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Abstract

This paper analyzes the effects of a varying patent strength on the incentives to undertake cost - saving innovations in a product - variety model with hierarchical preferences. If the majority of the agents does not derive profit income from patents, a reduction in patent strength can lead to an increase in overall innovation. The reason for this is that weaker patents transfer some of the cost savings to consumers and allow them to purchase a larger variety of goods which increases the market size and therefore the innovation incentives in sectors that produce more luxurious goods. Agents that do not derive income from patents prefer weaker patents and their preferred extent of patent protection can either increase or decrease in the size of the market.

1 Introduction

This paper analyzes the effects of varying patent strength on the incentives to undertake cost - saving innovations in a product variety model with hierarchical preferences. If labour and profit incomes are equally distributed, full patent enforcement is optimal and maximizes the variety of goods that are consumed. New effects however arise if the majority of consumers do not derive profit income from

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patents: when patent protection is strong, prices are high and these consumers can only afford to buy a limited amount of goods. When patent protection is lower, cost saving innovations lead to lower prices, which allows them to purchase a larger variety of goods, including more luxurious goods which they could not afford to buy before. This increase in demand for more luxurious goods increases the incentives to undertake cost - saving innovations in these sectors. Given that R&D cost functions (that are symmetric across sectors) are sufficiently convex and/or the size of the market and the realized cost reductions are sufficiently large, I show that overall innovation is maximal for an intermediate strength of patent protection and that the innovation maximizing patent strength decreases in the size of the market and the size of the cost reduction. Also total R&D spending can increase if patent protection is reduced below the maximal level. Depending on the shape of the R&D cost functions, the patent strength preferred by consumers that do not derive income from patents can decrease or increase in the size of the market. In the first (second) case, these consumers prefer a stronger (weaker) protection of patents in sectors supplying more luxurious goods.

R&D firms operating in more luxurious sectors tend to favour weaker overall patent protection as this increases the demand for their goods.

If lump sum transfers between income groups are feasible, full patent protection in combination with transfers is optimal.

2 Related literature

There are several models with hierarchical preferences replicating the empirical observation that richer agents consume a larger variety of goods than poorer ones (who spend most of their income on basic need goods like food). However, most of these papers analyze the effect of inequality on innovation and growth taking the extent of appropriability (e.g. patent policies) as given. Matsuyama (2002) studies "the rise of mass consumption societies" and assumes that productivity in a given sector increases due to learning by doing the speed of which is a positive function of market size. There are therefore no profit maximizing innovators in his model and intellectual property rights play no role. Murphy, Shleifer and Vishny (1989) analyze how an increase in agricultural productivity affects demand

and the incentives for mass production of manufactured goods depending on the distribution of income. While their model is static, Zweimüller (2000) looks at a dynamic model in which innovators can undertake cost - saving innovations and analyzes how inequality affects growth by changing the time path of demand faced by an innovator. In both models, it is assumed that marginal production costs of (nonagricultural) goods can be reduced if a given fixed cost (of innovation) is incurred and that a competitive fringe forces innovators to engage in limit pricing. In the model analyzed here, a different R&D technology is considered and firms can select their preferred level of R&D investment in a given sector and increase the probability with which an innovation occurs by investing more. Moreover, a new parameter measuring patent strength is introduced and affects the degree to which innovators can appropriate the benefits resulting from their innovations.

Compared to my other paper (Kiedaisch (2009)), there are several differences: while the latter analyzes the incentives to invent new goods, this paper analyzes the incentives to undertake cost - saving innovations for goods that are already consumed. While the analysis in the first chapter is more general in the sense that it is a full - fledged dynamic model allowing for substitutability between goods and endogenous markups, the focus here lies on the allocation of R&D activity across sectors, assuming that only one firm per sector can undertake R&D and that R&D productivity is declining in each sector. In order to keep the analysis simple, a two period model is used and it is assumed that preferences are strictly hierarchical and that there is limit pricing.

More somewhat related references are given in Kiedaisch (2009).

3 The model setup

There is a continuum of differentiated goods indexed by $j \in [0;\infty)$. Of each good, a given consumer i can either consume one or zero units (consuming more that one unit does not increase utility), so that $c_i(j) \in \{0;1\}$. Preferences are hierarchical and goods with a low index j represent basic need goods while goods become more luxurious if j increases. For simplicity it is assumed that utility is strictly hierarchical, so that consuming the good with index k only increases utility if all the goods with an index $j < k$ are consumed as well. Defining the "total consumption" N_i of agent i as the index of the highest - ranked good for which the agent consumes one unit of all goods with rank $0 \leq j \leq N_i$, his utility is given by[1]

(1) $U_i = U(N_i)$ with $U'(N) > 0$ and $U''(N) < 0$.

