Thessaloniki | September 24–28, 2018

Timoteo Carletti

Hopping in the crowd to unveil network topology

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Acknowledgements

M. Asllani

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D. Fanelli and F. Di Patti

FLORENTINA STUDIORUM UNIVERSITAS

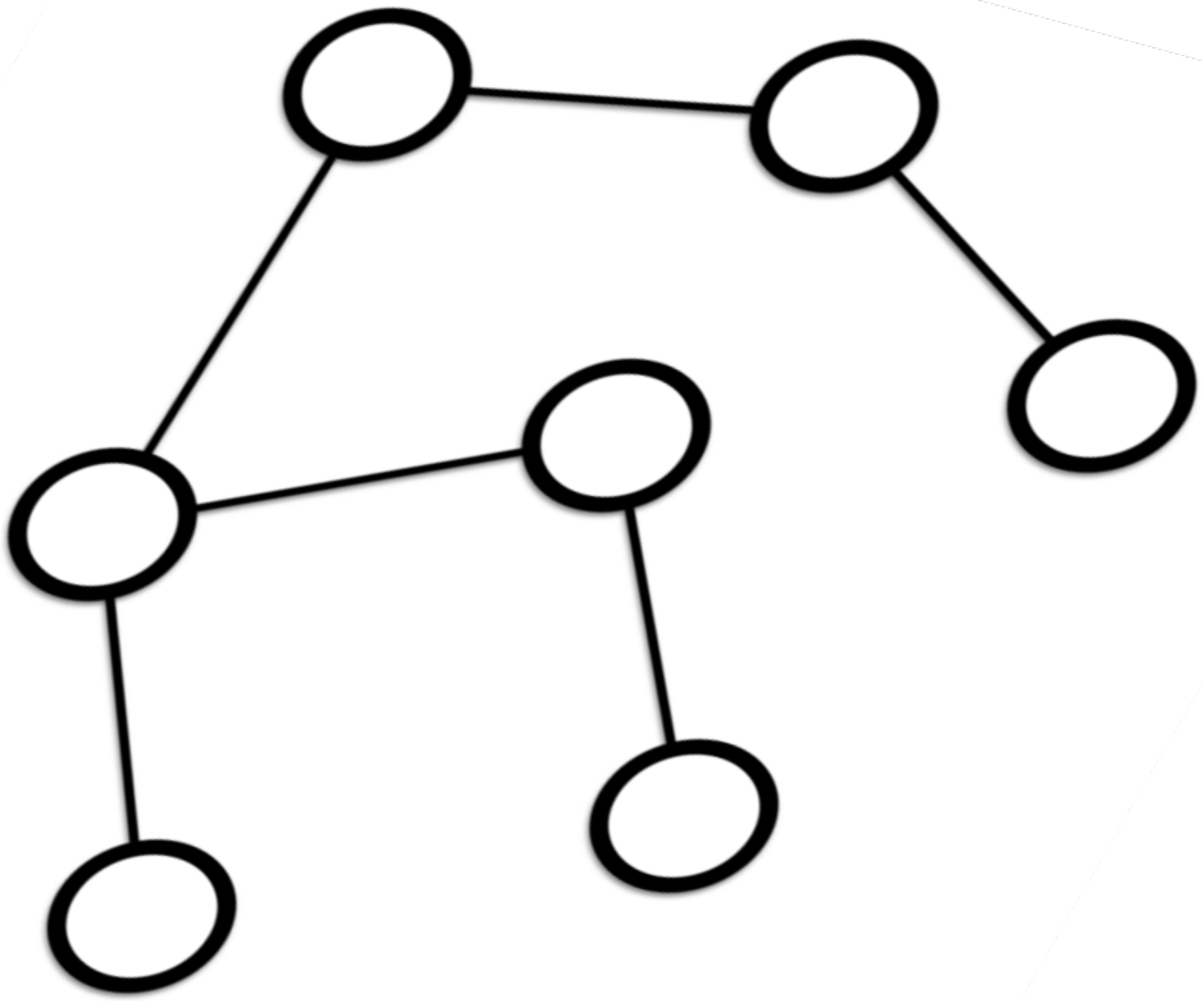
Francesco Piazza

UNIVERSITE D'ORLEANS

Random walk on networks

- Model of diffusions
- Ranking
- Community detection
- And many more …

