*) | 2239 | 6452 | 15.79 | 13.02 | 4.01 | 0.99 | 0.99 |
| **Transport** | | | | | | | |
| US airport 2010 | 1574 | 28236 | $1.19\,10^7$ | 79.30 | $1.19\,10^7$ | 0.01 | 1.00 |
| Road transportation network (Rome) | 3353 | 8870 | $2.40\,10^4$ | 120.05 | $2.39\,10^4$ | 0.08 | 0.28 |
| Road transportation network (Chicago) | 12982 | 39018 | 4.23 | $4.29\,10^{-4}$ | 4.54 | 0.04 | 0.19 |
| **Communication** | | | | | | | |
| e-mails network DNC | 2029 | 39264 | 28.00 | 2.00 | 26.37 | 0.53 | 0.89 |
| Enron email network (1999-2003) | 87273 | 1148072 | 85.14 | 14.54 | 71.05 | 0.30 | 0.99(∗) |
| e-mails network European institution | 265214 | 420045 | 76.02 | 6.09 | 70.30 | 0.30 | 0.84(∗) |
| **Citation** | | | | | | | |
| Citations to Milgram's 1967 paper (2002) | 395 | 1988 | 10.48 | 10.48 | 4.49 | 1.00 | 1.00 |
| Articles from Scientometrics (1978-2000) | 3084 | 10416 | 10.32 | 8.32 | 5.28 | 0.98 | 1.00 |
| Citation network DBLP | 12591 | 49743 | 21.50 | 16.82 | 8.45 | 0.87 | 1.00 |
| **Social** | | | | | | | |
| Hyper-network of 2004 US election blogs | 1224 | 19025 | 45.37 | 10.95 | 34.95 | 0.72 | 0.98 |
| Reply network of the news website Digg | 30398 | 87627 | 15.92 | 6.56 | 10.18 | 0.61 | 0.97 |
| Trust network from the website Epinions | 75879 | 508837 | 123.00 | 16.47 | 106.96 | 0.13 | 0.80(∗) |

# Conclusions

- Network non-normality can strongly modify the system behavior and create new patchy solutions (patterns).
- Network non-normality can be easily measured.
- Many real networks exhibit strong non-normality and thus challenge our comprehension of their dynamics.


- M. Asllani and T. Carletti, Topological resilience in non-normal networked systems, Physical Review E, **97**, (2018), 042302
- M. Asllani and T. Carletti, Universality of non-normality in real complex networks, arXiv:1803.11542

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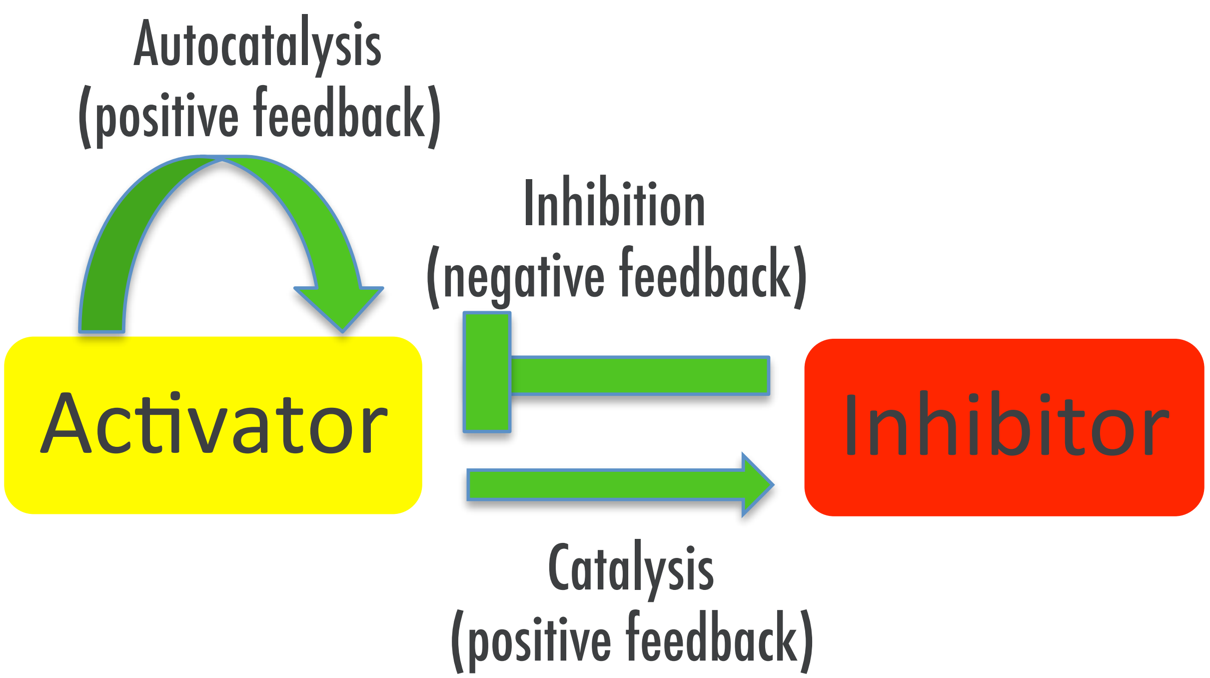
# One possible mechanism: Turing instability