There is a continuum of agents of mass L in the economy (the size of the population is given by L) and all have the same preferences as described above. The total labour endowment in the economy is given by L.

There exist "old" technologies that allow to produce one unit of any of the differentiated goods by using one unit of labour as an input. By undertaking R&D in a given sector, a "new" technology can be invented that allows to produce one unit of the good by using $c < 1$ units of labour. The R&D technology is symmetric across sectors (independent of j) and specified in the following way: in order to obtain a cost - reducing innovation in a given sector j with probability $0 \leq x \leq 1$, the R&D costs $R(x) = \frac{1}{k}(x+a)^k + F$ in terms of labour input have to be incurred[2]. The parameters a and F can be positive or negative and the fixed costs of undertaking R&D in a given sector are denoted by $\epsilon \equiv F + \frac{1}{k}a^k \geq 0$. It is assumed that $k > 1$ so that marginal R&D costs are increasing. If firms in different sectors undertake R&D at the same time, their success probabilities are assumed to be uncorrelated.

There are two periods. In the first ($t = 1$), R&D investments are undertaken and in the second ($t = 2$), differentiated goods are produced according to the available technologies and consumed.

The wage for one unit of labour is taken as the numeraire. The old technologies are in the public domain and there is perfect competition in the sectors in which an old technology is used so that the goods in these sectors are sold at the marginal cost of one (per unit). A firm that has successfully innovated and discovered the new technology for a given sector obtains a patent on it (assuming for the moment that no other firm has made the same innovation). Patents are however

[1] "Total consumption" is given by: $\int_{j=0}^{N_i} c_i(j)dj = \int_{j=0}^{N_i} 1dj = N_i$

[2] The analysis would be similar if instead it was assumed that a deterministic cost reduction equal to $x(1-c)$ could be obtained by incurring the R&D costs $R(x) = \frac{1}{k}(x+a)^k + F$.

only enforced with probability Δ. In the case of enforcement, a patent allows the inventor to exclusively use the new technology in the given sector, while the technology falls in the public domain if patents are not enforced. In the latter case, there is perfect competition and the corresponding good is supplied at the marginal cost of c.

Each agent has a wage income equal to the size of its labour endowment which is supplied inelastically. Firms undertaking R&D make profits or losses and these are distributed to agents that hold shares in R&D firms. As utility is concave in total consumption, individuals want to insure against income shocks. As it turns out that there is always a continuum of firms that undertake the same amount of R&D in equilibrium (see below) and as success probabilities are uncorrelated, agents can perfectly diversify their R&D stock portfolios and therefore fully insure against income shocks. An agent who owns a certain fraction of the total shares in R&D firms therefore gets the same fraction of the total expected profits (that are not stochastic as uncertainty chancels out in the aggregate).

There are K income groups indexed by $n \in \{1; K\}$, each containing a fraction β_n of the total population. Total per capita incomes y_n differ across income groups and increase in n ($y_1 < y_2 < ... < y_K$). The labour endowment of an individual in group n (which is equal to its wage income) is given by l_n and the profit income by π_n, so that $y_n = l_n + \pi_n$.

4 Equilibrium

As agents differ with respect to their total income and all consume along the hierarchy, richer agents have a larger total consumption in equilibrium ($N_1 < N_2 < ... < N_K$). The market size M^j for a given sector j that indicates the fraction of agents that consume one unit of good j is declining (step - wise) in j as only richer individuals consume more luxurious goods (that have a higher index j). The market size of a sector the goods of which are bought by all individuals belonging to an income group $h \geq n$ is denoted by M_n (so that $M_1 = L > M_2 > ... > M_K = \beta_K L$). The market size for sector j is therefore given by M_n if $N_{n-1} < j \leq N_n$.

If a firm holds a patent for the new technology in a sector (with $j \leq N_K$

so that $M^j > 0$) and if this patent is enforced, it can maximally charge a price equal to one as otherwise firms producing with the old technology (which allows to produce at the marginal cost of one) would enter the market. But is it optimal for the firm to charge the maximal possible price? Given consumers of at least one income group still have some income left after buying one of each goods with an index lower than j, an unconstrained monopolist in sector j could extract all this unspent income by charging a very high price[3] as consumers cannot derive utility from consuming goods with index $q > j$ if they do not consume good j. Moreover, increasing the price for the own good does not reduce demand as consumers are already satiated with goods that are less luxurious than good j, so that they do not want to consume more of them even if good j becomes more expensive. Therefore, unconstrained monopolists would like to charge an infinite price, and monopolists constrained by the threat of entry the maximal possible price of one.

Summing up, the price of a good is given by one in sectors in which only the old technology is available and in sectors in which the new technology is available and patents are enforced, and it is given by c in the sectors in which the new technology is available but in which the patents are not enforced.