Random walk on networks

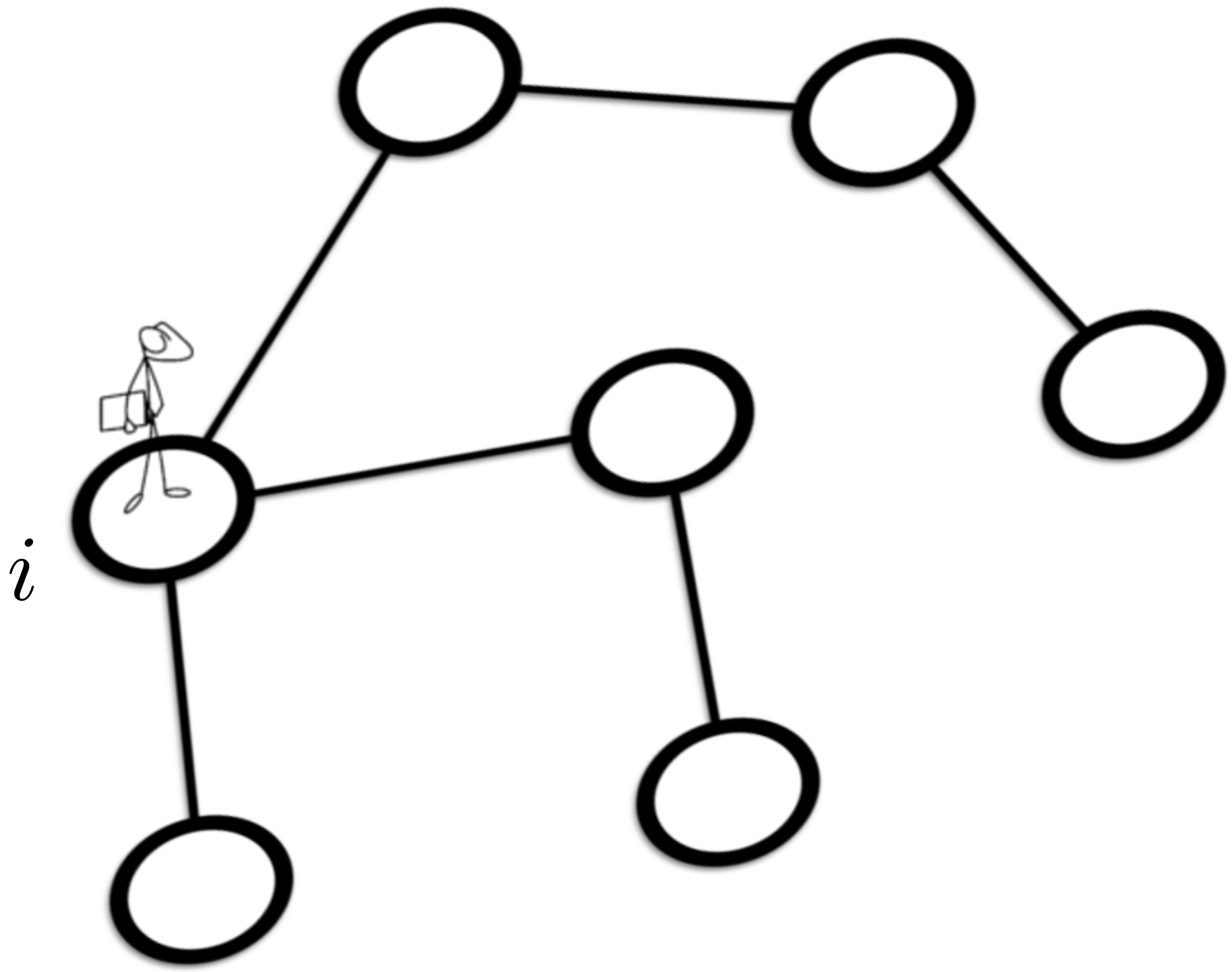


Number of nodes: Ω

Adjacency matrix: $A_{ij} = A_{ji} = 1$

Nodes degree: $k_i = \sum_j A_{ij}$

Random walk on networks



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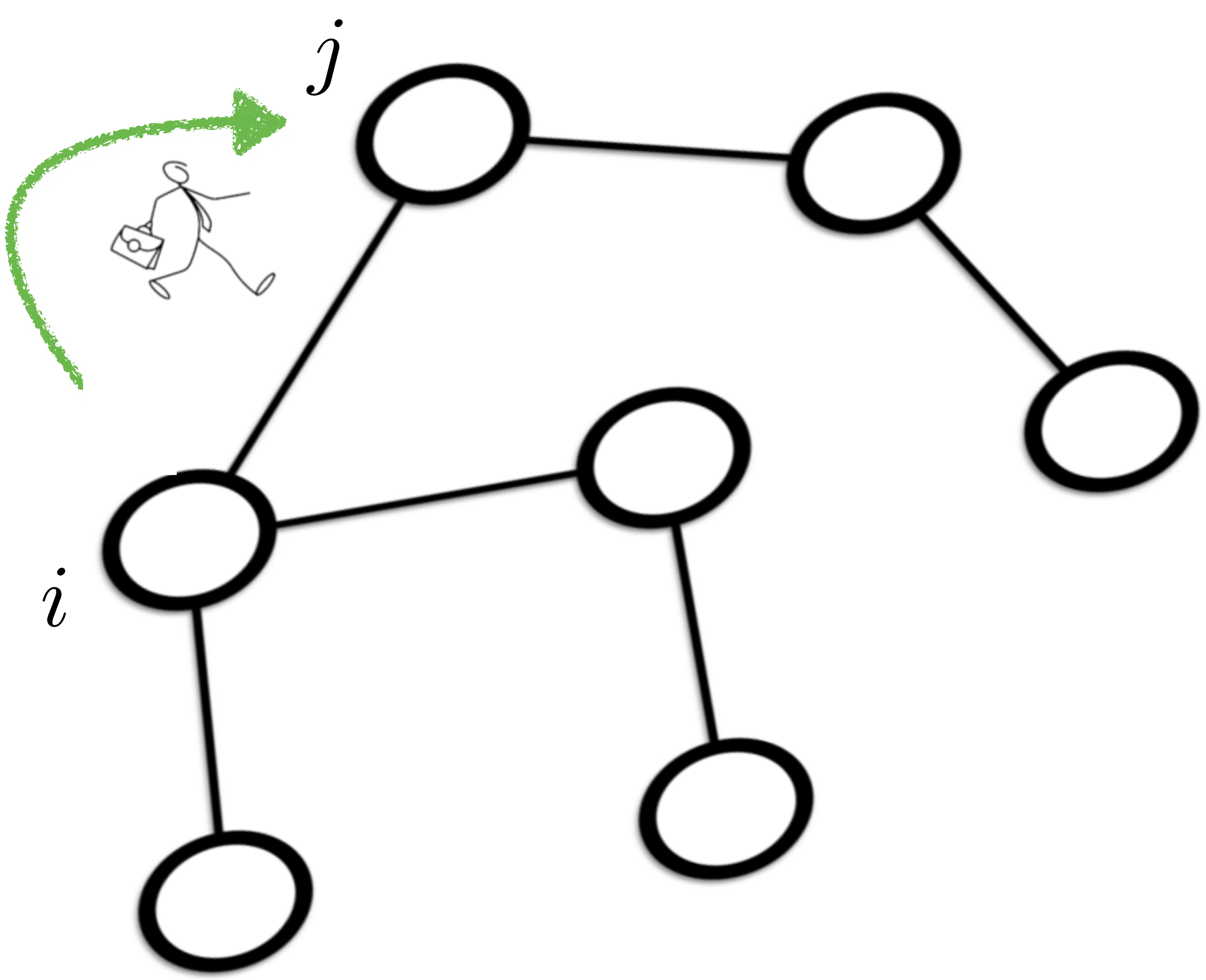

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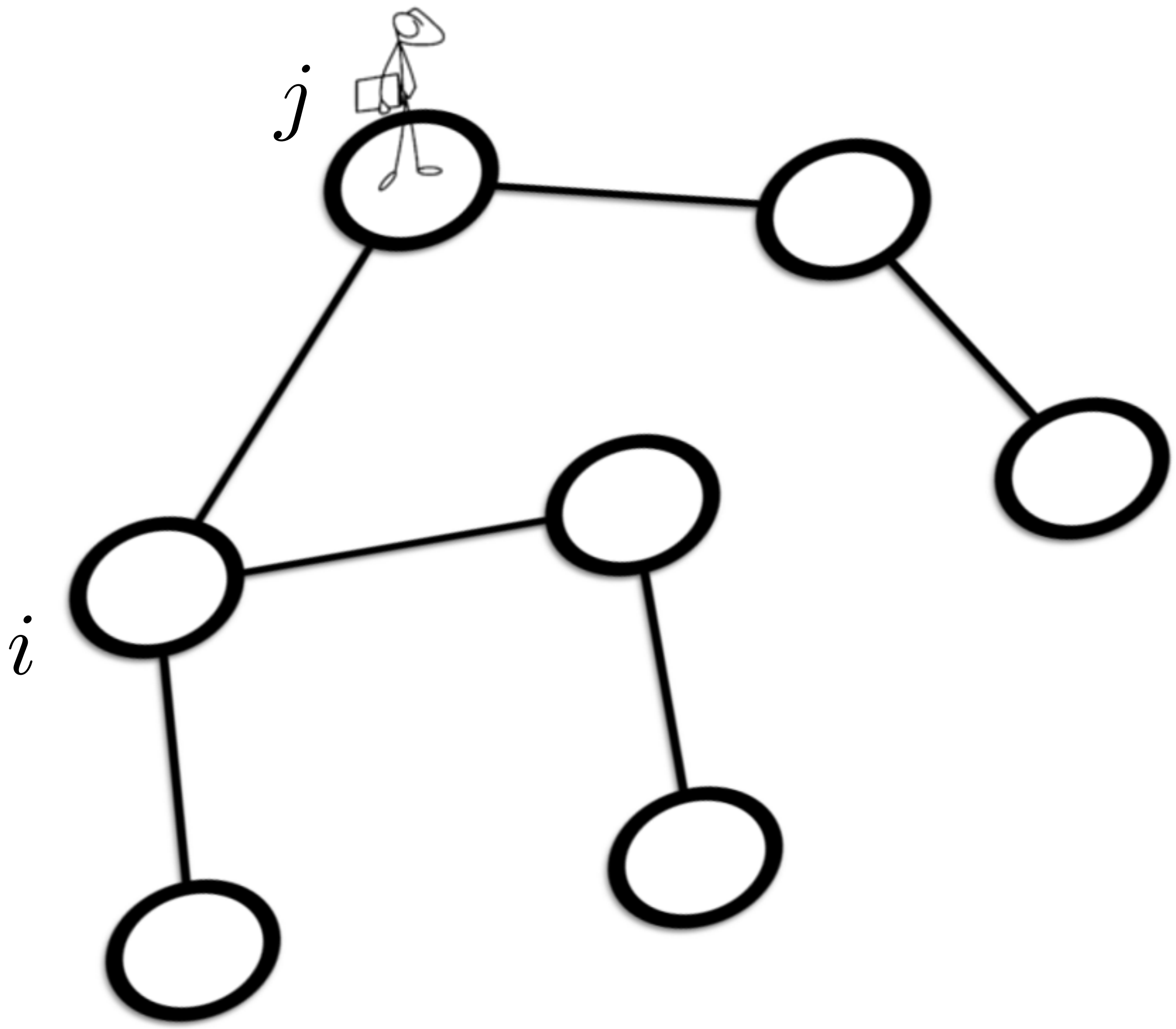

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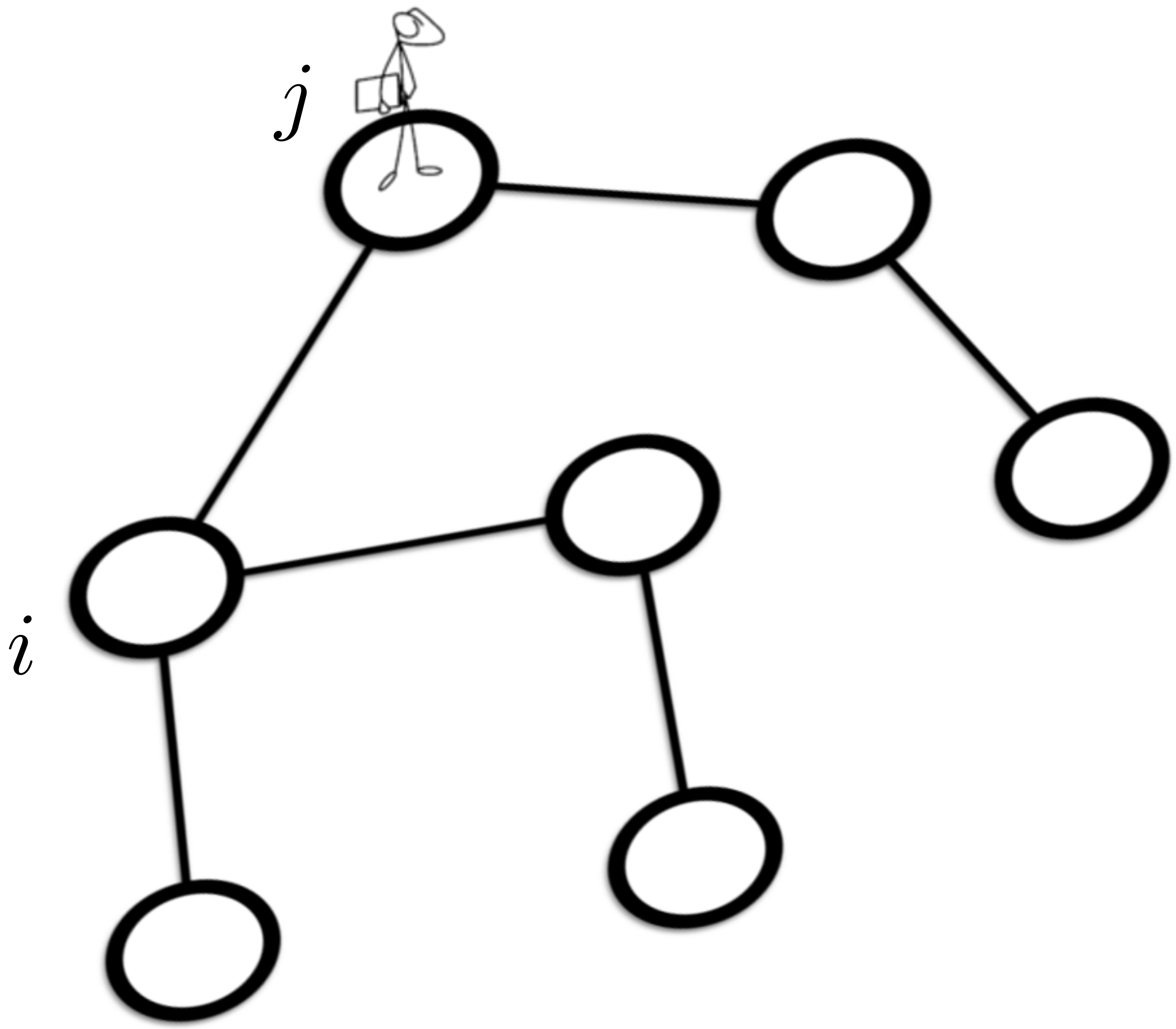

