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Patterns of Non-Normality in Networked Systems

UNIVERSITÉ DE NAMUR

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Namur Institute for Complex Systems

Acknowledgements

M. Asllani

Riccardo Muolo

D. Fanelli

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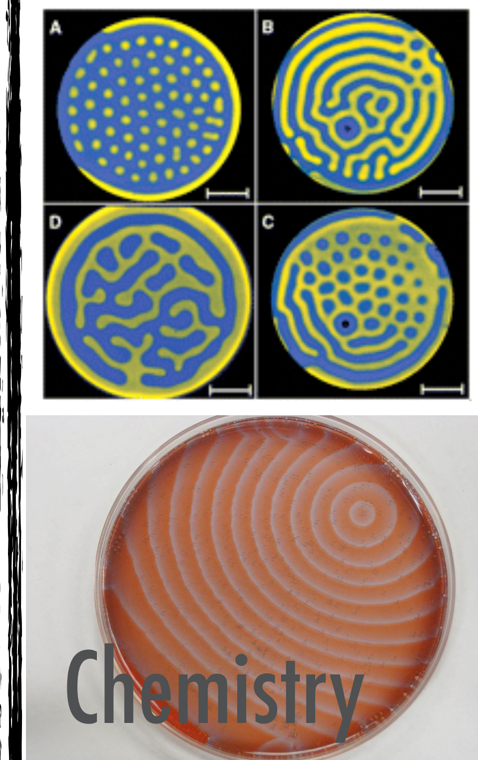
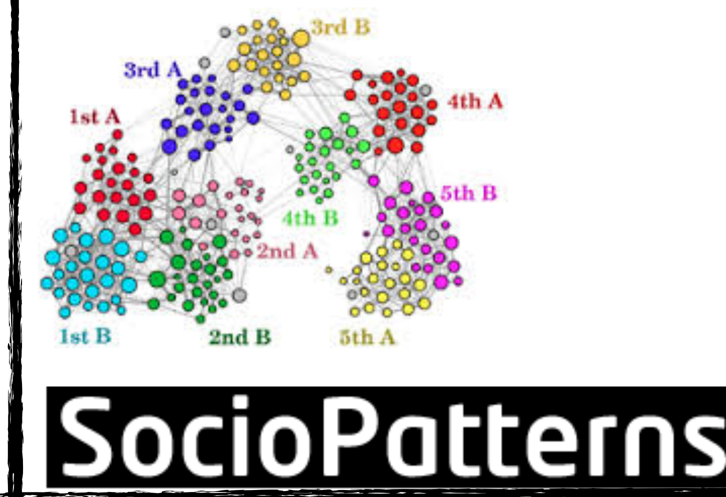
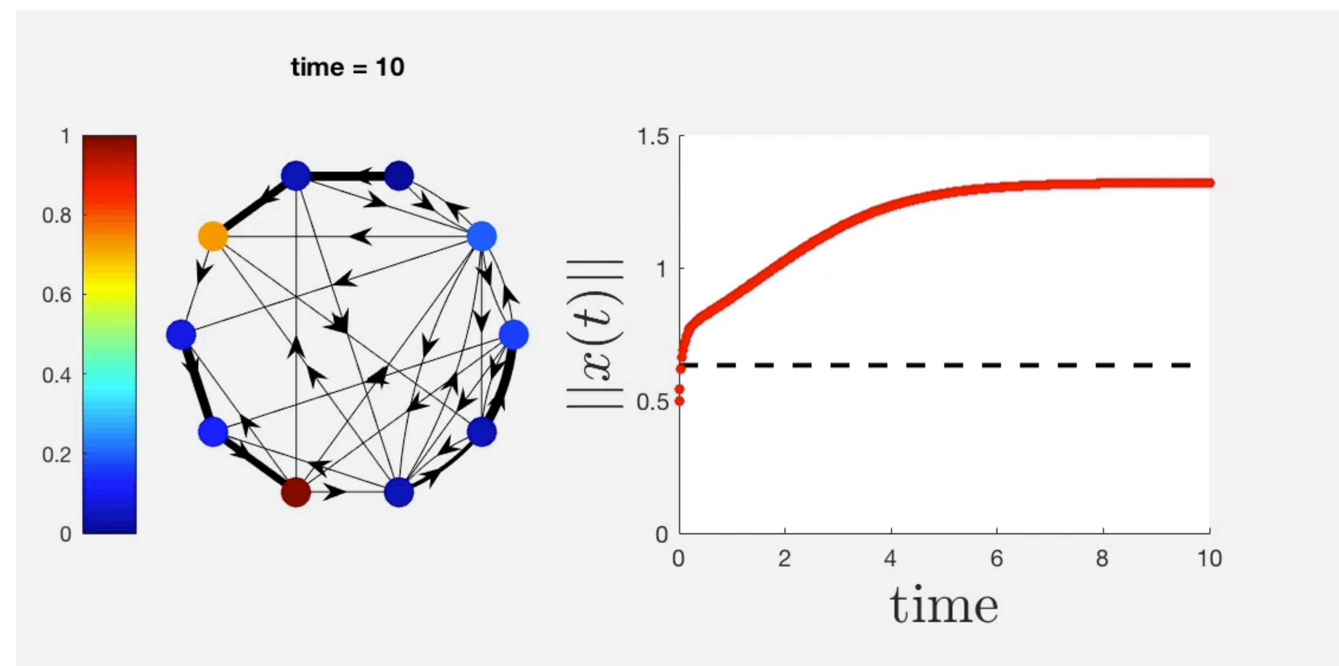
Patterns are everywhere