In the following, it is assumed that there is only one firm in each sector that is capable of doing R&D. An R&D firm in sector j chooses its R&D effort x_j by maximizing expected profits that are given by

$\Pi_j = x_j \Delta(1-c)M^j - \frac{1}{k}(x_j + a)^k - F$

Looking at the case where the fixed costs F and the parameter a are low enough[4] so that the firm does a positive amount of R&D, the profit maximizing research intensity x_j^* is (if it is interior) given by:

(2) $x_j^* = (\Delta(1-c)M^j)^{\frac{1}{k-1}} - a \leq 1$

Firms therefore invest more in R&D and obtain an innovation with a higher probability x if patents are enforced with a higher probability Δ, if the (correctly

[3] As the measure of a single good in total consumption is zero, the profit maximizing price is actually infinite.

[4] More exactly, the condition

$F + a\Delta(1-c)M^j < \left(\Delta(1-c)M^j\right)^{\frac{k}{k-1}} \left(\frac{k-1}{k}\right)$ which ensures that equilibrium profits are positive, needs to hold.

anticipated) market size M^j is larger, if the achieved cost reduction is larger (if c is smaller) and if R&D costs are less convex (that means if k is lower).

5 The case of an equal income distribution

In this section it is assumed that there is only one income group and that each agent has the same labour endowment $l_i = 1$ and holds the same amount of shares in R&D firms so that profits are equally distributed across agents ($\pi_i = \pi$). In order to obtain interior solutions, it is assumed that F and a are relatively small. Total consumption of any of the agents is given by N so that the market size in sectors $j \leq N$ is equal to the size of the population ($M^j = L$). In these sectors, the amount of R&D that firms undertake is therefore given by

(3) $x_j^* = (\Delta(1-c)L)^{\frac{1}{k-1}} - a.$

The total consumption N that results in equilibrium can now be derived by looking at the resource constraint of the economy which equates the total labour supply to the total demand for labour coming from the R&D - and the production sector:

(4) $L = N\frac{1}{k}(x+a)^k + NF + LxcN + L(1-x)N$

The left hand side of this equation represents the total supply of labour and the terms on the right hand side (from left to right) the demand for R&D labour and the demand for labour in the sectors producing with the new and the old technology. As an innovation occurs with probability x and firms in all sectors $0 \leq j \leq N$ undertake R&D, a cost reducing innovation occurs in $N\,x$ sectors while $N(1-x)$ sectors keep producing with the old technology. Solving the resource constraint for N gives:

(5) $N(x) = \frac{L}{L - Lx(1-c) + \frac{1}{k}(x+a)^k + F}$

Total per capita consumption N and therefore utility is maximal for the R&D effort $x^o = (L\,(1-c))^{\frac{1}{k-1}} - a$ and by comparison with equation (3) we see that this optimal effort can be obtained in equilibrium if $\Delta = 1$, that means if patents are fully enforced. Full patent protection is optimal here as it allows an innovator to appropriate the entire cost savings $L\,(1-c)$ obtained through an innovations, without leading to monopoly distortions (due to the assumption of $0-1$ - consumption). Moreover, there are no externalities and research is efficient as there is only one firm per industry that can undertake R&D.

In the case where there is free access into the R&D sectors, less than full patent protection is however optimal (see Appendix).

6 The case of two income groups

In the following, the case is considered in which all agents have the same labour endowment $l = 1$ and in which a small minority of rich people with population share $1 - \beta$ owns all shares in R&D firms and therefore gets all profit incomes. It is assumed that the population share of the rich and therefore the size of the market in sectors the goods of which are exclusively consumed by the rich is small enough (β large enough) relative to the fixed costs F so that firms do not find it profitable to undertake research in these sectors. This is the case if $F + a\Delta(1-c)(L\,(1-\beta)) > (\Delta(1-c)L(1-\beta))^{\frac{k}{k-1}}(\frac{k-1}{k})$ (**Condition 1**). However, it is assumed that $F + a\Delta(1-c)L < (\Delta(1-c)L)^{\frac{k}{k-1}}\,(\frac{k-1}{k})$ (**Condition 2**), implying that it is profitable to undertake R&D in "mass consumption" sectors in which the size of the market is equal to L. As the income of a poor agent is equal to one, her total consumption depends on the prices she has to pay for the different goods. As before, a fraction $x(1-\Delta)$ of the goods that are consumed by the poor (there is a measure N_P of these goods) is produced with the new technology and supplied at price c because patents are not protected, while the remaining fraction $1 - x(1-\Delta)$ is sold at the price of one as these goods are either produced with the old technology or protected by patents in case they are produced with the new technology. Equating income and expenditures, the budget constraint of a poor consumer can therefore be written as[5]:

(6) $1 = x(1-\Delta)cN_p + (1 - x(1-\Delta))N_P$ from which one obtains

[5] The analysis would be the same if patent policy was parametrized as a price cap on firms having invented the new technology and not by assuming a random enforcement of patents:

If the price cap is set equal to $\mu < 1$, expected profits are given by $\Pi = x(\mu - c)M - \frac{1}{k}\,(x+a)^k - F$ and are equal to those in the case of a varying probability of enforcement if $\mu = \Delta + c(1-\Delta)$.