Reaction




+

Diffusion

$u(x,y,t)$: Amount of activator at time t and position (x,y)

$v(x,y,t)$: Amount of inhibitor at time t and position (x,y)

$$\begin{cases} \dfrac{\partial u}{\partial t} = f(u,v) + D_u \nabla^2 u \\ \dfrac{\partial v}{\partial t} = g(u,v) + D_v \nabla^2 v \end{cases}$$

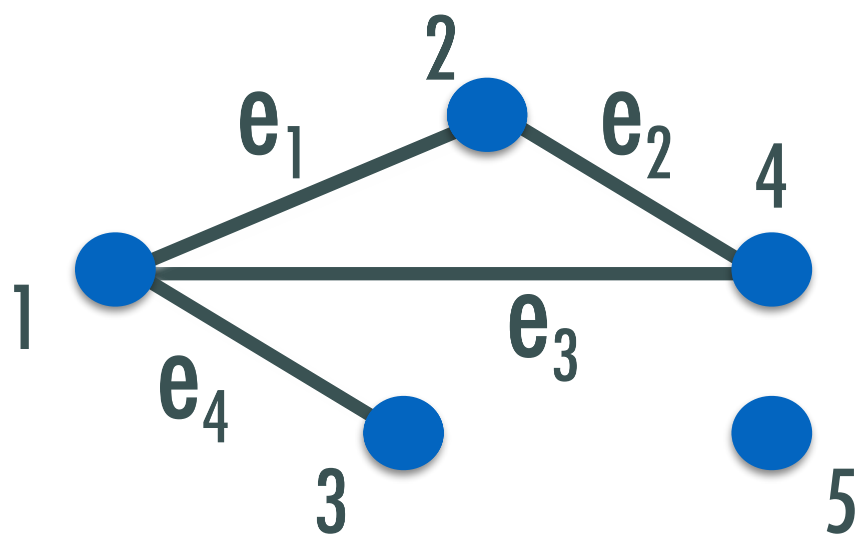
$$(x,y) \in \Omega$$

+ boundary conditions
+ initial condition

A.M.Turing, The chemical basis of morphogenesis, Phil. Trans. R Soc London B, 237, (1952), pp.37

# (complex) Networks: some definitions

A network is a set of nodes connected by links (edges)




Ex.: 5 nodes and 4 edges (undirected)

<u>Adjacency matrix</u>

$$A_{ij} = \begin{cases} 1 & \text{if nodes } i \text{ and } j \text{ are linked} \\ 0 & \text{otherwise} \end{cases}$$

The number of links entering (going out) from each node is called <u>in-degree</u> (<u>out-degree</u>)

Ex.: degree node 1 = 3
degree nodes 2 & 4 = 2
degree node 3 = 1
degree node 5 = 0