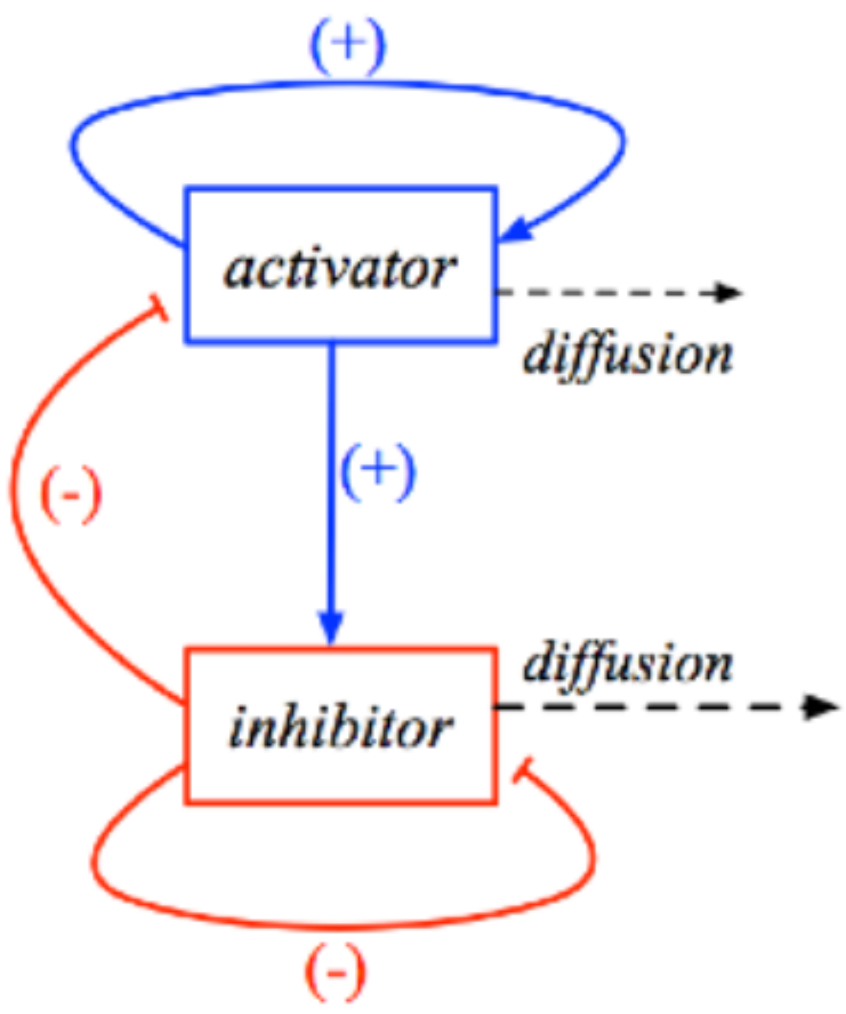
Internet

Twitter



Math models

Turing mechanism: diffusion driven instability



Assume the network to be symmetric

$u_i(t)$ Amount of activator in node i at time t

$v_i(t)$ Amount of inhibitor in node i at time t

$$\begin{cases} \dot{u}_i &= f(u_i, v_i) + D_u \sum_j L_{ij} u_j \\ \dot{v}_i &= g(u_i, v_i) + D_v \sum_j L_{ij} v_j \end{cases}$$

A.M.Turing, The chemical basis of morphogenesis, Phil. Trans. R Soc London B, 237, (1952), pp.37

Nakao H. and Mikhailov A. S., Turing patterns in network-organized activator–inhibitor systems, Nature Physics, 6, pp. 544 (2010)

Turing mechanism: diffusion driven instability

1) The spatially homogeneous solution:

$$(u_i, v_i) = (\hat{u}, \hat{v}) \quad \forall i = 1, \dots, n$$

is stable without diffusion: $D_u = D_v = 0$

2) The previous solution turns out to be unstable once the diffusion is in action

$$D_u > 0 \text{ and } D_v > 0$$

Small inhomogeneous initial perturbations are macroscopically amplified giving rise to patterns (spatially non homogenous solutions)

Turing mechanism:
elegant and simple, but unable to describe patterns onset in real scenarios.

- At least two diffusing species are needed;
- Activator and inhibitor are both necessary;
- The inhibitor must diffuse much faster than the activator

$$D_v \gg D_u$$

Turing mechanism:

elegant and simple, but unable to describe patterns onset in real scenarios.

Partial improvement: consider directed networks

nature COMMUNICATIONS

ARTICLE

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DOI: 10.1038/ncomms5517

The theory of pattern formation on directed networks

Malbor Asllani[1,2], Joseph D. Challenger[2], Francesco Saverio Pavone[2,3,4], Leonardo Sacconi[3,4] & Duccio Fanelli[2]

Still related to spectral properties …

Turing mechanism:

elegant and simple, but unable to describe patterns onset in real scenarios.

Go one step further and consider non-normal networks!

PHYSICAL REVIEW E **97**, 042302 (2018)

Topological resilience in non-normal networked systems

Malbor Asllani* and Timoteo Carletti

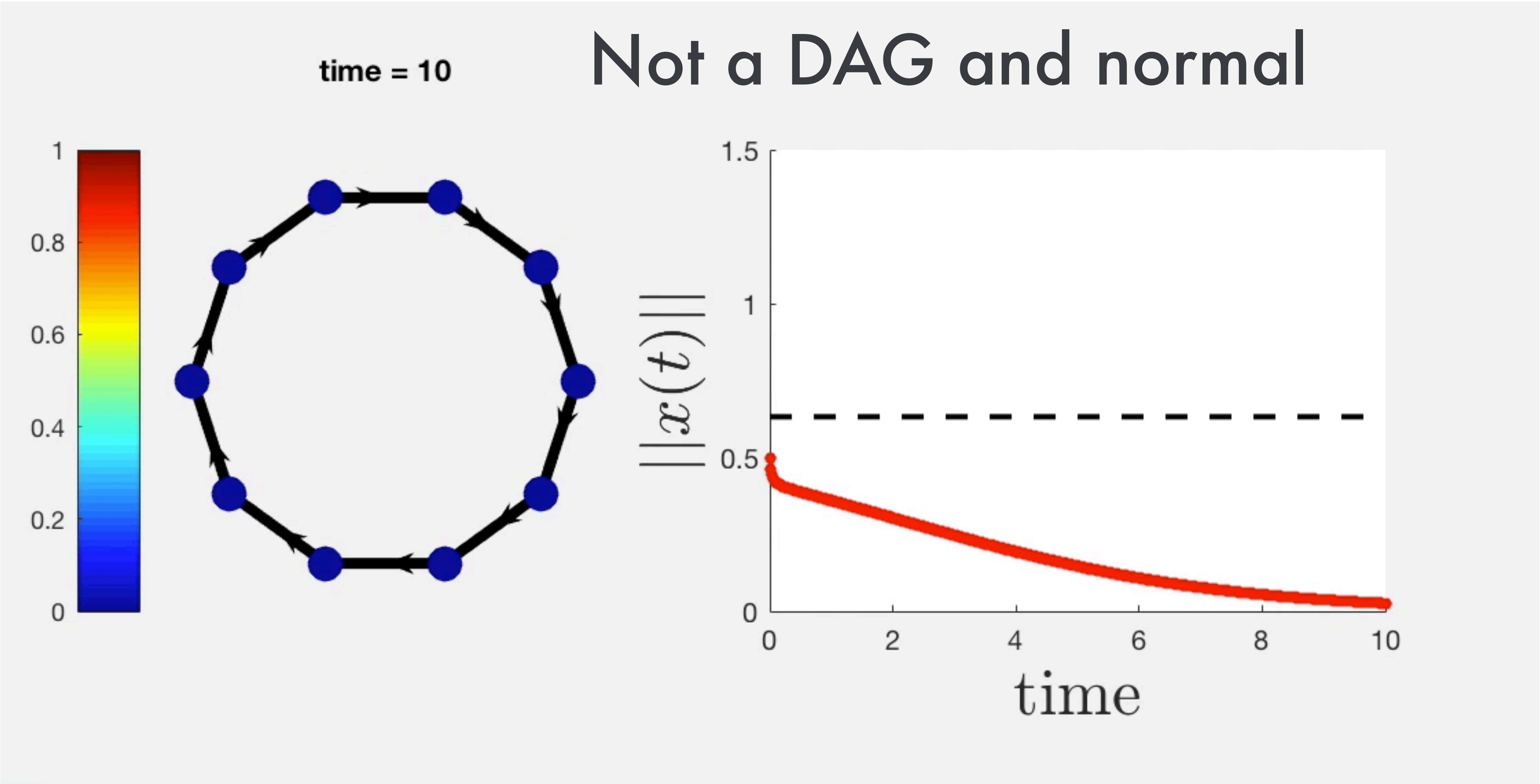
Department of Mathematics and naXys, Namur Institute for Complex Systems,
University of Namur, rempart de la Vierge 8, B 5000 Namur, Belgium

Definition (mathematical). A network is non-normal, if its adjacency matrix A_{ij} does satisfy $AA^T \neq A^T A$.