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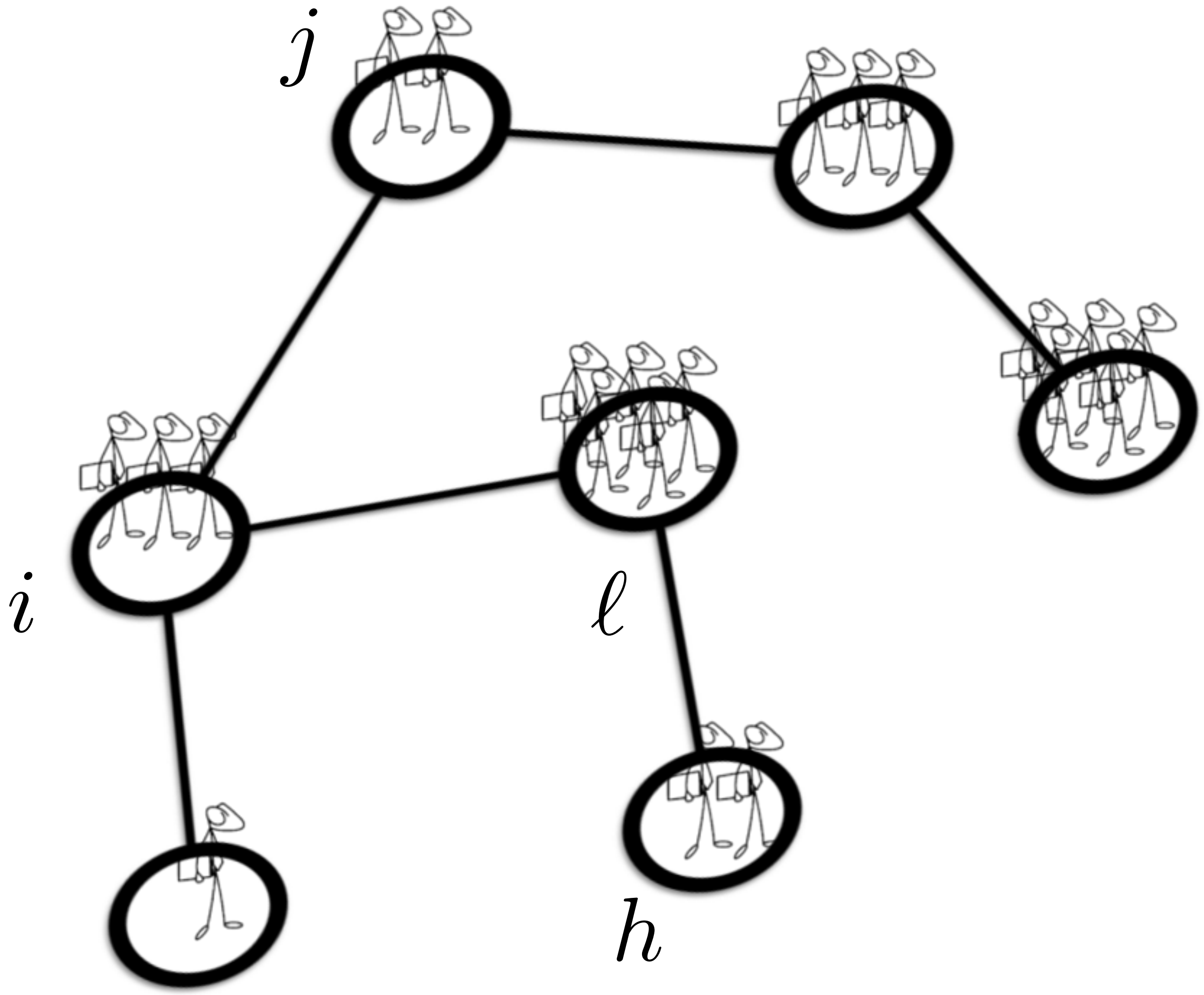
Probability to find the walker on node i at time t: $P_i(t)$

$P_i(t) \sim k_i$ (asymptotically)

Crowded Random walk on networks



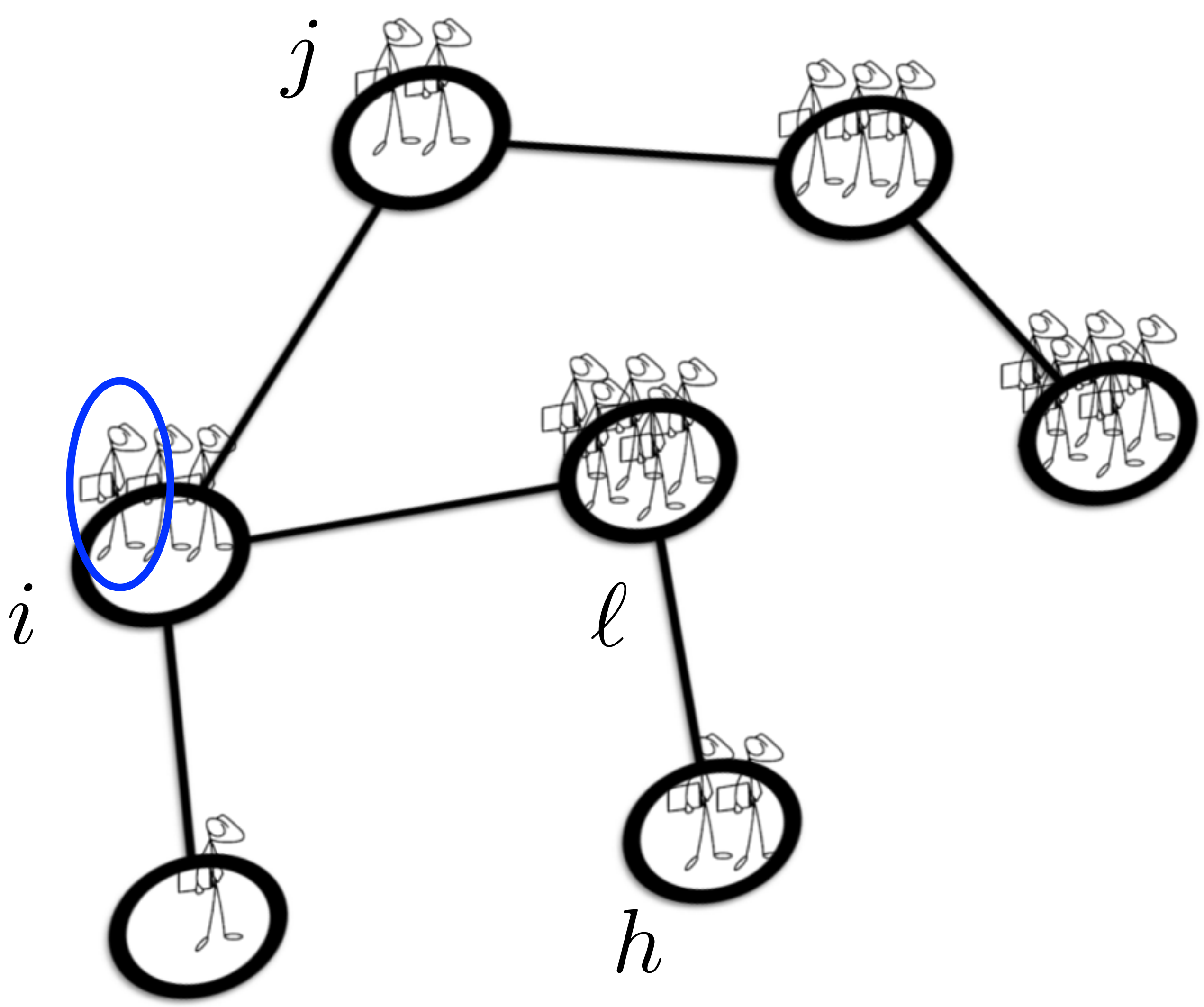
Node capacity: N

Number of walkers: $\beta\Omega$

Number walkers at node i: n_i

(N=5)