The budget constraint of a poor consumer is given by $1 = N_P(1-x) + N_P x\mu$ and - given that $\mu = \Delta + c(1-\Delta)$ - it defines the same N_P for a given x as the budget constraint from equation (6).

All results derived about the effects of a change in the probability of patent protection (Δ) on N_p and x would therefore also hold if patent strength was modeled as a price cap μ.

(7) $N_P = \frac{1}{1-x(1-\Delta)(1-c)}$

Inserting the equilibrium value of R&D $x_j^* = (\Delta(1-c)L)^{\frac{1}{k-1}} - a$, which is undertaken in mass consumption sectors (where $M^j = L$), one sees that total per capita consumption of the poor (N_P) is minimal if patents are fully enforced ($\Delta = 1$) and also if patents are not enforced at all ($\Delta = 0$), in which case $x = 0$. In both cases we get $N_P = 1$. Total consumption of the poor is therefore maximal for an intermediate strength of patent enforcement.

Proposition 1 *Given **Condition 2** is satisfied and a is not too negative, the patent strength Δ^* which locally maximizes the total per capita consumption N_P of the poor decreases (increases) in the size of the market L if $a > 0$ ($a < 0$) and is independent of L if $a = 0$.*

Proof. Deriving N_P as given by equation (7) with respect to Δ gives the following first order condition:

(8) $a + (\Delta(1-c)L)^{\frac{1}{k-1}} \left(\frac{1-k\Delta}{(k-1)\Delta}\right) = 0$

In the case where $a = 0$, the extent of patent protection that satisfies this first order condition is therefore given by $\Delta^* = \frac{1}{k}$. If $a > 0$, we get $\Delta^* > \frac{1}{k}$, and if $a < 0$, $\Delta^* < \frac{1}{k}$ results. The *sign* of the second order condition is given by: $sign\frac{\partial^2 N_P}{\partial \Delta^2} = sign\left\{(2-k(1+\Delta))(1-(1-\Delta)(1-c)x^*)\frac{((1-c)L)^{\frac{1}{k-1}}\Delta^{\frac{3-2k}{k-1}}}{(k-1)^2} + 2(1-c)\left(a + (\Delta(1-c)L)^{\frac{1}{k-1}}\left(\frac{1-k\Delta}{(k-1)\Delta}\right)\right)^2\right\}$. Inserting the first order condition (8) to replace the terms inside the last bracket, the *sign* is negative and there is a local maximum if $2 - k(1+\Delta^*) < 0$, that means if $\Delta^* > \frac{2-k}{k}$. This condition holds if a is positive (as then $\Delta^* > \frac{1}{k} > \frac{2-k}{k}$) or not too negative. By implicit differentiation of equation (8) we find that $sign\frac{d\Delta^*}{dM} = sign\frac{k\Delta^*-1}{2-k(1+\Delta)}$. Given that a is not too negative so that there is a local maximum, the denominator is negative so that $\frac{d\Delta^*}{dL} < 0$ results if $a > 0$ and $\frac{d\Delta^*}{dL} > 0$ if $a < 0$. In the case where $a > 0$ ($a < 0$) the extent of patent protection preferred by the poor therefore decreases (increases) in the size of the market, while it is independent of market size if $a = 0$.
■

If fixed costs F are relatively high so that firms in mass consumption sectors do not find it profitable to undertake R&D if patent strength is low, **Condition**

2 can be violated for the value of Δ that satisfies the first order condition (8). In this case, the preferred strength of patent protection is given by the minimal protection $\underline{\Delta}$ that allows R&D firms to break even ($\Delta^* = \underline{\Delta}$)[6]. As no one benefits if patents are only protected with a probability $\Delta < \underline{\Delta}$, even the poor want to sustain a minimal amount of protection in order to induce firms to undertake a positive amount of R&D.

A decrease in patent protection decreases profits for a given size of the market and decreasing Δ marginally below the maximal level cannot lead to an increase in total profits[7]. R&D firms that operate in sectors that satisfy basic needs and in which the size of the market is always given by one therefore prefer a maximal strength of patent protection ($\Delta = 1$). An R&D firm operating in a more luxurious sector in which there is no mass consumption if Δ is large however prefers to cut Δ if this induces a sufficient increase in total consumption of the poor to induce mass consumption in its sector. In fact, such a firm wants to reduce Δ only so much that its sector becomes the most luxurious mass consumption sector but not by more as a further decrease in the strength of patent protection does not increase demand further and only reduces profits. R&D firms in more luxurious sectors (with a higher index j) therefore prefer a lower extent of patent protection than firms in sectors satisfying more basic needs[8]. However, no firm ever prefers a lower extent of patent protection than the poor, as mass consumption cannot be increased by lowering Δ below the level preferred by the poor.