Definition (operational). It must contain a large Direct Acyclic Graph (DAG).

$$||x(t)||^2 = \sum_{i=1}^{M} [x_i(t)]^2$$

Dynamical behaviour:



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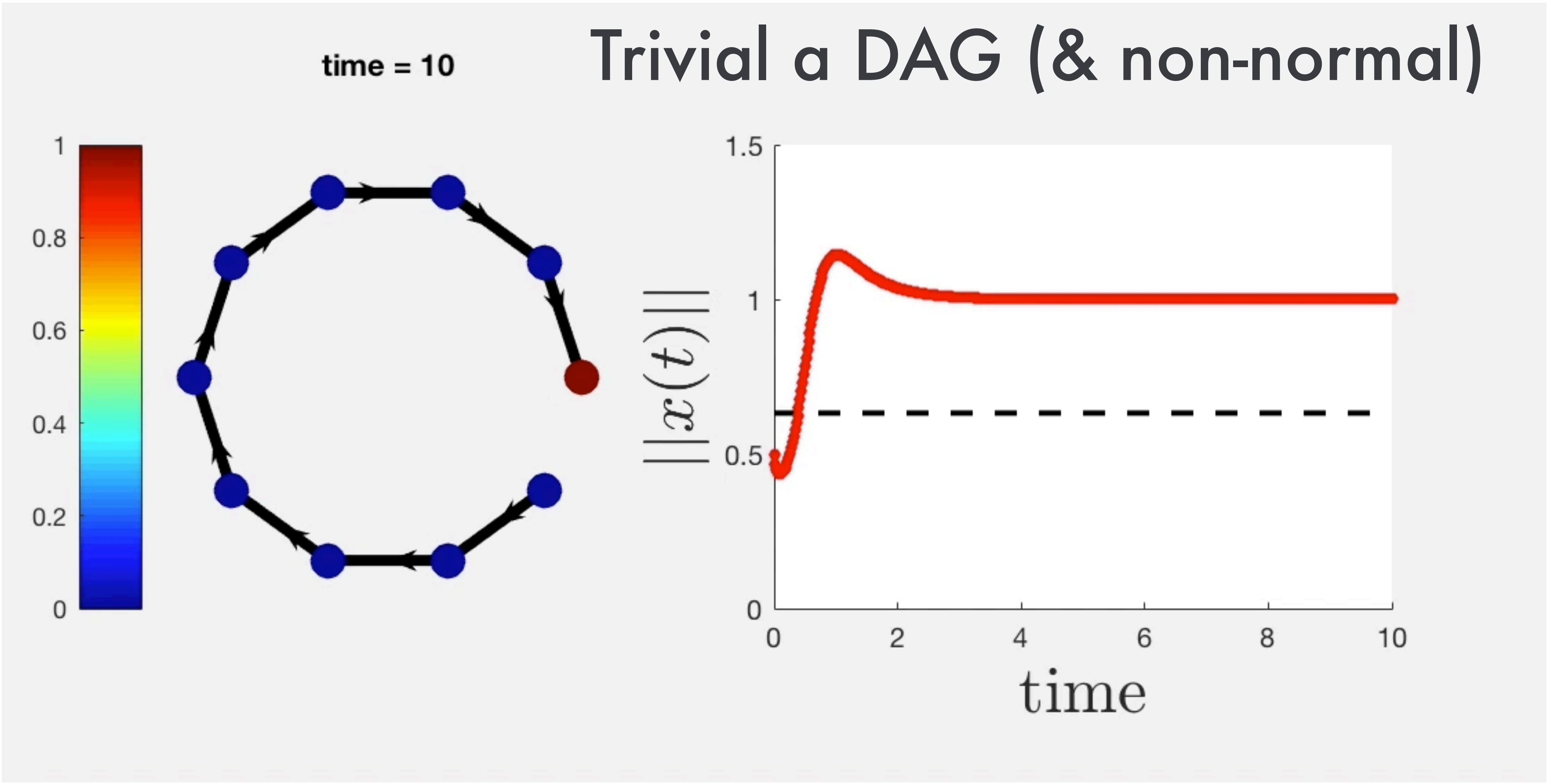
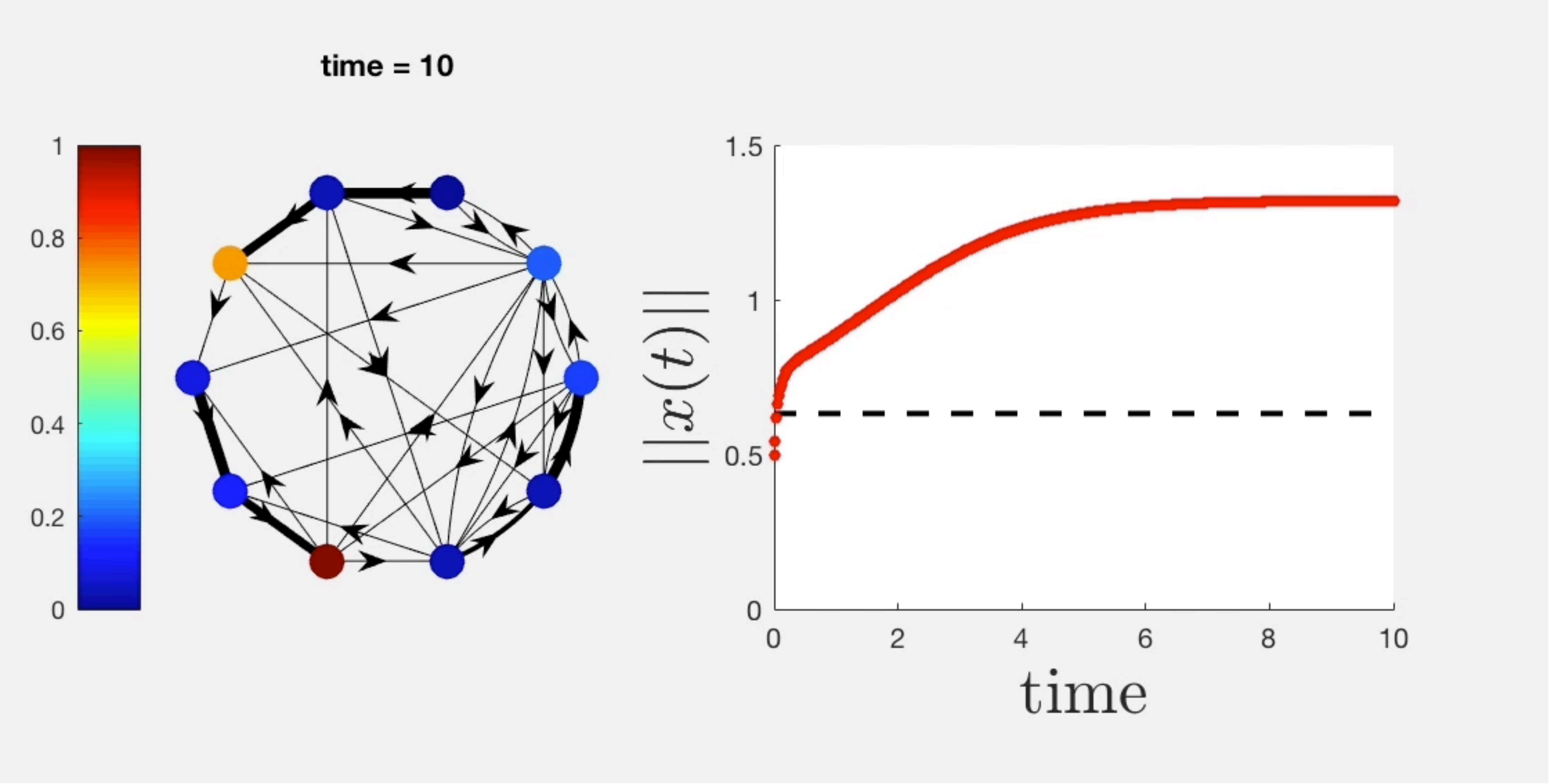
Dynamical behaviour:



Non-normal network

Dynamical behaviour:

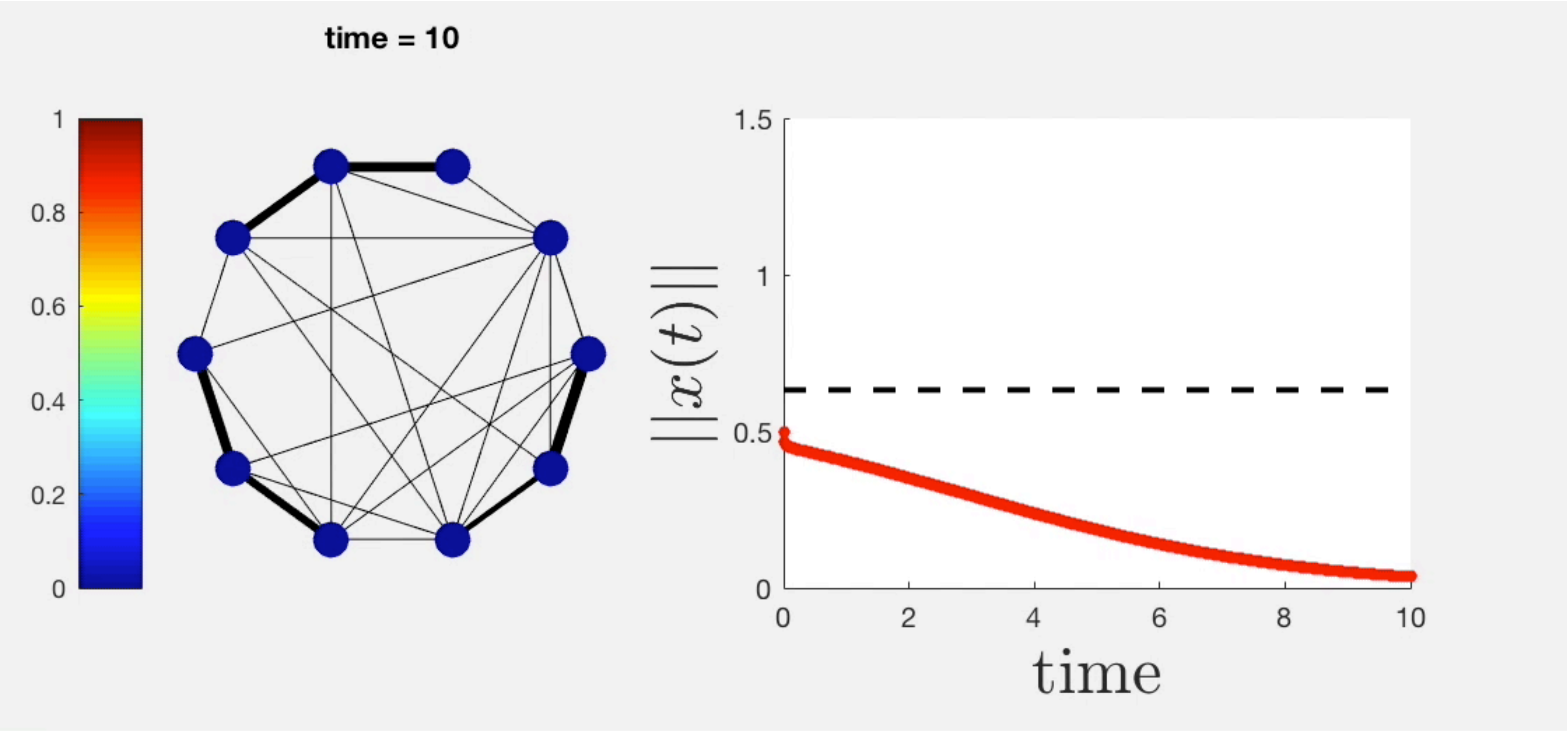
$$||x(t)||^2 = \sum_{i=1}^{M} [x_i(t)]^2$$



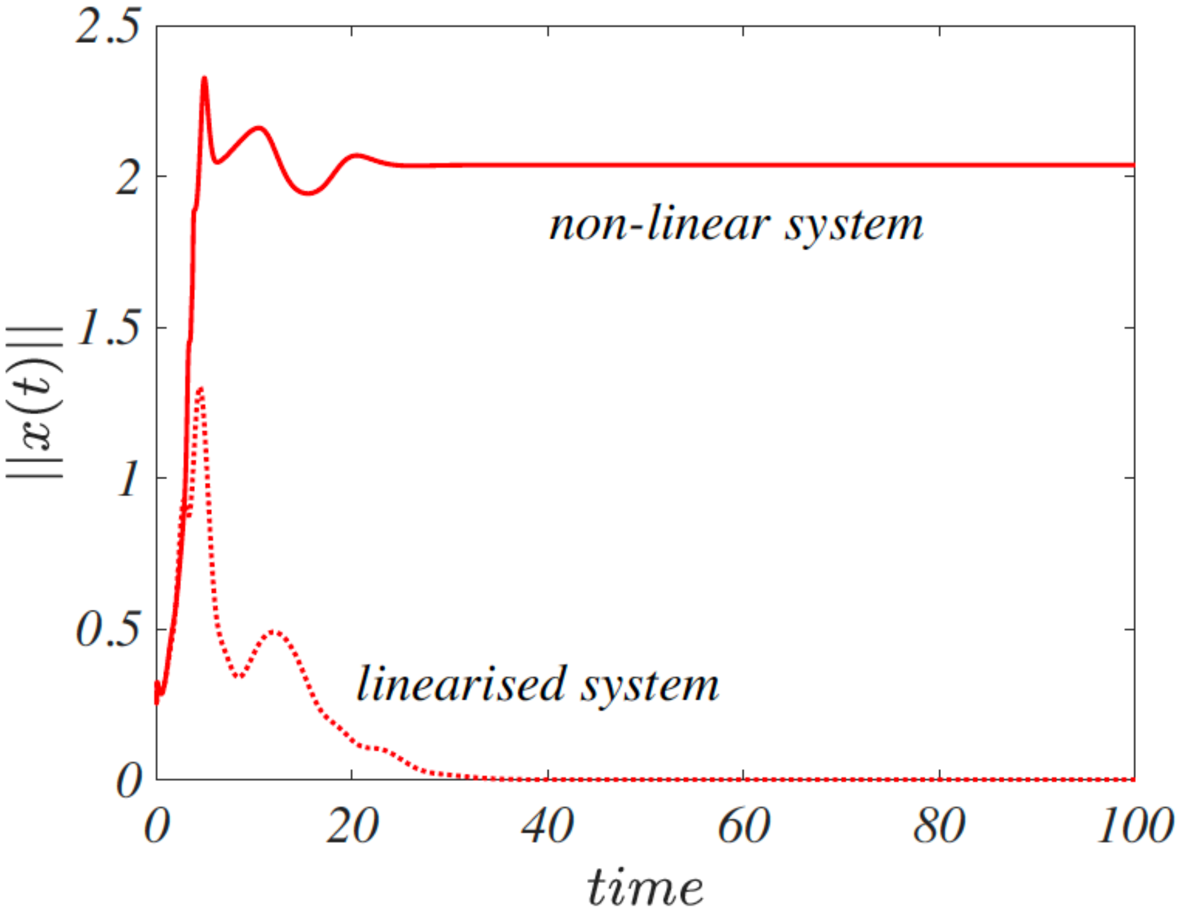
Normal network (symmetric version of the previous one)