A network is said to be <u>complex</u> if the <u>degree distribution is not trivial</u>, i.e. not constant (lattice) nor Poissonian (random, Erdős-Rényi)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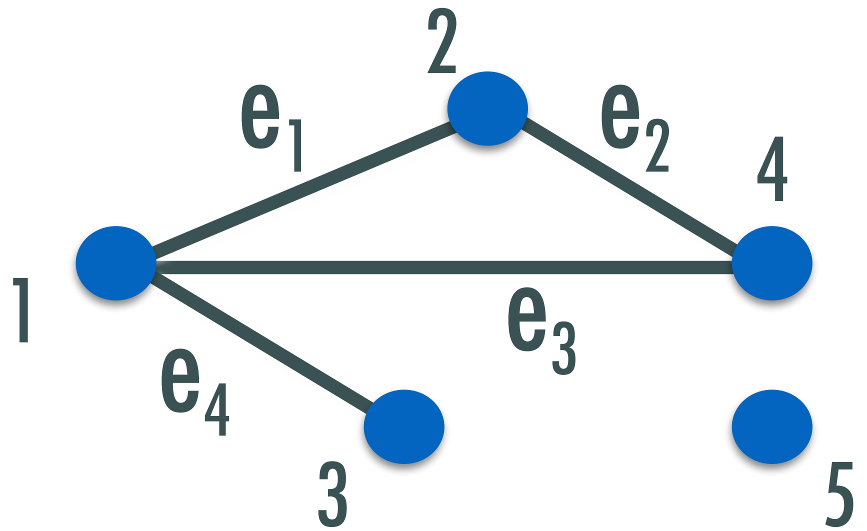
# Reaction term:

$$\begin{cases} \dot{u}_i(t) &= f(u_i(t), v_i(t)) \\ \dot{v}_i(t) &= g(u_i(t), v_i(t)) \quad \forall i = 1, \dots, n \text{ and } t > 0. \end{cases}$$

At each node i=1,...,n, "species" u and v react through some non-linear functions f and g depending on the quantities available at node i-th (metapopulation assumption)

# Diffusion term:

Diffusive transport of species into a certain node i is given by the sum of incoming fluxes to node i from other connected nodes j, fluxes are proportional to the concentration difference between the nodes (Fick's law).




Ex.: consider the amount of u in node 1,
u can enter from 2, 3 and 4
u can leave 1 to go to 2, 3 and 4

$$u_2 + u_3 + u_4 - 3u_1 = \sum_j A_{1j} u_j - k_1 u_1 = \sum_j \left(A_{1j} - \delta_{1j} k_j\right) u_j := \sum_j L_{1j} u_j$$