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[6] The threshold $\underline{\Delta}$ is defined in the following way:
$\Pi_j = x_j^* \underline{\Delta}(1-c) - \frac{1}{k}(x_j^* + a)^k - F = 0$
with $x_j^* = (\underline{\Delta}(1-c))^{\frac{1}{k-1}} - a$
Expected profits of firms in sectors where the market size is given by L are zero for this level of patent protection (and negative if $\Delta < \underline{\Delta}$).

[7] Deriving expected profits in sector j with respect to Δ, we obtain: $\frac{\partial \Pi_j(\Delta, M^j)}{\partial \Delta} = (1 - c)M^j x^*(\Delta, M^j) > 0$
Total profits in all mass consumption sectors (in which $M^j = 1$) are given by $\Pi_{tot} = \frac{\Pi(\Delta)}{1 - x^*(1-c)(1-\Delta)}$. Deriving with respect to Δ, we find that $sign \left.\frac{\partial \Pi_{tot}}{\partial \Delta}\right|_{\Delta=1} = sign \frac{1}{k} > 0$.

[8] If patent protection can be made conditional on the sectors, a firm in sector j wants full protection for the own sector but a lower protection in sectors $i < j$ in order to induce mass consumption in the own sector.

6.1 The effect of varying patent strength on overall R&D

While a decrease in patent protection (Δ) has the direct effect of reducing the R&D effort x_j^* for a given size of the market, it can (given that $\underline{\Delta} < \Delta^* < \Delta$) lead to an increase in the current consumption of the poor and therefore an increase in the total number of sectors in which there is mass consumption and in which it is profitable to undertake R&D. It is therefore not a priori clear how overall innovation activity and R&D spending depend on the extent of patent protection Δ. Let us define "total innovation" I as

(9) $I \equiv N_P(\Delta) \cdot x^*(\Delta) = \frac{x^*(\Delta)}{1-x^*(\Delta)\cdot(1-\Delta)(1-c)}$

Proposition 2 *Given $a = 0$, **Conditions 1** and **2** hold. If $\frac{1}{(1-c)L(k-1)^{1-\frac{1}{k}}} < 1$, there is an inverted - U relation between Δ and I and the strength of patent protection $\widetilde{\Delta}$ for which I is maximal decreases in k and L and increases in c. If $\frac{1}{(1-c)L(k-1)^{1-\frac{1}{k}}} > 1$, I increases in Δ and is maximal if $\Delta = 1$.*

Proof. Inserting $x^* = (\Delta(1-c)L)^{\frac{1}{k-1}}$ into (9) and deriving with respect to Δ, we find that $sign\frac{\partial I}{\partial \Delta} < 0$ (> 0) if $\Delta > \frac{1}{(1-c)L(k-1)^{1-\frac{1}{k}}} \equiv \widetilde{\Delta}$ ($\Delta < \widetilde{\Delta}$). Given $\widetilde{\Delta} < 1$, I is therefore maximal if $\Delta = \widetilde{\Delta}$ and decreases in $L(1-c)$ and also in k as $\widetilde{\Delta}$ decreases in k for $k > 1$. ■

It can be shown that $\lim_{k\to\infty} \widetilde{\Delta}(k) = 0$, so that the extent of patent protection $\widetilde{\Delta}$ that maximizes total innovation I is always interior ($\widetilde{\Delta} < 1$) if k is large enough, but also if the cost reduction $L(1-c)$ achieved by an innovation is large enough. The more convex the R&D cost functions (that are the same across sectors) are - that means that larger k is - the lower is the patent protection that maximizes total innovation. The intuition for this result is the following: if R&D productivity is declining more quickly at the industry level (if k is larger), a reduction in patent protection implies a lower reduction of the probability to innovate (x^*) in mass consumption sectors while it can still lead to a considerable increase in total consumption of the poor (given that $\Delta > \Delta^*$) and induce firms in new sectors to undertake R&D. And this effect is stronger the larger the size of the market and the stronger the cost reduction and therefore the price cut for consumers if patents are not protected is. Put differently: the faster R&D productivity decreases in a given sector, the cheaper it becomes to obtain a given amount of total innovation by doing a bit of R&D in many sectors compared to doing a lot of R&D in a few sectors.