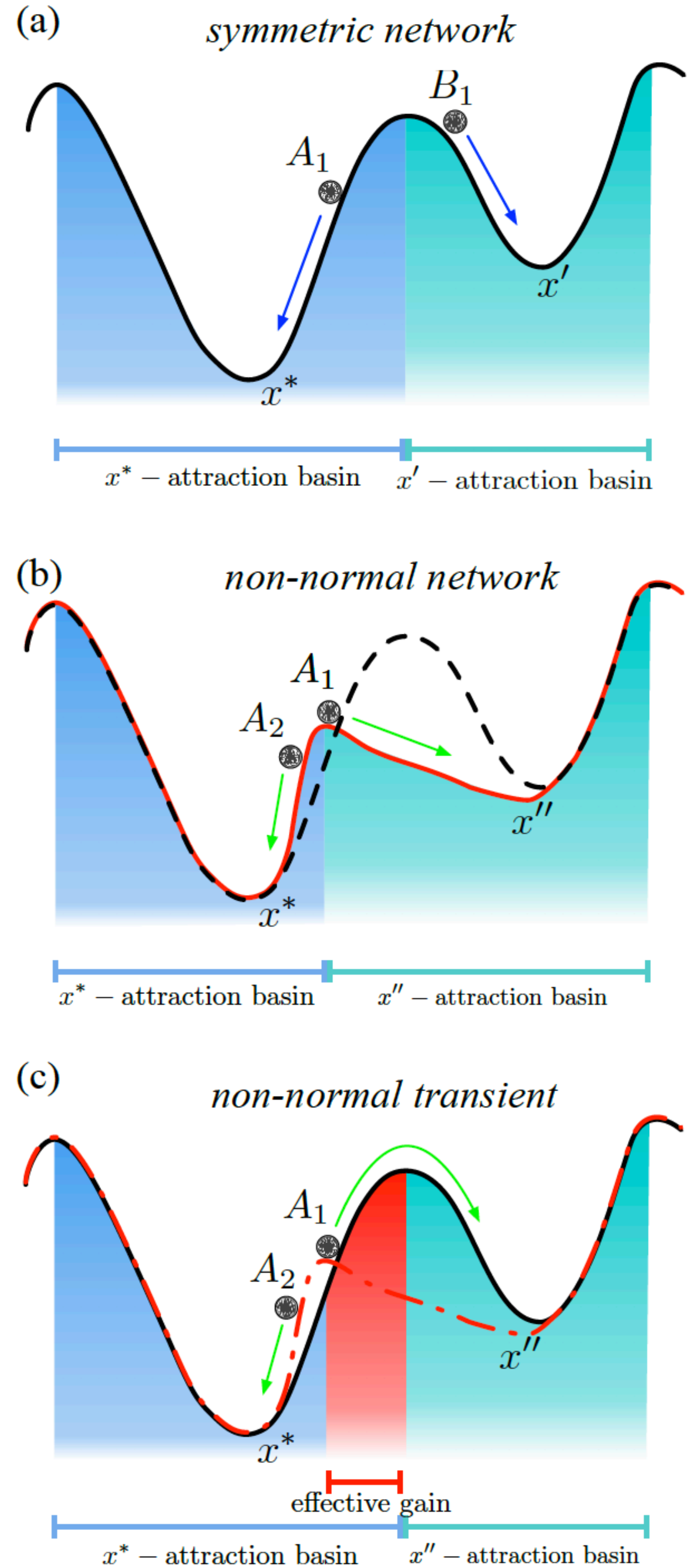
$$||x(t)||^2 = \sum_{i=1}^{M} [x_i(t)]^2$$

Dynamical behaviour:



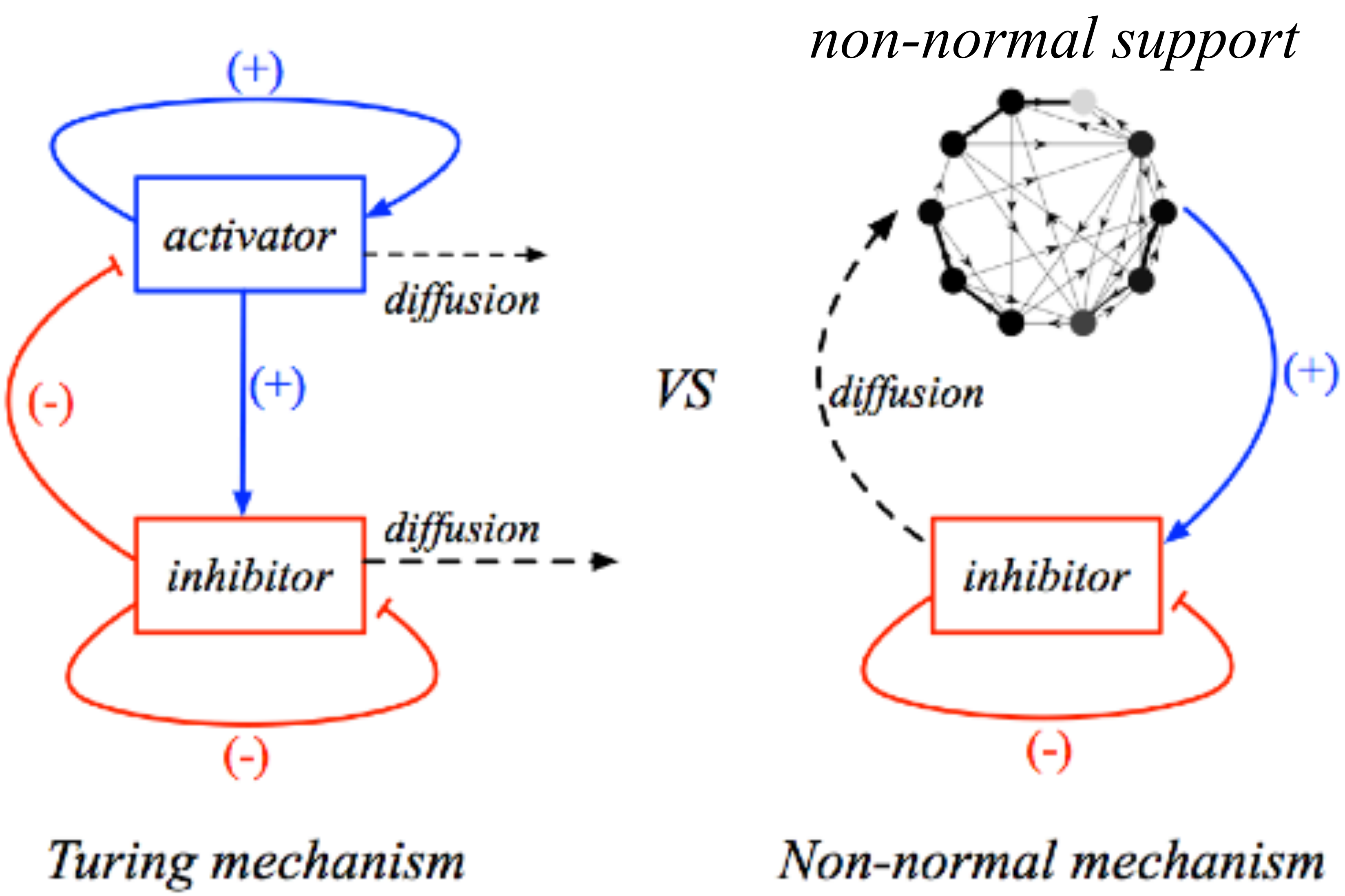
Network non-normality reduces the stability of the homogeneous equilibrium.





