

January the 28th, 2019, Hasselt, Belgium

Timoteo Carletti

Complex behaviours
from simple models



Acknowledgements

“Belgian” team:

R. Lambiotte

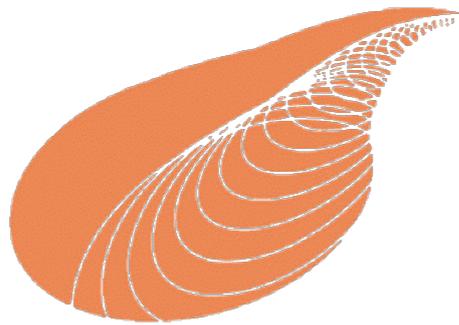
M. Asllani, N. Kouvaris (post docs)

J. Petit (PhD)

A. Bellière, G. Planchon, R. Muolo (Master students)

Italian team:

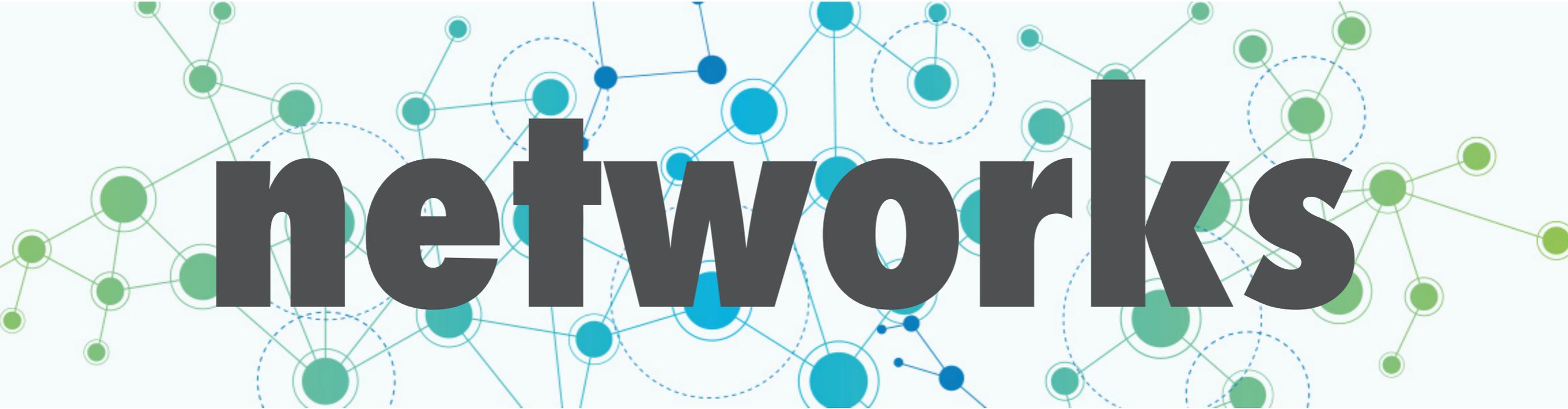
D. Fanelli, D.M. Busiello, C. Cianci, M. Galanti, F. Miele,
F. Di Patti