L is called Laplacian matrix of the network

## The model:

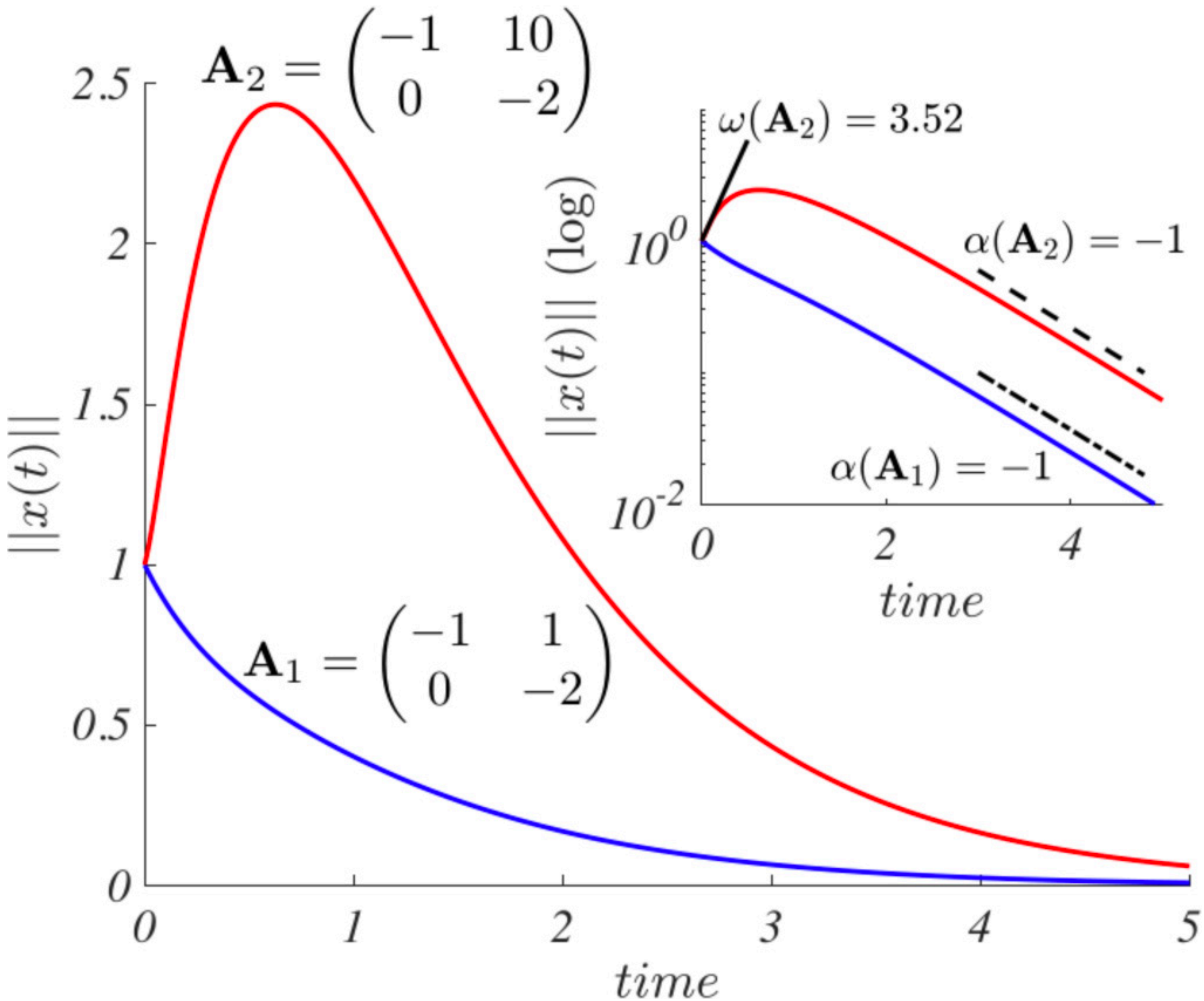
$$\begin{cases} \dot{u}_i(t) & = f(u_i(t), v_i(t)) + D_u \sum_{j=1}^{n} L_{ij} u_j(t) \\ \dot{v}_i(t) & = g(u_i(t), v_i(t)) + D_v \sum_{j=1}^{n} L_{ij} v_j(t) \quad \forall i = 1, \dots, n \text{ and } t > 0. \end{cases}$$

Du and Dv are the diffusion coefficients of species u and v

Observe that because the network is undirected, the matrices A and L are symmetric