Crowded Random walk on networks



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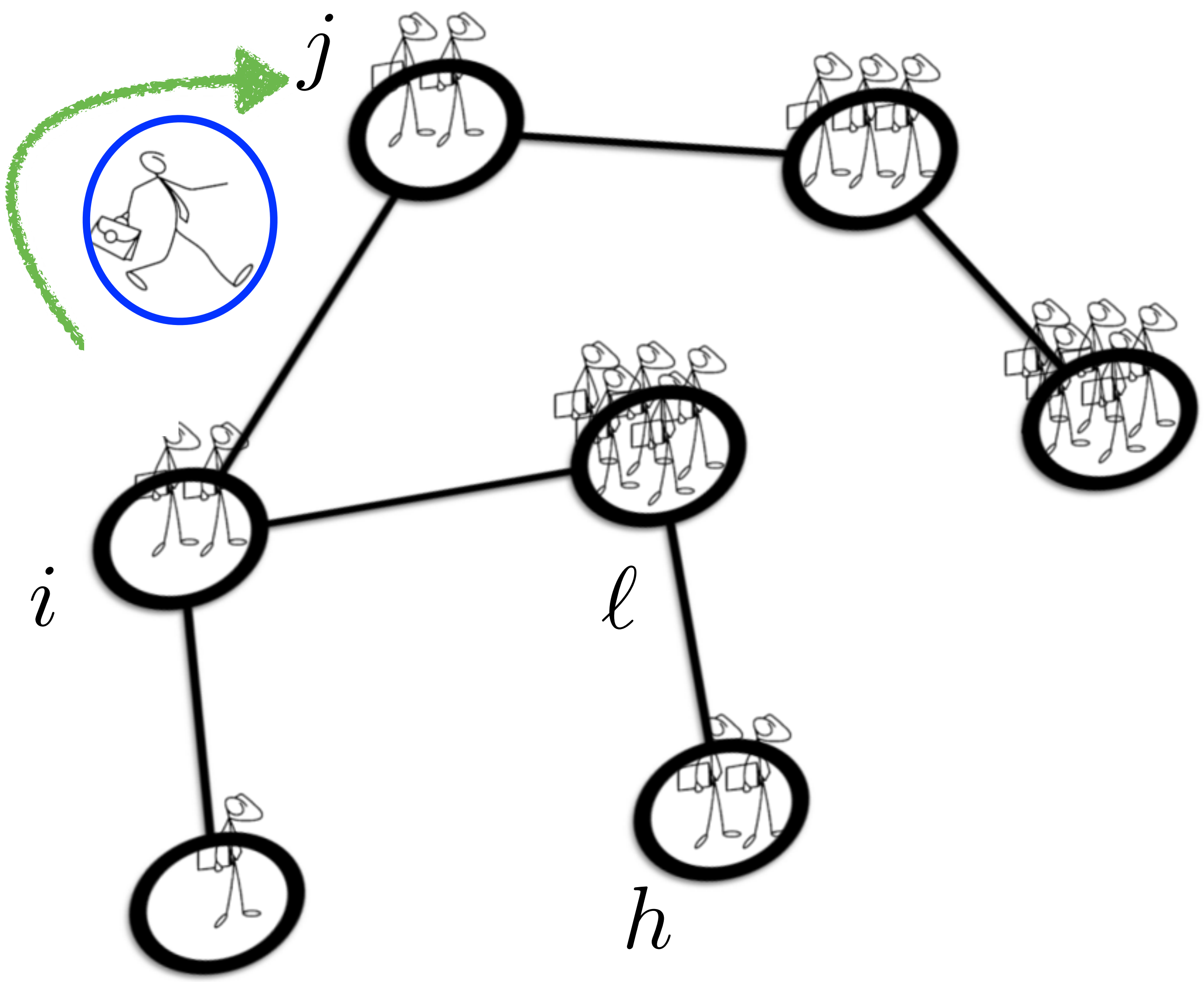

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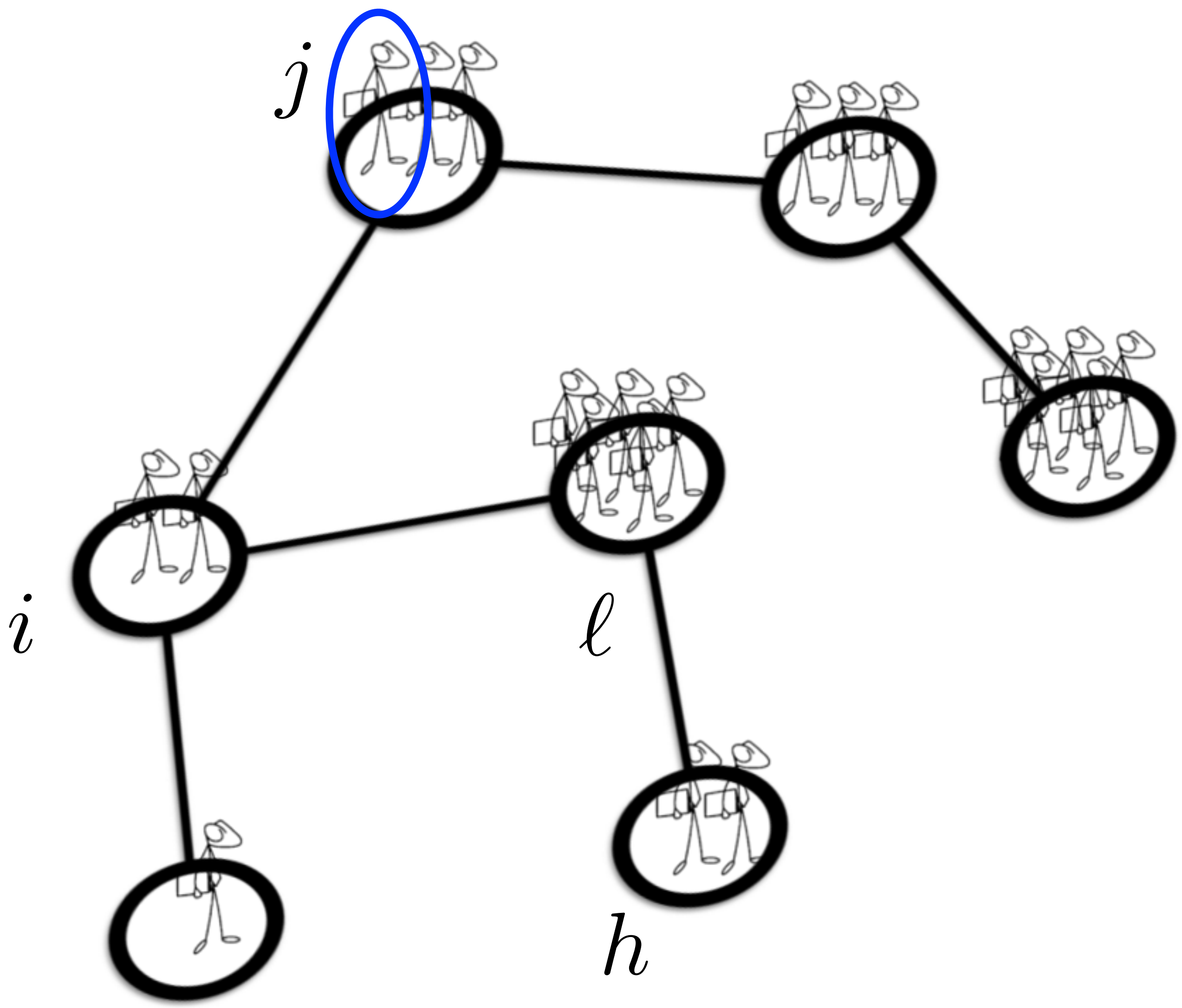

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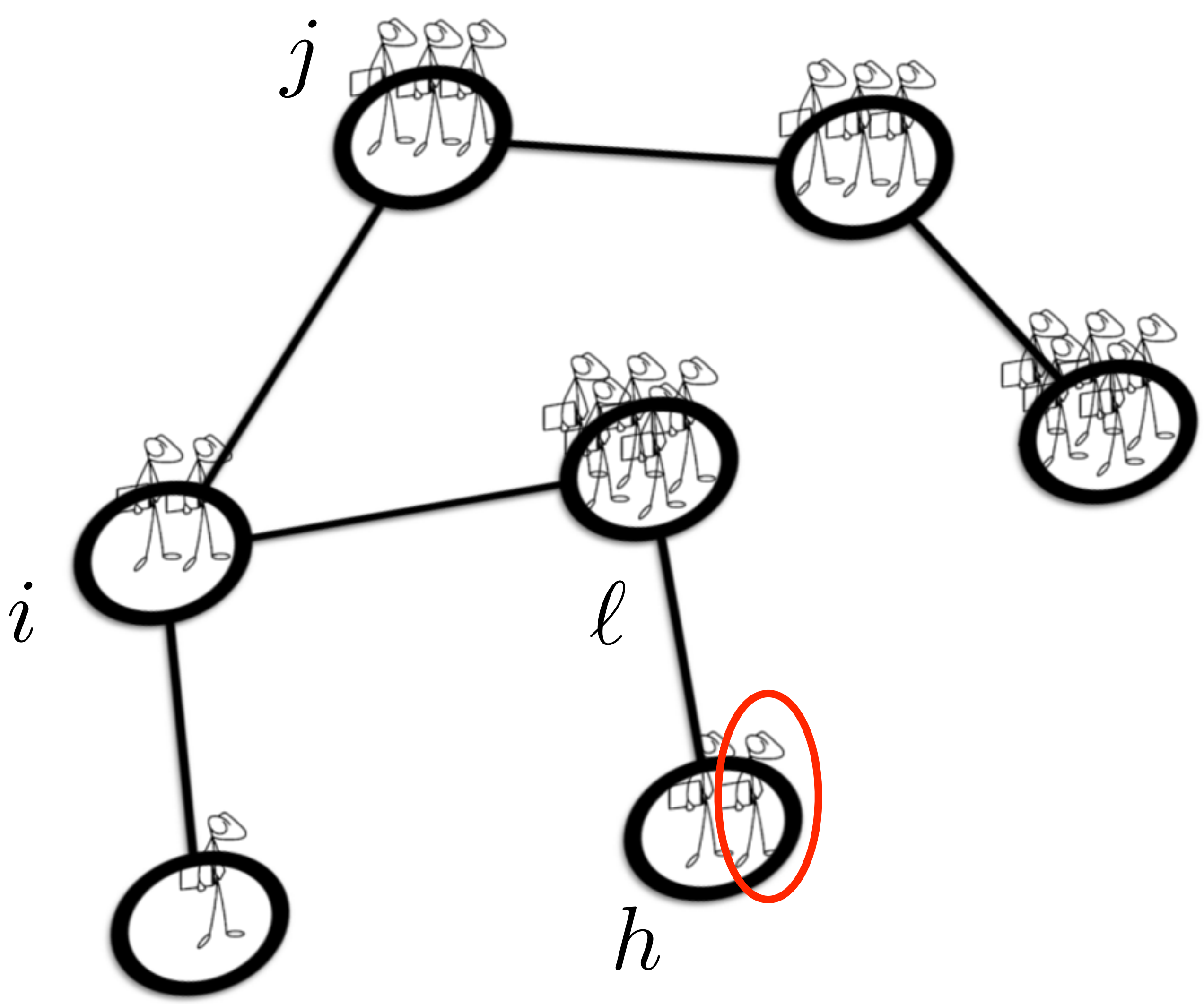