IAP VII/19 - DYSCO



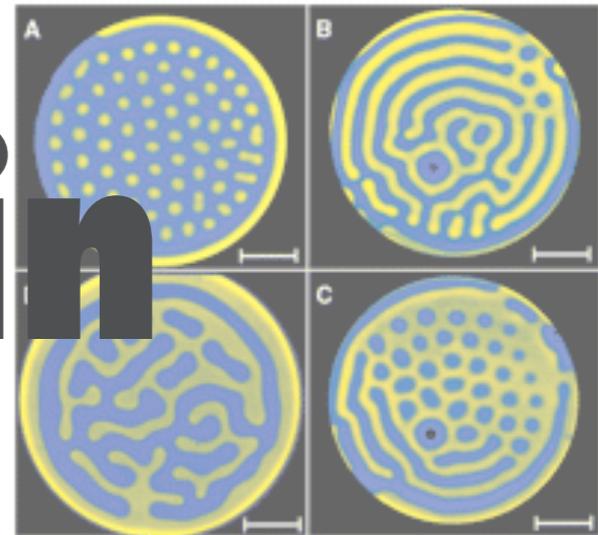
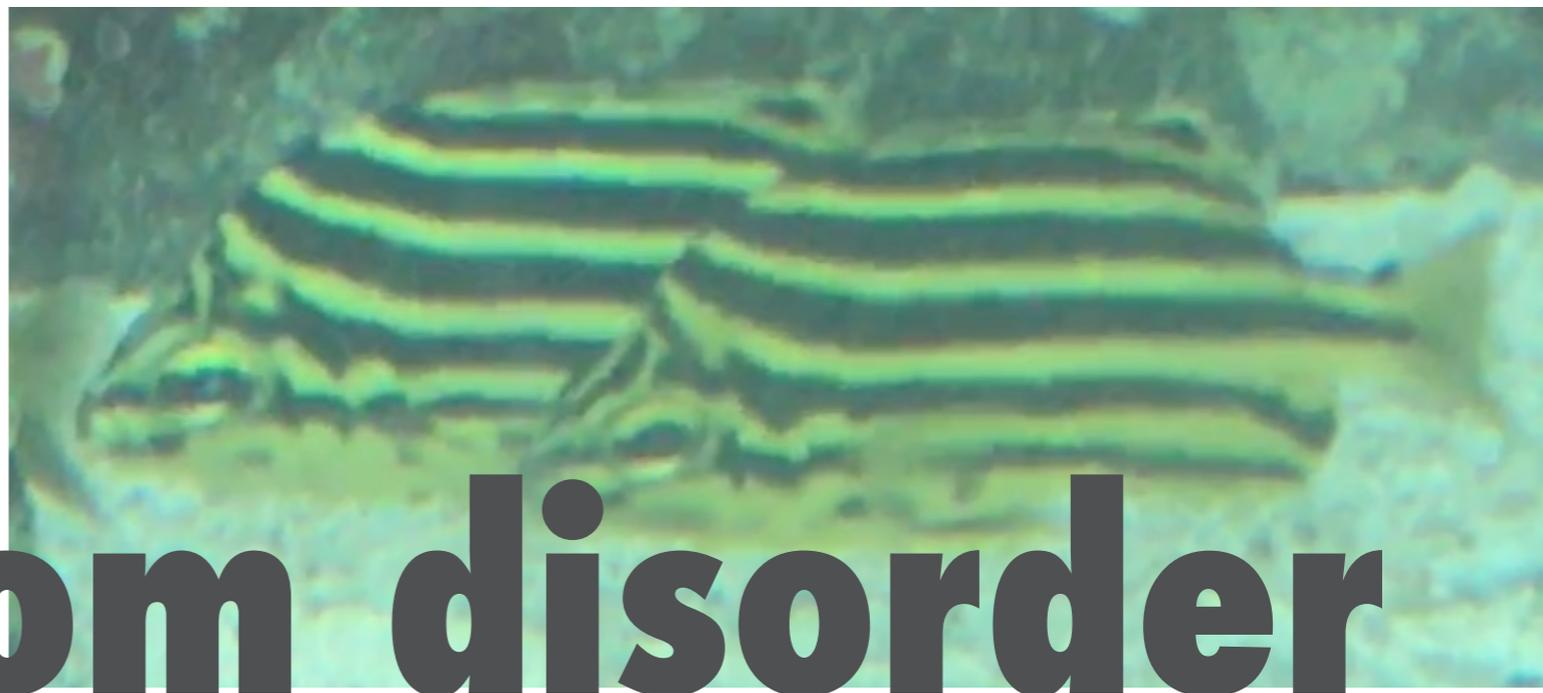
interactions

A background graphic featuring a network diagram with various nodes and connections. The nodes are represented by circles in shades of green and blue, connected by thin lines. Some nodes are highlighted with larger, dashed circles. The overall style is clean and modern, typical of a technical or academic presentation.