# Linear asymptotically stable networked systems

$$\dot{\vec{x}} = A\vec{x}$$



# Non-normal networks are "the rule"

| Network name | nodes | links | $\omega$ | $\omega-\alpha$ | $\alpha_\epsilon$ | $d_F$ | $d_F$ | $\Delta$ |
|---|---|---|---|---|---|---|---|---|
| **Foodwebs** | | | | | | | | |
| Charca de Maspalomas, Gran Canaria [20, 21] | 21 | 82 | $5.40\,10^5$ | $2.97\,10^4$ | $5.10\,10^5$ | $7.04\,10^5$ | 0.69 | 0.66 |
| Crystal River (2) [21, 22] | 21 | 100 | $1.94\,10^3$ | $8.86\,10^2$ | $1.06\,10^3$ | $3.26\,10^3$ | 0.90 | 0.72 |
| Crystal River (1) [21, 22] | 21 | 125 | $2.52\,10^3$ | $1.31\,10^3$ | $1.21\,10^3$ | $4.42\,10^3$ | 0.93 | 0.80 |
| Narragansett Bay Model [21, 23] | 32 | 220 | 6.83 | 0.84 | 0.40 | 9.95 | 0.79 | 0.79 |
| Lower Chesapeake (Summer carbon flows) [21, 24] | 34 | 178 | $1.06\,10^5$ | $3.34\,10^4$ | $7.24\,10^4$ | $1.92\,10^5$ | 0.88 | 0.76 |
| Middle Chesapeake (Summer carbon flows) [21, 24] | 34 | 209 | $1.65\,10^5$ | $3.47\,10^4$ | $1.30\,10^5$ | $2.57\,10^5$ | 0.77 | 0.75 |
| Upper Chesapeake (Summer carbon flows) [21, 24] | 34 | 215 | $6.23\,10^4$ | $7.76\,10^3$ | $5.45\,10^4$ | $1.05\,10^5$ | 0.81 | 0.68 |
| Chesapeake (Summer carbon flows) [21, 25] | 36 | 177 | $4.11\,10^5$ | $9.87\,10^4$ | $3.13\,10^5$ | $6.48\,10^5$ | 0.84 | 0.71 |
| St Marks River (Florida) Estuary [21, 26] | 51 | 356 | 139.50 | 28.45 | 111.51 | 201.34 | 0.80 | 0.70 |
| Everglades Graminoid Marshes [21, 27] | 66 | 916 | $1.44\,10^3$ | $1.04\,10^3$ | 407.94 | $2.82\,10^3$ | 0.98 | 0.95 |
| Foodweb cypress wetlands South Florida (wet season) [28, 29] | 128 | 2016 | 296.71 | 132.11 | 167.46 | 266.65 | 1.00 | 0.83 |
| Foodweb cypress wetlands South Florida (dry season) [28, 29] | 128 | 2137 | 217.60 | 152.50 | 82.20 | 223.41 | 1.00 | 0.89 |
| Foodweb Little Rock Lake (Wisconsin, US) [28, 30] | 183 | 2494 | 21.69 | 14.69 | 10.02 | 12.56 | 0.93 | 0.95 |
| **Biological** | | | | | | | | |
| Secondary-structure elements adjacency for large proteins (PDB, serine protease inhibitor 1EAW) [8] | 53 | 123 | 2.90 | 2.90 | 2.22 | 14.04 | 1.00 | 1.00 |
| Secondary-structure elements adjacency for large proteins (PDB, immunoglobulin 1A4J) [8] | 95 | 213 | 2.68 | 2.68 | 2.22 | 18.63 | 1.00 | 0.95 |
| Secondary-structure elements adjacency for large proteins (PDB, oxidoreductase 1AOR) [8] | 99 | 212 | 3.27 | 3.27 | 2.46 | 19.36 | 1.00 | 0.97 |
| Local subnetwork of neurons within C. Elegans rostral ganglia [31] | 130 | 764 | 7.99 | 2.46 | 6.26 | 10.50 | 0.92 | 0.76 |
| Transcriptional regulation network (Escherichia coli) [32] | 423 | 578 | 5.11 | 4.11 | 2.52 | 31.45 | 0.93 | 0.81 |
| Metabolic network (Caenorhabditis Elegans) [33, 34] | 453 | 4596 | 13.44 | 12.44 | 6.89 | 14.25 | 1.00 | 0.98 |
| Transcriptional regulation networks (Saccharomyces cerevisiae) [35] | 688 | 1079 | 4.99 | 3.66 | 2.19 | 26.17 | 0.99 | 0.99 |
| Pairwise proteins interaction *Homo sapiens* (large) [33, 36] | 2239 | 6452 | 15.79 | 13.02 | 4.01 | 46.98 | 0.99 | 0.99 |