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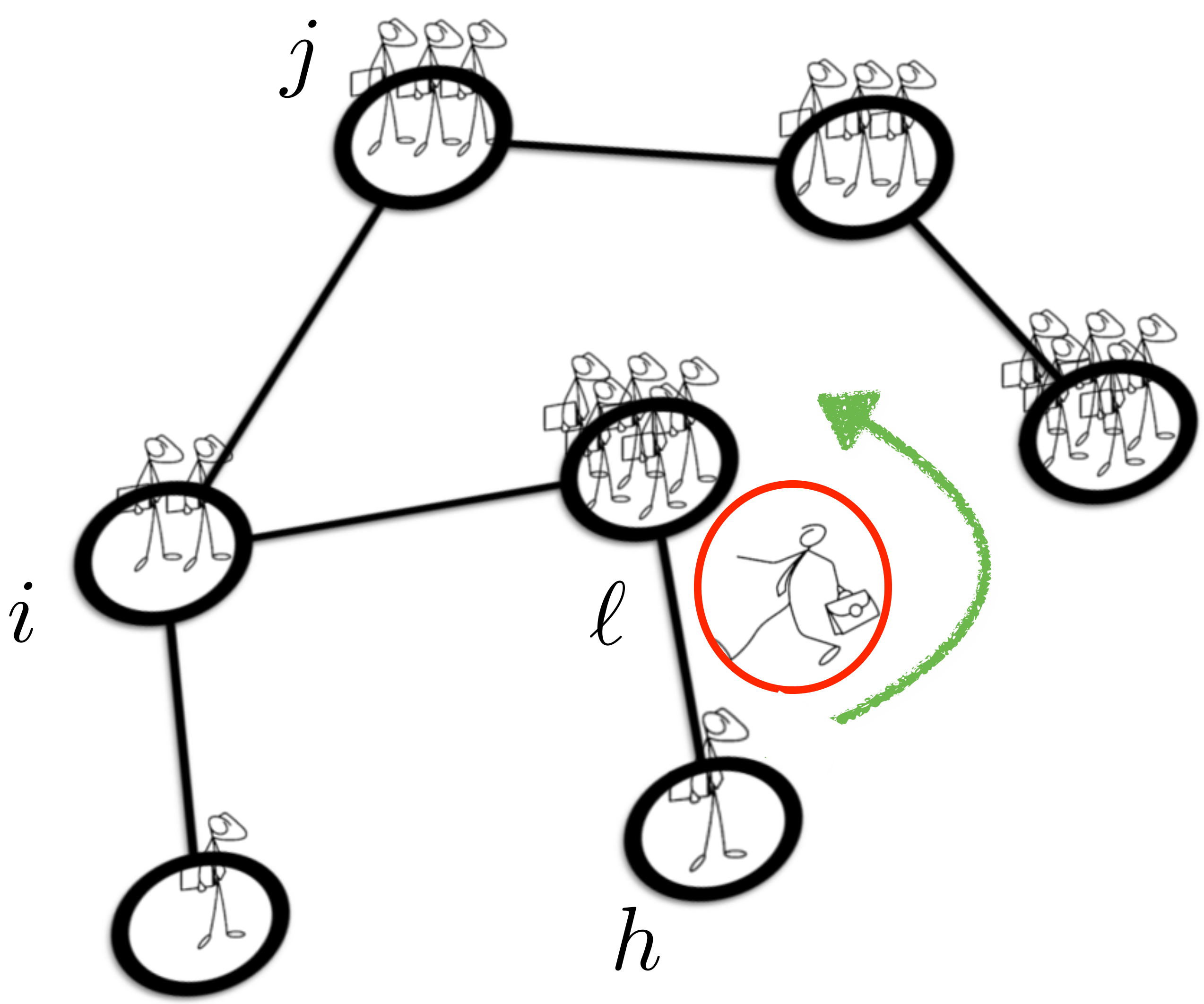

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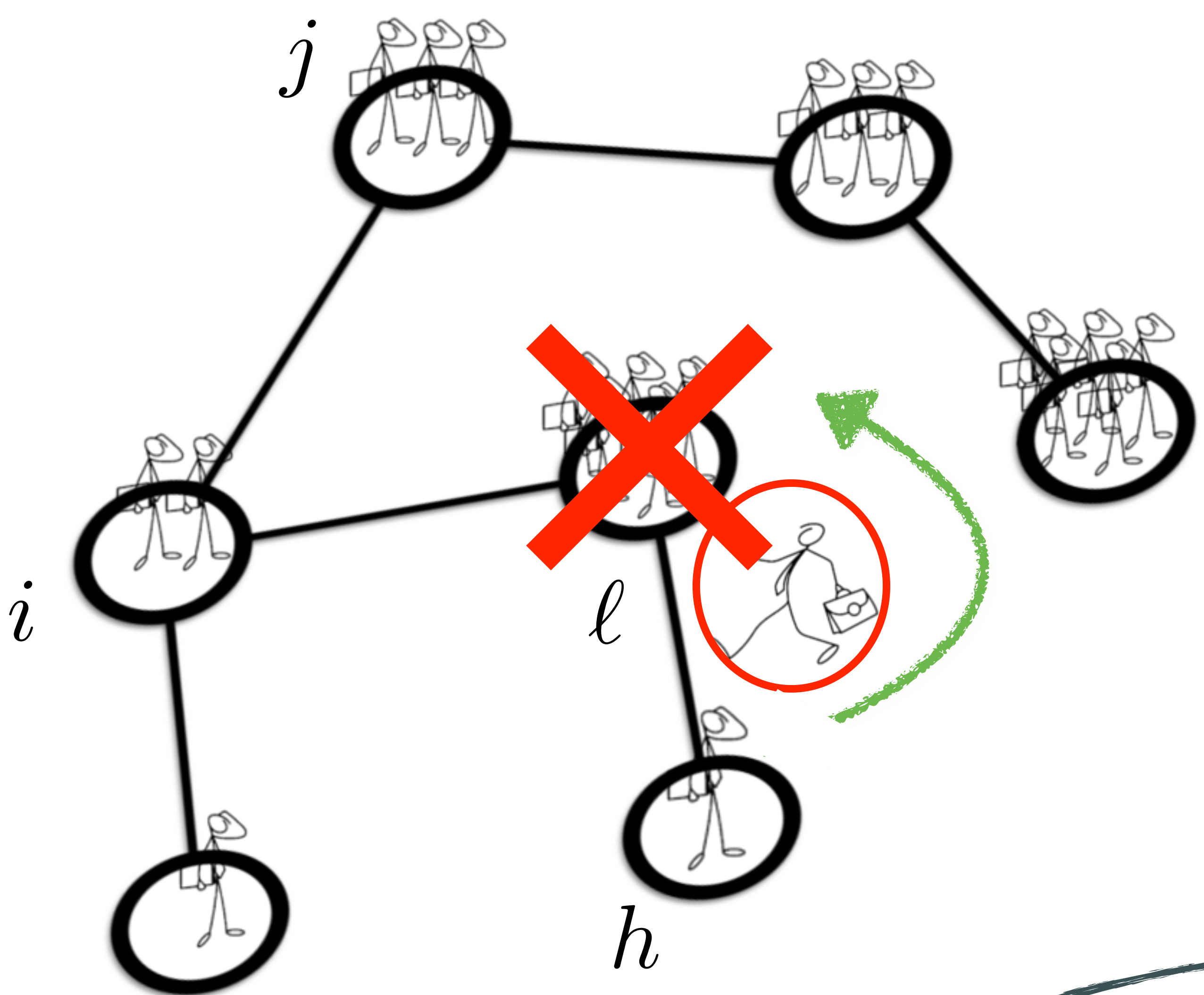