Because of a strong transient amplification

Beyond the Turing mechanism



- The homogeneous equilibrium remains stable;
- One species is enough;
- No need for activator/inhibitor;
- No need for separate diffusion scales: $D_u \sim D_v$

Non-normal networks are "the rule"

Network name	nodes	links	ω	$\omega - \alpha$	α_ϵ	Δ	$\hat{d}_F$
Foodwebs							
Cypress wetlands South Florida (wet)	128	2016	296.71	132.11	167.46	0.83	1.00
Cypress wetlands South Florida (dry)	128	2137	217.60	152.50	82.20	0.89	1.00
Little Rock Lake (Wisconsin, US)	183	2494	21.69	14.69	10.02	0.95	0.93
Biological							
Transcriptional regulation network (*E. coli*)	423	578	5.11	4.11	2.52	0.81	0.93
Metabolic network (*C. Elegans*)	453	4596	13.44	12.44	6.89	0.98	1.00
Pairwise proteins interaction (*Homo sapiens*)	2239	6452	15.79	13.02	4.01	0.99	0.99
Transport							
US airport 2010	1574	28236	$1.19\,10^7$	79.30	$1.19\,10^7$	0.01	1.00
Road transportation network (Rome)	3353	8870	$2.40\,10^4$	120.05	$2.39\,10^4$	0.08	0.28
Road transportation network (Chicago)	12982	39018	4.23	$4.29\,10^{-4}$	4.54	0.04	0.19
Communication							
e-mails network DNC	2029	39264	28.00	2.00	26.37	0.53	0.89
Enron email network (1999-2003)	87273	1148072	85.14	14.54	71.05	0.30	0.99(*)
e-mails network European institution	265214	420045	76.02	6.09	70.30	0.30	0.84(*)
Citation							
Citations to Milgram's 1967 paper (2002)	395	1988	10.48	10.48	4.49	1.00	1.00
Articles from Scientometrics (1978-2000)	3084	10416	10.32	8.32	5.28	0.98	1.00
Citation network DBLP	12591	49743	21.50	16.82	8.45	0.87	1.00
Social							
Hyper-network of 2004 US election blogs	1224	19025	45.37	10.95	34.95	0.72	0.98
Reply network of the news website Digg	30398	87627	15.92	6.56	10.18	0.61	0.97
Trust network from the website Epinions	75879	508837	123.00	16.47	106.96	0.13	0.80(*)

Conclusions

- Network non-normality can strongly modify the system behavior and create new patchy solutions (patterns).

- Many real networks exhibit strong non-normality and thus challenge our comprehension of their dynamics.

- Network non-normality can be easily measured.

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Thank you

- M. Asllani and T. Carletti, Topological resilience in non-normal networked systems, Physical Review E, **97**, (2018), 042302

- M. Asllani, R. Lambiotte and T. Carletti, Structure and dynamical behaviour of non-normal networks, arXiv: 1803.11542

- M. Asllani, R. Muolo, T. Carletti, D. Fanelli and Ph. Maini, Giant deterministic patterns beyond the Turing mechanism, preprint

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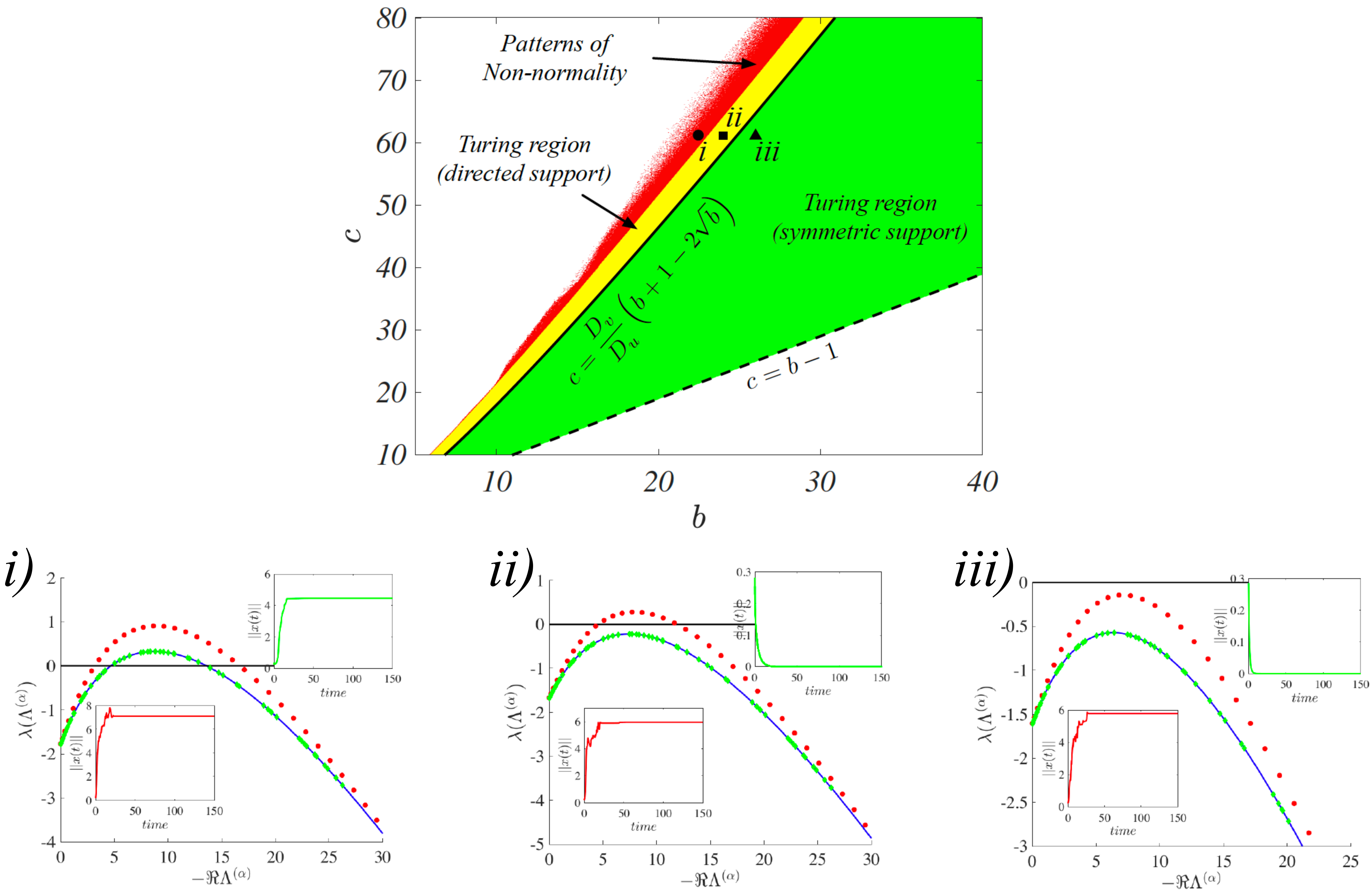
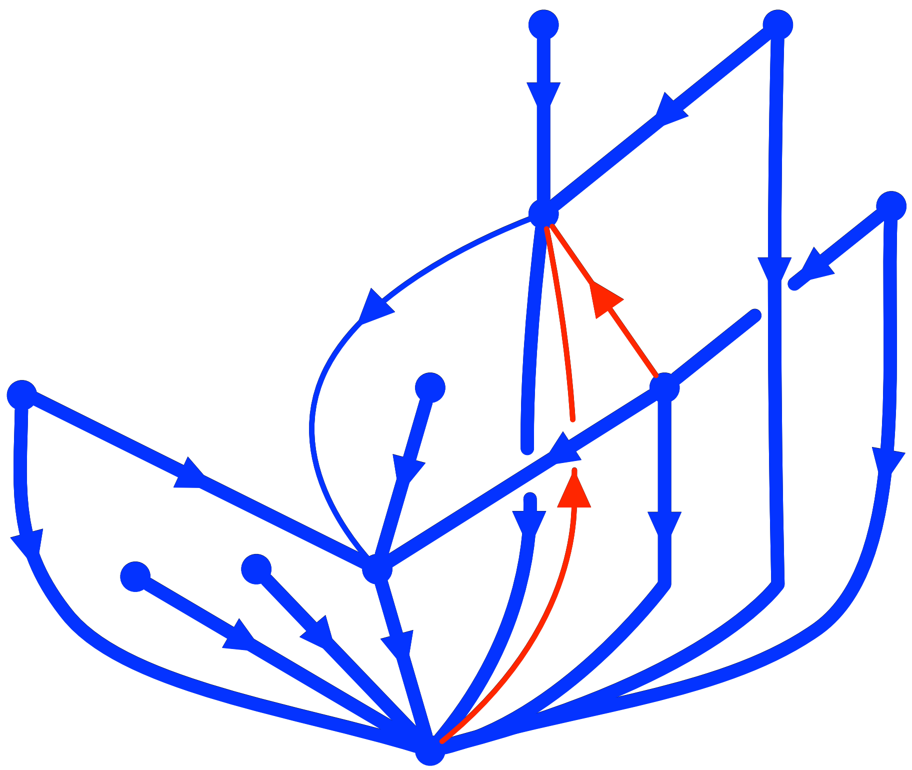
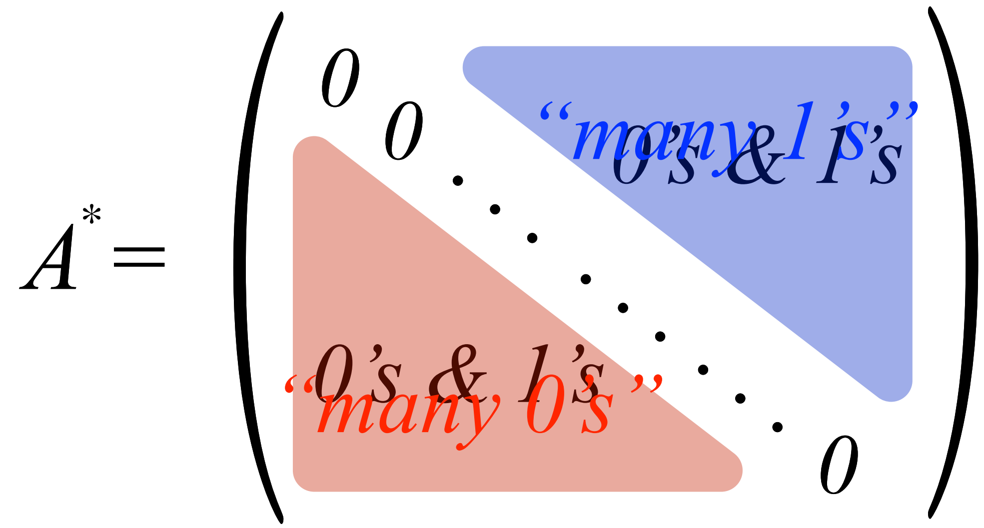