Overall R&D spending Q is given by

(10) $Q \equiv N_p(\Delta) \cdot R(\Delta) = \frac{\frac{1}{k}(x^*(\Delta)+a)^k+F}{1-x^*(\Delta)\cdot(1-\Delta)(1-c)}$

Deriving Q with respect to Δ and looking at the simple case where $a = 0$ $L = 1$ and $c = 0$, we obtain: $sign\frac{\partial Q}{\partial \Delta}(\Delta = 1) = sign\left(\frac{1}{k-1} - \frac{1}{k} - F\right)$. Expected profits in a given sector where $\Delta = 1$ are given by $\Pi = x^* - \frac{1}{k}(x^*)^k - F = 1 - \frac{1}{k} - F$ and as they cannot be negative, the condition $F \leq 1 - \frac{1}{k}$ must be satisfied. Taking the largest possible value $F = 1 - \frac{1}{k}$, we obtain $sign\frac{\partial Q}{\partial \Delta}(\Delta = 1) < 0$ if $k > 2$. A small reduction in patent protection below the maximal value ($\Delta = 1$) can therefore lead to an increase in overall R&D spending Q if F and k are sufficiently large. In the case considered, it is therefore only possible that overall R&D expenditures rise if patent protection falls if there are large enough fixed costs F of undertaking R&D. The mechanism at work here is that a decrease in Δ leads to an increase in the total consumption of the poor and therefore in the number of sectors in which R&D is undertaken and in which the fixed costs have to be incurred. While a decrease in Δ reduces the R&D effort and therefore the variable R&D costs in all mass consumption sectors, these costs and in addition the fixed costs have to be born in more sectors and this makes it possible that total R&D spending increases.

7 Conditioning patent strength on market size

In the following, the case of K income groups is considered and it is assumed that only the richest group owns all shares in R&D firms. Patent protection Δ is allowed to be sector specific and to depend on the size of the market M^j. The budget constraints of the different income groups are given as follows:

(11) $y_1 = l_1 = N_1(1 - x_1^*(1 - \Delta_1)(1 - c))$

$y_2 = l_2 = N_1(1 - x_1^*(1 - \Delta_1)(1 - c)) + (N_2 - N_1)(1 - x_2^*(1 - \Delta_2)(1 - c))$

$y_3 = l_3 = N_1(1 - x_1^*(1 - \Delta_1)(1 - c)) + (N_2 - N_1)(1 - x_2^*(1 - \Delta_2)(1 - c)) + (N_3 - N_2)(1 - x_3^*(1 - \Delta_3)(1 - c))$

$y_n = l_n = l_{n-1} + (N_n - N_{n-1})(1 - x_n^*(1 - \Delta_n)(1 - c))$
$y_K = l_{K-1} + (N_K - N_{K-1})(1 - x_K^*(1 - \Delta_K)(1 - c))$

The left hand sides stand for the per capita incomes while the right hand sides sum the expenditures of the different income groups. The market size for goods with index $0 \leq j \leq N_1$ is given by $M_1 = L$, and by $M_n = L\left(1 - \sum_{i=1}^{n-1} \beta_i\right)$ for goods with index $N_{n-1} < j \leq N_n$. Patent strength and equilibrium R&D effort in sectors with market size M_n are denoted by Δ_n and x_n^* (the latter being a function of M_n and Δ_n).

Looking at the budget constraints we see that current consumption N_n of an agent from income group n who does not have any profit income is maximized if patent policy in sectors with market size M_l $(l \leq n)$ is such that it maximizes $x_l^*(1-\Delta_l)$. Given a certain size of the market, the problem is therefore the same as the one already analyzed in the case of two income groups (where N_p in equation (7) is also maximal if $x^*(1 - \Delta)$ if maximal). Replacing L by M_l we can therefore directly use the results from *Proposition 1* to derive the market size dependent patent strength $\Delta_l^*(M_l)$ preferred by agents without profit income. Given that a is not too negative, so that there is a local maximum, $\Delta^*(M)$ therefore decreases (increases) in the size of the market if $a > 0$ $(a < 0)$ while it is independent of market size if $a = 0$. It should be noted that in equilibrium market size decreases step - wise in the index j that reflects how luxurious a good is so that the results obtained here can also be rephrased in the following way:

The sector specific strength of patent protection preferred by agents without profit income increases (decreases) the more luxurious a sector is (the larger j is) if $a > 0$ $(a < 0)$ while it is independent of j if $a = 0$.

If there are considerable fixed costs of undertaking R&D and if the preferred patent strength $\Delta_n^*(M_n)$ derived above is so low in sector n that firms do not find it profitable to engage in R&D, the extent of patent protection preferred by agents that do not hold shares of R&D firms is given by the minimum amount $\underline{\Delta}_n$ that is necessary to make R&D firms break even. As $\underline{\Delta}_n$ decreases in the size of the market M_n, the preferred protection is now always larger in sectors supplying more luxurious goods[9].