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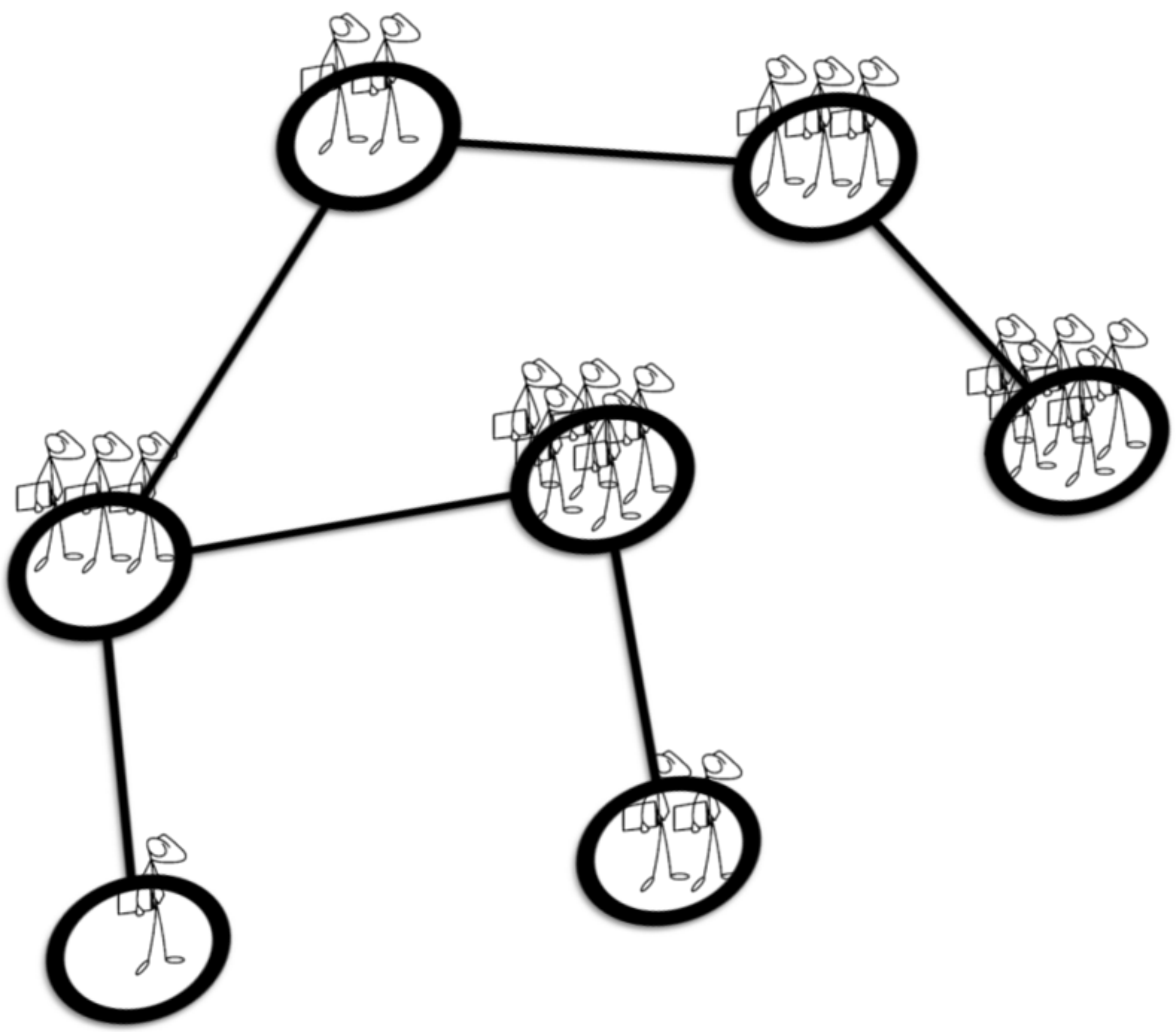
Transition probability form i to j :

$$T(n_i - 1, n_j + 1 | n_i, n_j) = \frac{A_{ji}}{k_i} \frac{n_i}{N} \frac{(N - n_j)}{N}$$

Crowded Random walk on networks



$$P(n_1, \dots, n_\Omega, t)$$

Probability to find the system in the state $(n_1, \dots, n_\Omega)$ at time t

$$\langle n_i \rangle \equiv \sum_{\mathbf{n}} n_i P(n_1, \dots, n_\Omega, t)$$

Average number of walkers at node i at time t

Master equation

$$\frac{d}{dt}P(\mathbf{n},t) = \sum_{\mathbf{n}'} \left[T(\mathbf{n}|\mathbf{n}')P(\mathbf{n}',t) - T(\mathbf{n}'|\mathbf{n})P(\mathbf{n},t)\right] ,$$

n′ should be compatible with **n**

Transitions probabilities: $T(n_i - 1, n_j + 1 | n_i, n_j) = \frac{1}{k_i}\frac{n_i}{N}\frac{(N-n_j)}{N}$

iff $A_{ij} = 1$

$$T(n_i + 1, n_j - 1 | n_i, n_j) = \frac{1}{k_j}\frac{n_j}{N}\frac{(N-n_i)}{N}$$

Rescale time: $\tau = t/N$

Density of walkers in node i at time t: $\rho_i = \lim_{N\to\infty} \langle n_i \rangle / N$

A new non-linear transport operator

$$\frac{\partial}{\partial t}\rho_i = \sum_{j=1}^{\Omega} \Delta_{ij} \left[\rho_j \left(1 - \rho_i\right) - \frac{k_j}{k_i} \rho_i \left(1 - \rho_j\right) \right]$$

$$\Delta_{ij} = A_{ij}/k_j - \delta_{ij}$$

Random walk Laplacian

A new non-linear transport operator

$$\beta = \sum_{i=1}^{\Omega} \rho_i(t)$$

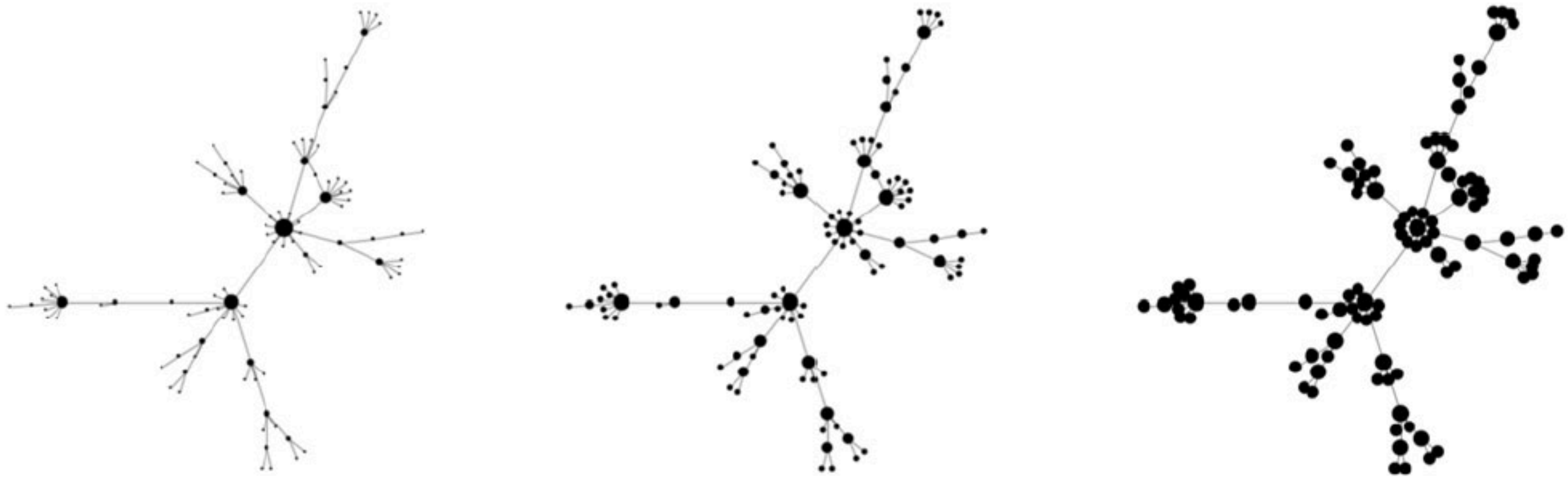
The total "mass" is preserved

$$\rho_i^{\infty} = \frac{a k_i}{1 + a k_i}$$

(asymptotically)

Application I: topological vs functional hubs

$$\rho_i^\infty = \frac{ak_i}{1 + ak_i}$$



$\beta \rightarrow 0$ $\beta = 0.3\,\Omega$ $\beta = 0.7\,\Omega$

FIG. 1. Asymptotic distribution of walkers diffusing on a scale-free network under different crowding conditions, as specified by β. Standard diffusion is recovered in the limit $\beta \rightarrow 0$ and displayed in the leftmost picture. Nodes are drawn with a size proportional to the corresponding asymptotic density, ρ_i^∞.