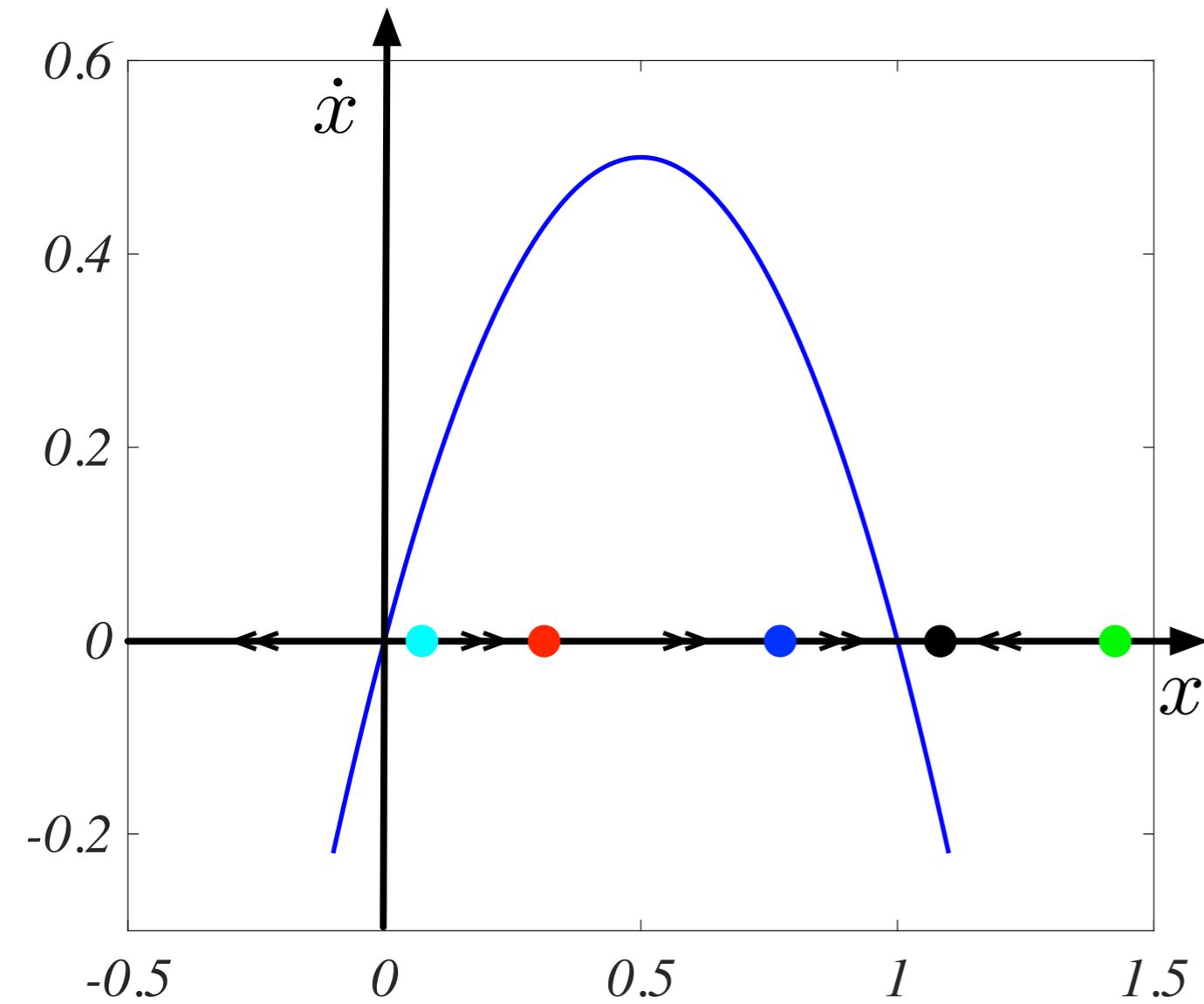
networks

**Order from disorder