Patterns of Non-normality
Turing region (directed support)
Turing region (symmetric support)
$c = \frac{D_v}{D_u}\left(b+1-2\sqrt{b}\right)$
$c = b - 1$
i
ii
iii
c
b
i)
ii)
iii)
$\lambda(\Lambda^{(\alpha)})$
$-\Re\Lambda^{(\alpha)}$
$||x(t)||$
time

Non-normal networks: DAG backbone





A (simple) measure of non-normality

$$\Delta := |K^< - K^>|/K$$

$$K^< = \sum_{i<j} A^*_{ij} \qquad K^> = \sum_{i>j} A^*_{ij}$$

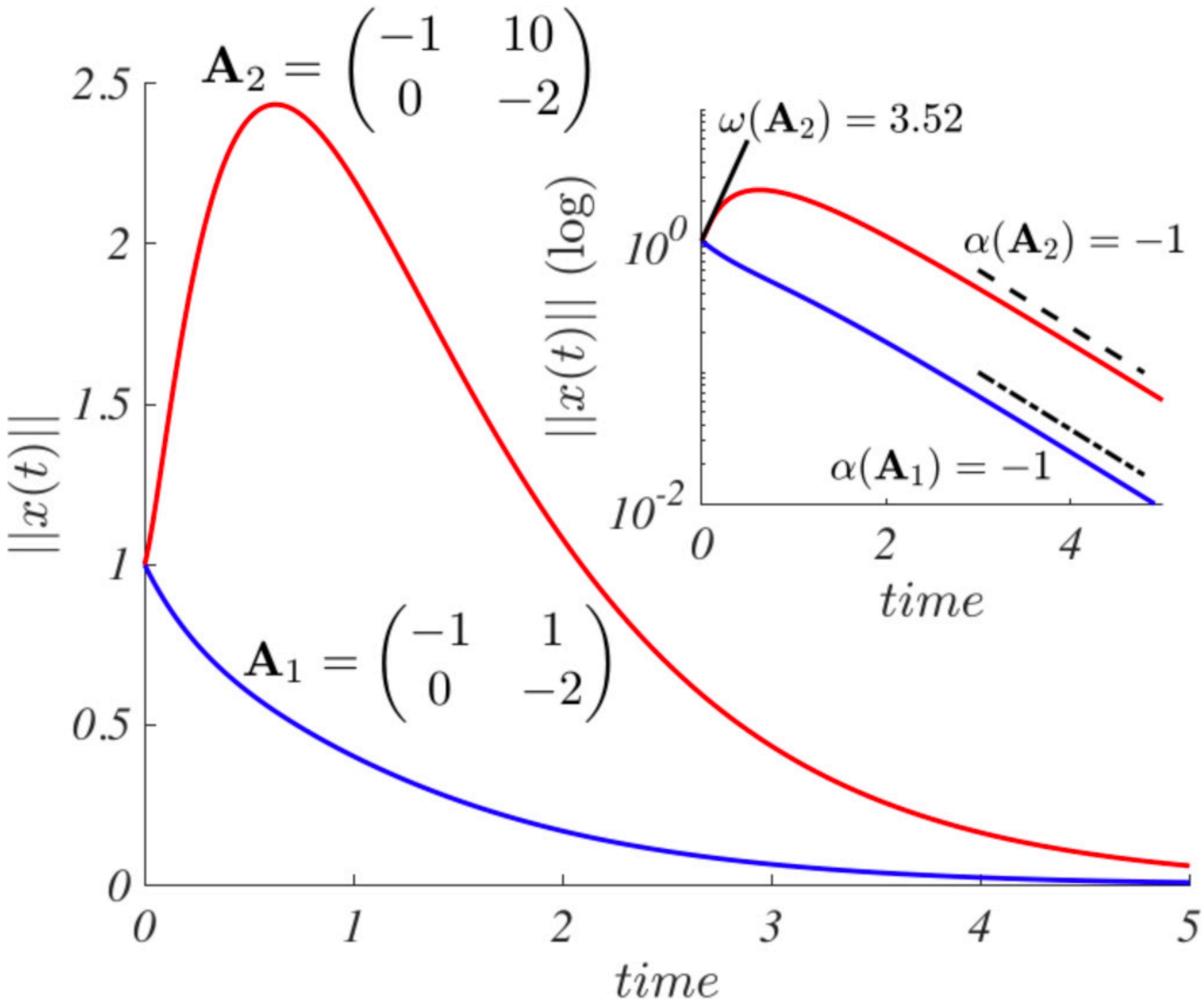
$$K = K^< + K^>$$

Adjacency matrix directed network

$$A_{ij} = \begin{cases} 1 & \text{if there exists } j \to i \\ 0 & \text{otherwise} \end{cases}$$