Application (II): reconstruction of p(k)

- ✓ The "mass" conservation provides $$\beta = \sum_k p(k) \frac{a(\beta)k}{1+a(\beta)k}$$

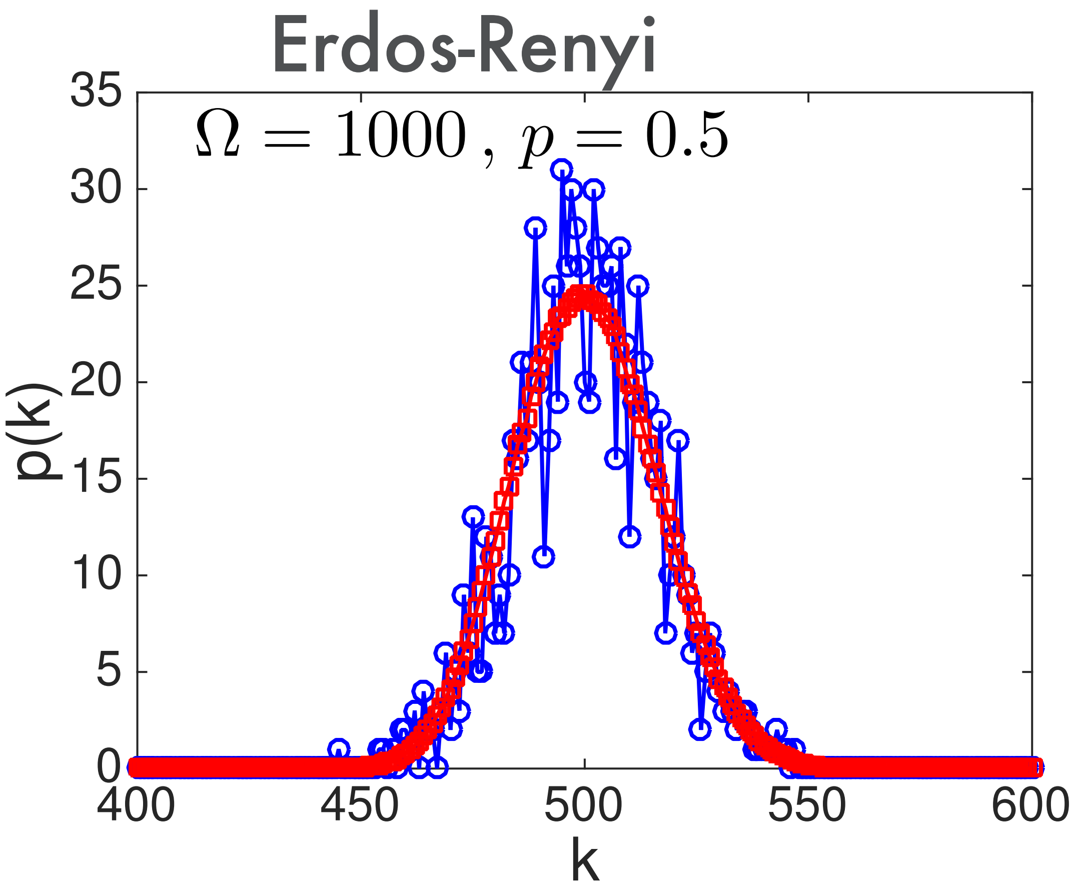
- ✓ Let β_1 walkers to diffuse in the crowded network
- ✓ From a <u>single node</u> measure ρ_1^∞ and k_1
- ✓ Compute $$a_1 := a(\beta_1) = \frac{\rho_1^\infty}{1-\rho_1^\infty}\frac{1}{k_1}$$

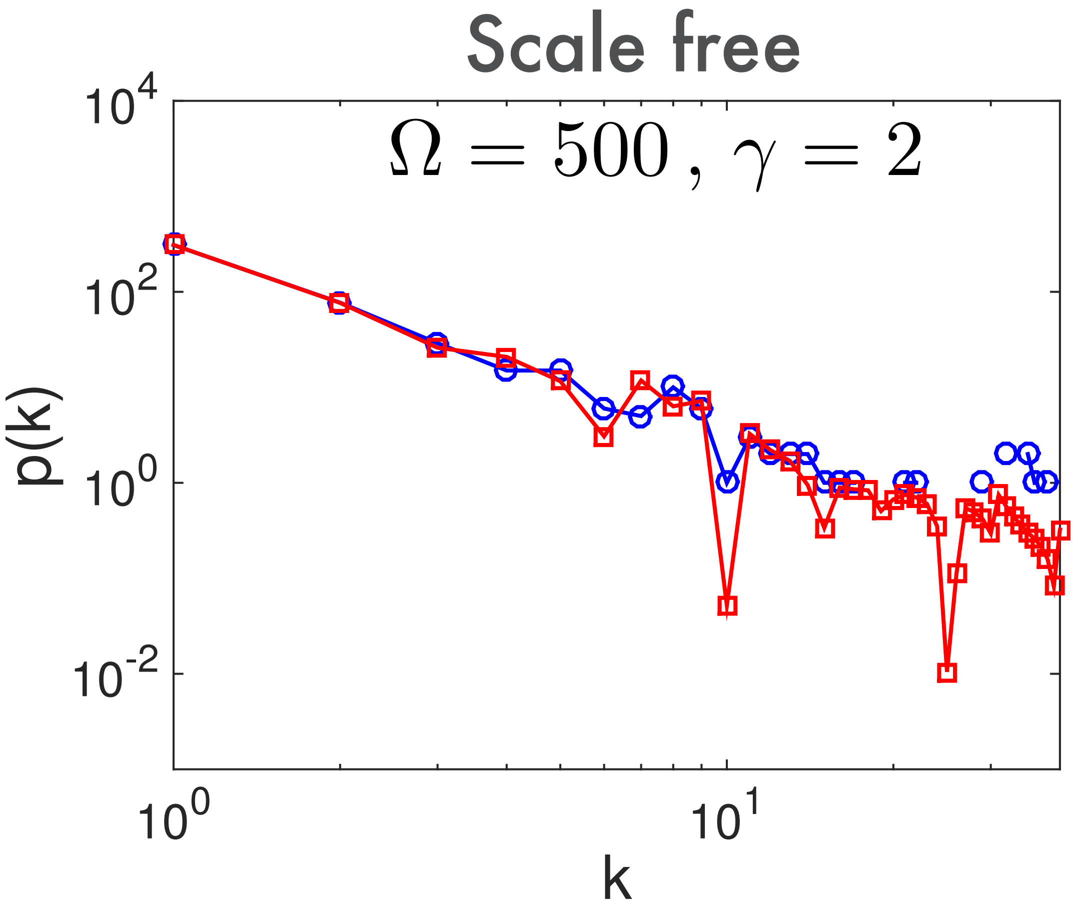
"experiment"

- ✓ Repeat s times the experiment and compute $a_s = a(\beta_s)$
- ✓ Solve

$$\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_s \end{pmatrix} = \mathbf{F} \begin{pmatrix} p(1) \\ \vdots \\ p(s) \end{pmatrix} \qquad F_{lr} = \frac{ra_l}{1+ra_l}$$

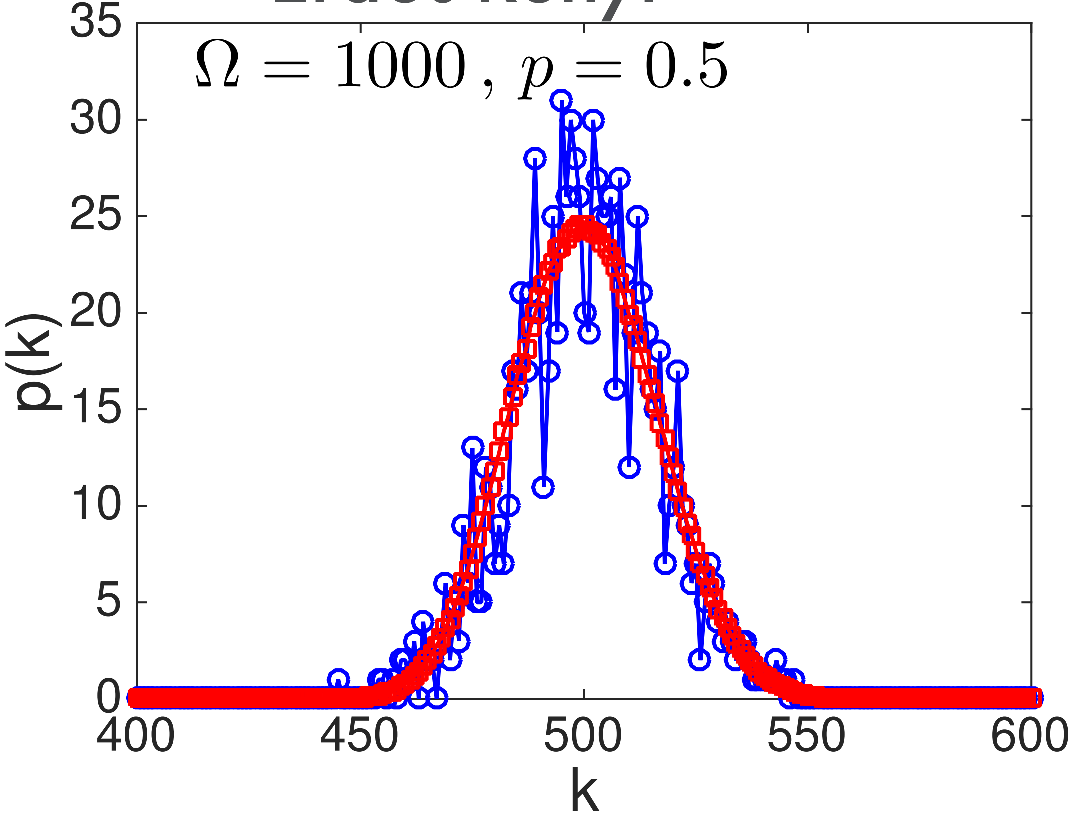
Application (II): reconstruction of p(k)