| Human binary protein-protein interactions [33, 37] | 3133 | 6726 | 8.31 | 7.31 | 4.54 | 55.12 | 0.98 | 0.80 |
| **Transport** | | | | | | | | |
| Preferred Routes Database (NFDC, US FAA) [38, 39] | 1226 | 2615 | 5.73 | 0.48 | 5.62 | 30.67 | 0.88 | 0.69 |
| US airport 2010 [39, 40] | 1574 | 28236 | $1.19\,10^7$ | 79.30 | $1.19\,10^7$ | $4.76\,10^6$ | 1.00 | 0.01 |
| Directed road transportation network 1999 (Rome, Italy) [39, 41] | 3353 | 8870 | $2.40\,10^4$ | 120.05 | $2.39\,10^4$ | $2.80\,10^4$ | 0.28 | 0.08 |
| Directed road transportation network (Chicago region, USA) [39, 42, 43] | 12982 | 39018 | 4.23 | $4.29\,10^{-4}$ | 4.54 | 37.53 | 0.19 | 0.04 |
| **Communication** | | | | | | | | |
| Sent messages among students University of California, Irvine [44, 45] | 1899 | 59835 | 36.38 | 2.12 | 34.60 | 26.11 | 0.87 | 0.33 |
| e-mails network Democratic National Committee (2016) [44] | 2029 | 39264 | 28.00 | 2.00 | 26.37 | 24.13 | 0.89 | 0.53 |
| Wikipedia who-votes-on-whom network [46–48] | 8297 | 103689 | 74.42 | 29.28 | 45.92 | 90.07 | 0.99 | 0.85 |
| Enron email network (1999-2003) [44, 49] | 87273 | 1148072 | 85.14 | 14.54 | 71.05 | 160.07(∗) | 0.99(∗) | 0.30 |
| e-mails network European institution [44, 50] | 265214 | 420045 | 76.02 | 6.09 | 70.30 | 434.63(∗) | 0.84(∗) | 0.30 |
| **Citation** | | | | | | | | |
| Citations to Milgram's 1967 Psychology Today paper (2002) [51] | 395 | 1988 | 10.48 | 10.48 | 4.49 | 18.57 | 1.00 | 1.00 |
| Citations to Small & Griffith and Descendants (2001) [51] | 1059 | 4922 | 14.34 | 13.34 | 6.60 | 32.53 | 1.00 | 0.99 |
| Articles from or citing Scientometrics, 1978-2000 (2002) [51] | 3084 | 10416 | 10.32 | 8.32 | 5.28 | 55.49 | 1.00 | 0.98 |
| Articles citing and by AH Zewail, 1970-2002 (2002) [51] | 6752 | 54253 | 24.57 | 21.84 | 11.63 | 82.13 | 1.00 | 0.90 |
| Citation network DBLP [52, 53] | 12591 | 49743 | 21.50 | 16.82 | 8.45 | 112.06 | 1.00 | 0.87 |
| Cora citation network [52] | 23166 | 91500 | 16.11 | 7.12 | 9.91 | 150.96 | 0.99 | 0.74 |
| **Social** | | | | | | | | |
| College students in a course about leadership [8] | 32 | 96 | 3.74 | 0.62 | 3.56 | 4.90 | 0.87 | 0.63 |
| Inmates in prison [8] | 67 | 182 | 3.75 | 0.41 | 3.76 | 6.86 | 0.84 | 0.55 |
| Highschool Illinois [54, 55] | 70 | 366 | 10.35 | 0.76 | 10.00 | 11.32 | 0.94 | 0.46 |
| Friendship network Australian National University campus [55, 56] | 217 | 2672 | 53.98 | 3.10 | 51.24 | 45.24 | 0.94 | 0.34 |
| Network of innovation spread among physicians (1966) [55, 57] | 241 | 1098 | 6.45 | 0.72 | 6.11 | 14.02 | 0.90 | 0.63 |
| Hyperlinks network of 2004 US election blogs [58, 59] | 1224 | 19025 | 45.37 | 10.95 | 34.95 | 34.32 | 0.98 | 0.72 |
| Reply network of the social news website Digg [44, 60] | 30398 | 87627 | 15.92 | 6.56 | 10.18 | 168.28 | 0.97 | 0.61 |
| Trust network from the online social network Epinions [61, 62] | 75879 | 508837 | 123.00 | 16.47 | 106.96 | 221.01(∗) | 0.86(∗) | 0.55 |