Linear asymptotically stable networked systems

$$\dot{\vec{x}} = A\vec{x}$$



Network name	nodes	links	ω	$\omega-\alpha$	α_ϵ	d_F	$\dot{d}_F$	Δ
Foodwebs								
Charca de Maspalomas, Gran Canaria [20, 21]	21	82	$5.40\,10^{5}$	$2.97\,10^{4}$	$5.10\,10^{5}$	$7.04\,10^{5}$	0.69	0.66
Crystal River (2) [21, 22]	21	100	$1.94\,10^{3}$	$8.86\,10^{2}$	$1.06\,10^{3}$	$3.26\,10^{3}$	0.90	0.72
Crystal River (1) [21, 22]	21	125	$2.52\,10^{3}$	$1.31\,10^{3}$	$1.21\,10^{3}$	$4.42\,10^{3}$	0.93	0.80
Narragansett Bay Model [21, 23]	32	220	6.83	0.84	0.40	9.95	0.79	0.79
Lower Chesapeake (Summer carbon flows) [21, 24]	34	178	$1.06\,10^{5}$	$3.34\,10^{4}$	$7.24\,10^{4}$	$1.92\,10^{5}$	0.88	0.76
Middle Chesapeake (Summer carbon flows) [21, 24]	34	209	$1.65\,10^{5}$	$3.47\,10^{4}$	$1.30\,10^{5}$	$2.57\,10^{5}$	0.77	0.75
Upper Chesapeake (Summer carbon flows) [21, 24]	34	215	$6.23\,10^{4}$	$7.76\,10^{3}$	$5.45\,10^{4}$	$1.05\,10^{5}$	0.81	0.68
Chesapeake (Summer carbon flows) [21, 25]	36	177	$4.11\,10^{5}$	$9.87\,10^{4}$	$3.13\,10^{5}$	$6.48\,10^{5}$	0.84	0.71
St Marks River (Florida) Estuary [21, 26]	51	356	139.50	28.45	111.51	201.34	0.80	0.70
Everglades Graminoid Marshes [21, 27]	66	916	$1.44\,10^{3}$	$1.04\,10^{3}$	407.94	$2.82\,10^{3}$	0.98	0.95
Foodweb cypress wetlands South Florida (wet season) [28, 29]	128	2016	296.71	132.11	167.46	266.65	1.00	0.83
Foodweb cypress wetlands South Florida (dry season) [28, 29]	128	2137	217.60	152.50	82.20	223.41	1.00	0.89
Foodweb Little Rock Lake (Wisconsin, US) [28, 30]	183	2494	21.69	14.69	10.02	12.56	0.93	0.95
Biological								
Secondary-structure elements adjacency for large proteins (PDB, serine protease inhibitor 1EAW) [8]	53	123	2.90	2.90	2.22	14.04	1.00	1.00
Secondary-structure elements adjacency for large proteins (PDB, immunoglobulin 1A4J) [8]	95	213	2.68	2.68	2.22	18.63	1.00	0.95
Secondary-structure elements adjacency for large proteins (PDB, oxidoreductase 1AOR) [8]	99	212	3.27	3.27	2.46	19.36	1.00	0.97
Local subnetwork of neurons within C. Elegans rostral ganglia [31]	130	764	7.99	2.46	6.26	10.50	0.92	0.76
Transcriptional regulation network (Escherichia coli) [32]	423	578	5.11	4.11	2.52	31.45	0.93	0.81
Metabolic network (Caenorhabditis Elegans) [33, 34]	453	4596	13.44	12.44	6.89	14.25	1.00	0.98
Transcriptional regulation networks (Saccharomyces cerevisiae) [35]	688	1079	4.99	3.66	2.19	26.17	0.99	0.99
Pairwise proteins interaction *Homo sapiens* (large) [33, 36]	2239	6452	15.79	13.02	4.01	46.98	0.99	0.99
Human binary protein-protein interactions [33, 37]	3133	6726	8.31	7.31	4.54	55.12	0.98	0.80
Transport								
Preferred Routes Database (NFDC, US FAA) [38, 39]	1226	2615	5.73	0.48	5.62	30.67	0.88	0.69
US airport 2010 [39, 40]	1574	28236	$1.19\,10^{7}$	79.30	$1.19\,10^{7}$	$4.76\,10^{6}$	1.00	0.01
Directed road transportation network 1999 (Rome, Italy) [39, 41]	3353	8870	$2.40\,10^{4}$	120.05	$2.39\,10^{4}$	$2.80\,10^{4}$	0.28	0.08
Directed road transportation network (Chicago region, USA) [39, 42, 43]	12982	39018	4.23	$4.29\,10^{-4}$	4.54	37.53	0.19	0.04
Communication								
Sent messages among students University of California, Irvine [44, 45]	1899	59835	36.38	2.12	34.60	26.11	0.87	0.33
e-mails network Democratic National Committee (2016) [44]	2029	39264	28.00	2.00	26.37	24.13	0.89	0.53
Wikipedia who-votes-on-whom network [46–48]	8297	103689	74.42	29.28	45.92	90.07	0.99	0.85
Enron email network (1999-2003) [44, 49]	87273	1148072	85.14	14.54	71.05	160.07(*)	0.99(*)	0.30
e-mails network European institution [44, 50]	265214	420045	76.02	6.09	70.30	434.63(*)	0.84(*)	0.30
Citation								
Citations to Milgram's 1967 Psychology Today paper (2002) [51]	395	1988	10.48	10.48	4.49	18.57	1.00	1.00
Citations to Small & Griffith and Descendants (2001) [51]	1059	4922	14.34	13.34	6.60	32.53	1.00	0.99
Articles from or citing Scientometrics, 1978-2000 (2002) [51]	3084	10416	10.32	8.32	5.28	55.49	1.00	0.98
Articles citing and by AH Zewail, 1970-2002 (2002) [51]	6752	54253	24.57	21.84	11.63	82.13	1.00	0.90
Citation network DBLP [52, 53]	12591	49743	21.50	16.82	8.45	112.06	1.00	0.87
Cora citation network [52]	23166	91500	16.11	7.12	9.91	150.96	0.99	0.74
Social								
College students in a course about leadership [8]	32	96	3.74	0.62	3.56	4.90	0.87	0.63
Inmates in prison [8]	67	182	3.75	0.41	3.76	6.86	0.84	0.55
Highschool Illinois [54, 55]	70	366	10.35	0.76	10.00	11.32	0.94	0.46
Friendship network Australian National University campus [55, 56]	217	2672	53.98	3.10	51.24	45.24	0.94	0.34
Network of innovation spread among physicians (1966) [55, 57]	241	1098	6.45	0.72	6.11	14.02	0.90	0.63
Hyperlinks network of 2004 US election blogs [58, 59]	1224	19025	45.37	10.95	34.95	34.32	0.98	0.72
Reply network of the social news website Digg [44, 60]	30398	87627	15.92	6.56	10.18	168.28	0.97	0.61
Trust network from the online social network Epinions [61, 62]	75879	508837	123.00	16.47	106.96	221.01(*)	0.86(*)	0.55