As only individuals of the richest income group consume the most luxurious goods in the sectors $N_{K-1} < j \leq N_K$, no other income group is affected by the strength of patents in this sector and, given the patent policies in all other sectors, the total consumption of agents of the richest income group is maximal if patents are fully protected in these sectors (that means if $\Delta_K = 1$). The reasoning here is the same as in the case of one income group analyzed above.

8 First best

While the sections above studied the distributional implications of patent policies and derived the extent of patent protection preferred by individuals with a specific labour endowment or wealth (in the form of shares in R&D firms), they did not analyze which policy is efficient if beside patent policy transfers between agents can be used to address distributional concerns. Let us now consider a social planner who maximizes a weighted sum of utilities, putting more weight on agents belonging to a higher "income group". Using the notation from the last section, the resource constraint of the economy which equates total labour supply to total labour demand in the production and R&D sectors is given by:

(12) $L = N_1 M_1(x_1 c + 1 - x_1) + (N_2 - N_1) M_2(x_2 c + 1 - x_2) +$
$(N_3 - N_2) M_3(x_3 c + 1 - x_3) + ... + N_1(\frac{1}{k}(x_1 + a)^k + F) +$
$(N_2 - N_1)(\frac{1}{k}(x_2 + a)^k + F) + (N_3 - N_2)(\frac{1}{k}(x_3 + a)^k + F) + ...$

Any solution to the social planner problem must be a Pareto optimum and maximizes the utility and therefore the total per capita consumption N_l of any given income group l given the utilities $(U(N_{-l}))$ of all other groups. Solving the resource constraint for a certain N_l and maximizing with respect to the different x_n (the R&D intensities in the different sectors characterized by market sizes M_n) taking all other N_{-l} as given results in the first order conditions: $x_n^o =$

[9] The threshold $\underline{\Delta}_n$ is defined in the following way:
$\Pi_n = x_n^* \underline{\Delta}_n M_n(1-c) - \frac{1}{k}(x_n^* + a)^k - F = 0$
with $x_n^* = (\underline{\Delta}_n M_n(1-c))^{\frac{1}{k-1}} - a$
Expected profits of firms in sectors where the market size is given by M_n are zero for this level of patent protection (and negative if $\Delta < \underline{\Delta}$). It can easily be seen that $\frac{d\underline{\Delta}_n}{dM_n} < 0$ if $\Pi_n = 0$.

$((1-c)M_n)^{\frac{1}{k-1}} - a$. Given that market size is large enough relative to the fixed costs[10] these conditions give the optimal R&D intensities which are equal to those obtained in a decentralized equilibrium if patents are fully enforced (that means if $\Delta = 1$). Full patent protection is therefore efficient also in the case of many income groups if transfer payments between groups can be used in order to deal with distributional concerns.

But even if it is feasible to use lump - sum transfer payments so that patent policies need not be used to tackle distributional concerns, it can be important for policymakers to understand the distributional implications of different patent policies as this for example allows to identify which groups loose from a shift to a system of strong patent protection and might have to be compensated with appropriate transfer payments from other groups.

9 Discussion

Important assumptions of the model are that preferences are strictly hierarchical and that consumers are satiated after consuming one unit of a good from a given sector. In order to relax the assumption of strictly hierarchical preferences, a weighting function could be introduced so that utility is given by $U_i = \int\limits_{j=0}^{\infty} j^{-\gamma} c_i(j)dj$, where again $c_i(j) \in \{0, 1\}$. This specification makes goods to some extent substitutable, and puts a larger relative welfare weight on basic need goods (with a low index j) the larger the parameter γ is, that means the "steeper" the hierarchy is (see Föllmi and Zweimüller (2006)). If γ is very large, the analysis would be the same as in the case of a strict hierarchy, but things would change if γ becomes lower and goods are more substitutable. Then, R&D firms in more luxurious sectors might find it profitable to engage in R&D and to sell at a price below the limit price of one in order to attract demand from sectors satisfying more basic needs. And as consumers are not bound to buy one unit of each basic need good before consuming more luxurious goods, decreasing patent protection could not boost demand for more luxurious goods as much as it does

[10] The condition $x_n^o M_n(1-c) - \frac{1}{k}(x_n^o + a)^k - F > 0$ needs to hold in order to obtain $x_n^o > 0$. If it is violated we have $x_n^o = 0$.

in the model with strictly hierarchical preferences. Because of this, a decrease in γ would probably make it less likely that a decrease in patent protection can lead to more overall innovation.