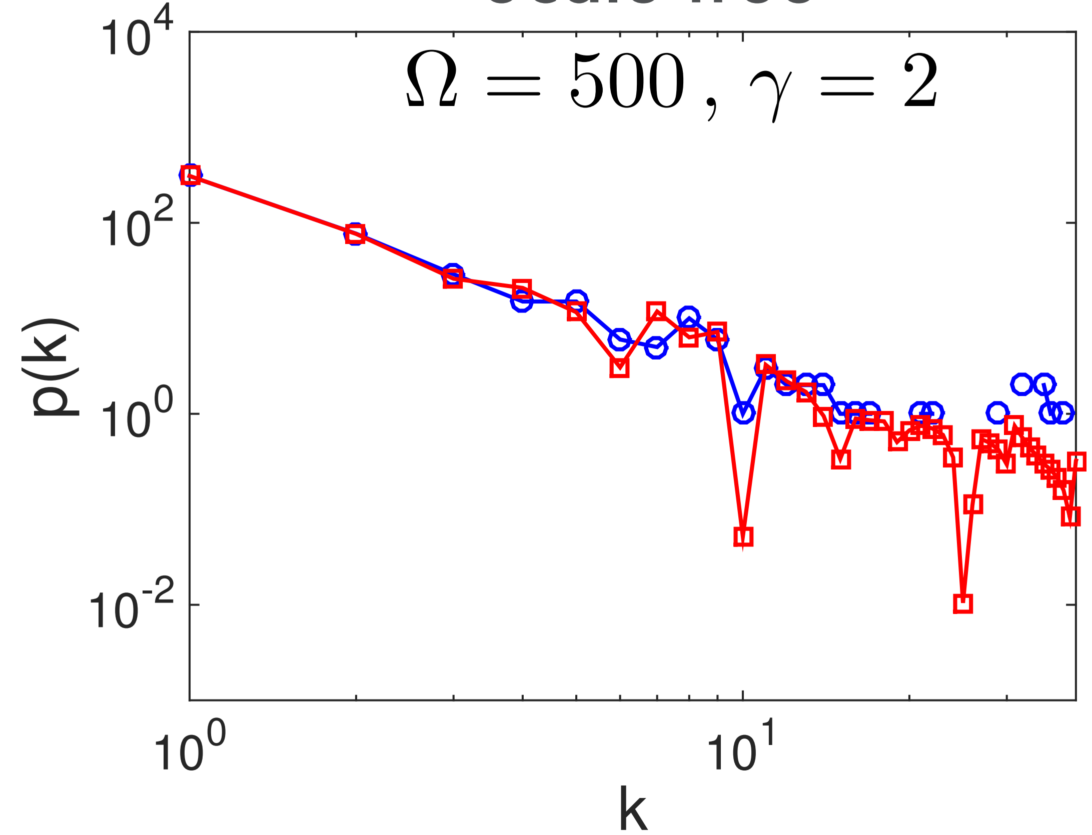


Application (II): reconstruction of p(k)

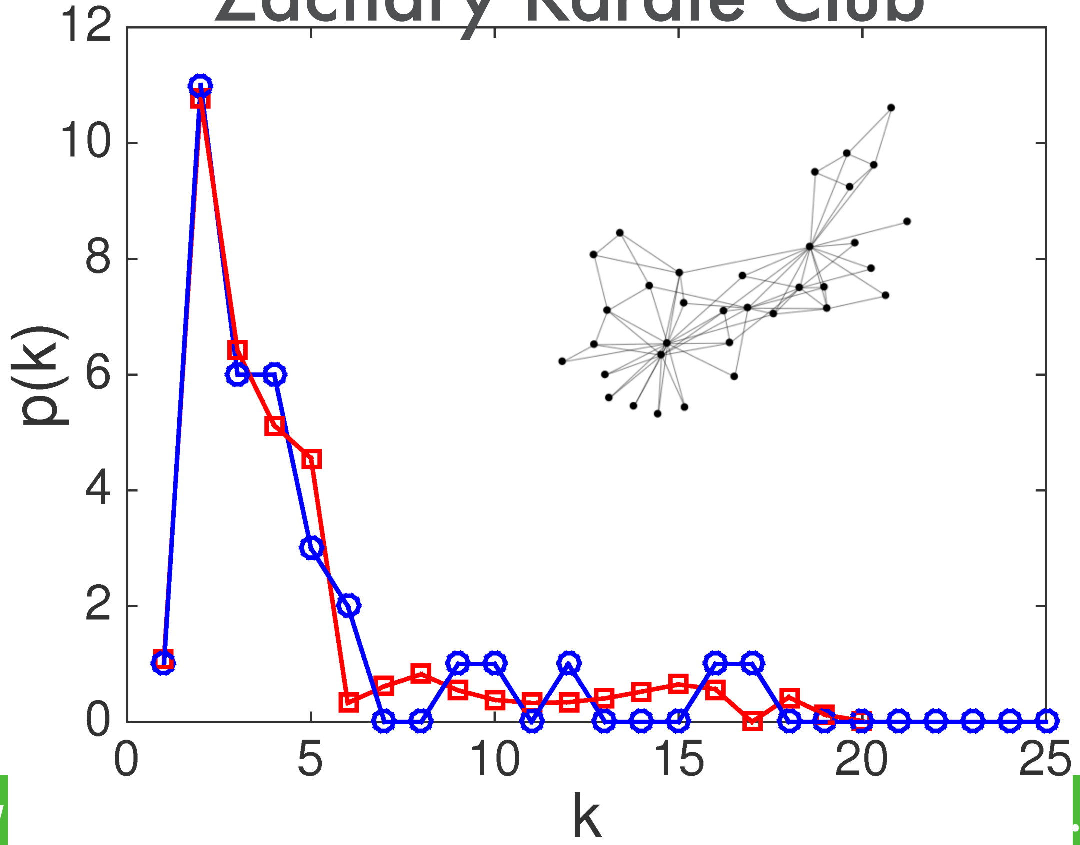
Erdos-Renyi



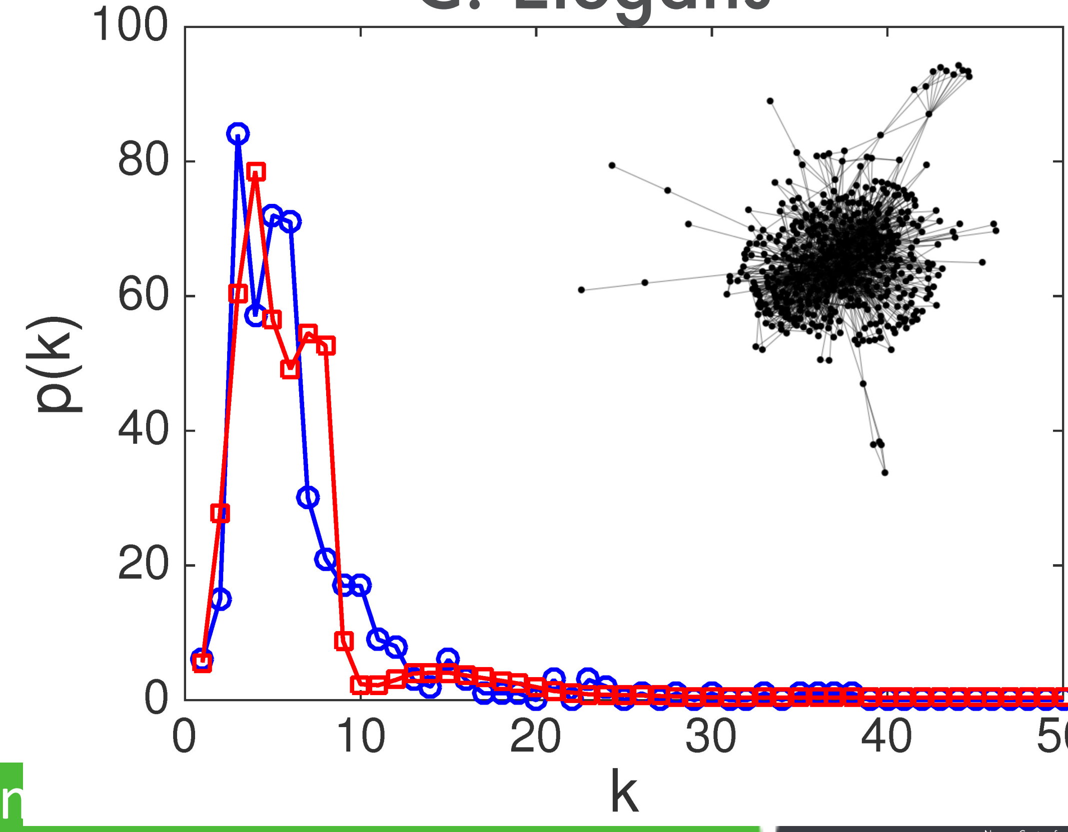
Scale free



Zachary Karate Club



C. Elegans



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PHYSICAL REVIEW LETTERS **120**, 158301 (2018)

Hopping in the Crowd to Unveil Network Topology

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(Received 29 November 2017; published 9 April 2018)

We introduce a nonlinear operator to model diffusion on a complex undirected network under crowded conditions. We show that the asymptotic distribution of diffusing agents is a nonlinear function of the nodes' degree and saturates to a constant value for sufficiently large connectivities, at variance with standard diffusion in the absence of excluded-volume effects. Building on this observation, we define and solve an inverse problem, aimed at reconstructing the *a priori* unknown connectivity distribution. The method gathers all the necessary information by repeating a limited number of independent measurements of the asymptotic density at a single node, which can be chosen randomly. The technique is successfully tested against both synthetic and real data and is also shown to estimate with great accuracy the total number of nodes.

DOI: 10.1103/PhysRevLett.120.158301

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