The assumption of $0-1$ consumption could be relaxed by introducing a concave "baseline utility function" (See Föllmi and Zweimüller (2008)) allowing the consumption of a continuous amount of a given good j. In such a more general model, patent protection would lead to monopoly pricing and static distortions and consumers would consume more of a given good if patents were not enforced. This effect would make it less likely that a decrease in patent protection can lead to a higher rate of total innovation as the price cuts from which consumers benefit if patents are not enforced would not only induce consumers to spend more on more luxurious goods but also to spend more on nonpatented basic need goods. However, this offsetting effect would become smaller and less important the faster consumers become satiated with a given good (that means the faster marginal utility decreases in the consumption of a given good).

The assumption that innovators cannot appropriate any surplus created by their innovations if they are not protected by patents is clearly not very realistic as there are other means of appropriation like capacity constraints, lead time or secrecy that are even preferred to patent protection in certain industries. Especially in the case of cost - saving innovations that is analyzed in this paper, it is often hard to enforce patents as a given good can legally be produced by firms other than the patent holder if they use a nonpatented technology. In order to discover infringement, the production processes of potentially infringing firms have to be supervised which might prove difficult. But even if there are other means of appropriation that firms use, introducing these in the model would not lead to different qualitative results and changing the degree of appropriability would affect inequality and innovation in the same way as a change in the extent of patent protection does. However, if other forms of appropriability are used besides patents it might be more difficult to influence the extent of appropriability through (antitrust) policy.

If R&D causes positive unpriced spillovers (proportional to $N \cdot x$) to other sectors, it might become more likely that weaker patents lead to more overall

R&D by stimulating mass consumption in more luxurious sectors. But again, increasing mass consumption by transferring income from the rich to the poor would probably be preferable.

10 References

Föllmi, Reto and Josef Zweimüller (forthcoming); "Structural Change, Engel's Consumption Cycles, and the Kaldor Facts of Economic Growth", Journal of Monetary Economics

Föllmi, Reto and Josef Zweimüller (2006); "Income Distribution and Demand-Induced Innovations," Review of Economic Studies, Blackwell Publishing, vol. 73(4), pages 941-960, October.

Kiedaisch, Christian (2009); "Patent Policy and Income Distribution", working paper, Toulouse School of Economics

Matsuyama, Kiminori (2002) "The Rise of Mass Consumption Societies," Journal of Political Economy, 110, 1035-1070.

Murphy, K.M., A. Shleifer, and R. Vishny. (1989). "Income Distribution, Market Size, and Industrialization," Quarterly Journal of Economics 104, 537-564.

11 Appendix: Free access into the R&D sectors

Let us now assume that more than one firm can undertake R&D in a given sector j. For simplicity it is assumed that $a = 0$ and $F = 0$ and that there is the probability x that at least one of the firms makes the innovation if the total amount of R&D labour hired by all firms is given by $R(x) = \frac{1}{k}x^k$. It is assumed that the probability with which a firm i exclusively obtains a patent in a given sector is given by γx, where γ indicates the share of the total R&D labour of the sector that is hired by the firm.

If there is free entry into the R&D sectors, entry occurs until the average costs of innovating are equal to the expected benefits from innovating, that means until $\frac{1}{k}x^{k-1} = \Delta(1-c)M^j$. The amount of R&D that is undertaken in each sector if there is free entry is therefore given by $x_f = (k\Delta(1-c)M^j)^{\frac{1}{k-1}}$, which is larger than the amount of R&D undertaken in the case where only one firm is capable of doing R&D in a given sector ($x_f > x_j^* = (\Delta(1-c)M^j)^{\frac{1}{k-1}}$ as $k > 1$). The reason for this is that an entrant does not take into account that the R&D costs of other firms rise if he undertakes R&D, so that he enters even if the marginal costs (if seen from the perspective of the entire sector) exceed the marginal benefits, as long as the latter are higher than the average costs. In the case of full patent protection ($\Delta = 1$), the total costs of innovating are equal to the total benefits ($\frac{1}{k}x^k = x(1-c)L$) so that total per capita consumption is given by $N = 1$ (see equation (5)), which is the same as in the case where no R&D is undertaken in any sector. As firms in each sector engage in such a fierce patent race that expected profits are zero and consumers do not benefit from a reduction in prices, no additional income (surplus) is created which would allow consumers to purchase a larger variety of goods. And anticipating this, no firm engages in R&D in sectors with an index $j > 1$. As the R&D effort x_f in each sector increases in the probability of patent protection, there is an inverted - U relation between patent protection Δ and total per capita consumption N. If patents are not enforced ($\Delta = 0$) or if they are fully enforced ($\Delta = 1$), total per capita consumption is minimal ($N = 1$) while it reaches its maximum if $\Delta = \frac{1}{k} < 1$ (in which case

$x_f = x_o = (L(1-c))^{\frac{1}{k-1}})$. An intermediate strength of patent protection is therefore optimal as it offsets the excessive R&D incentives resulting from free entry.