is a leitmotif in**

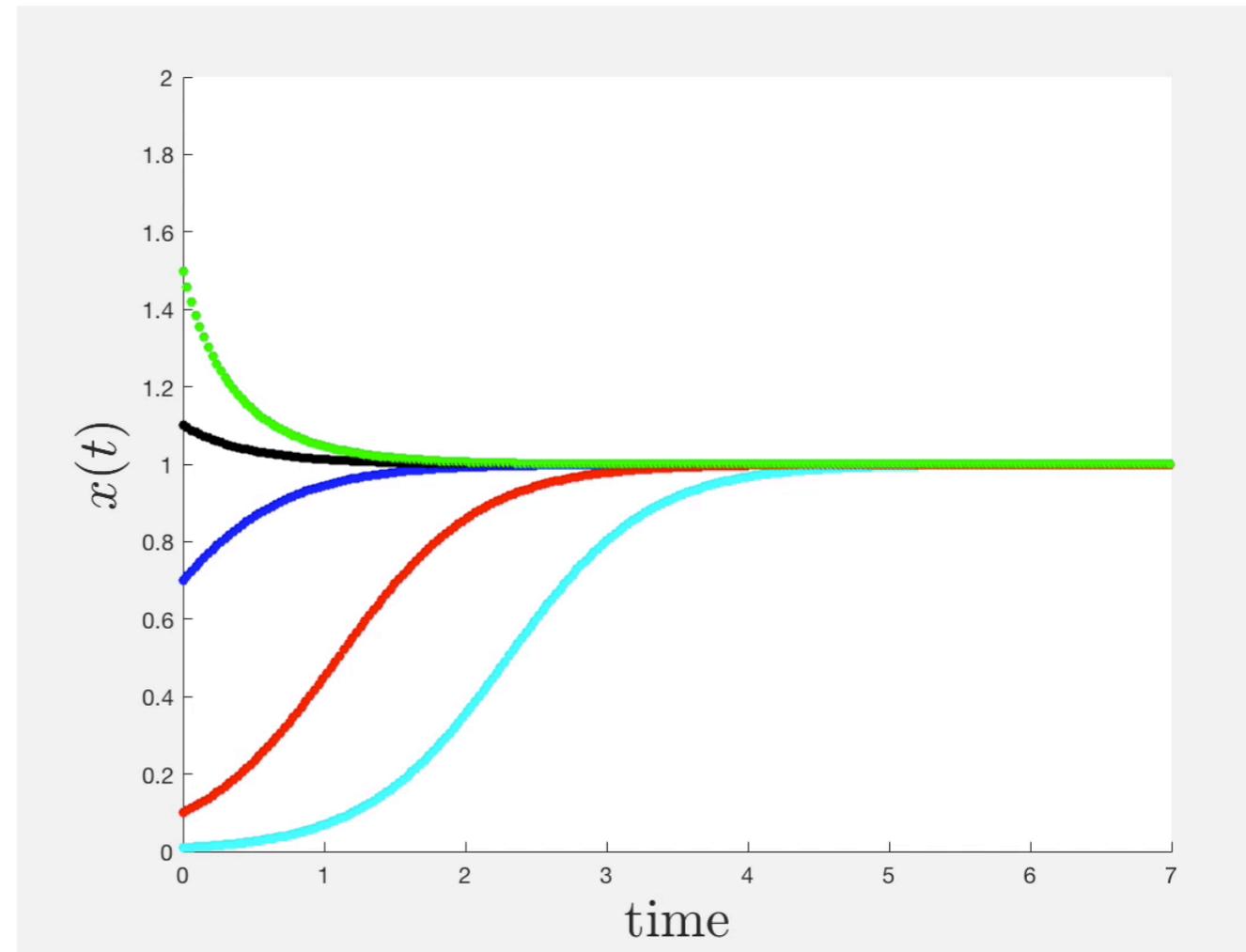
Nature



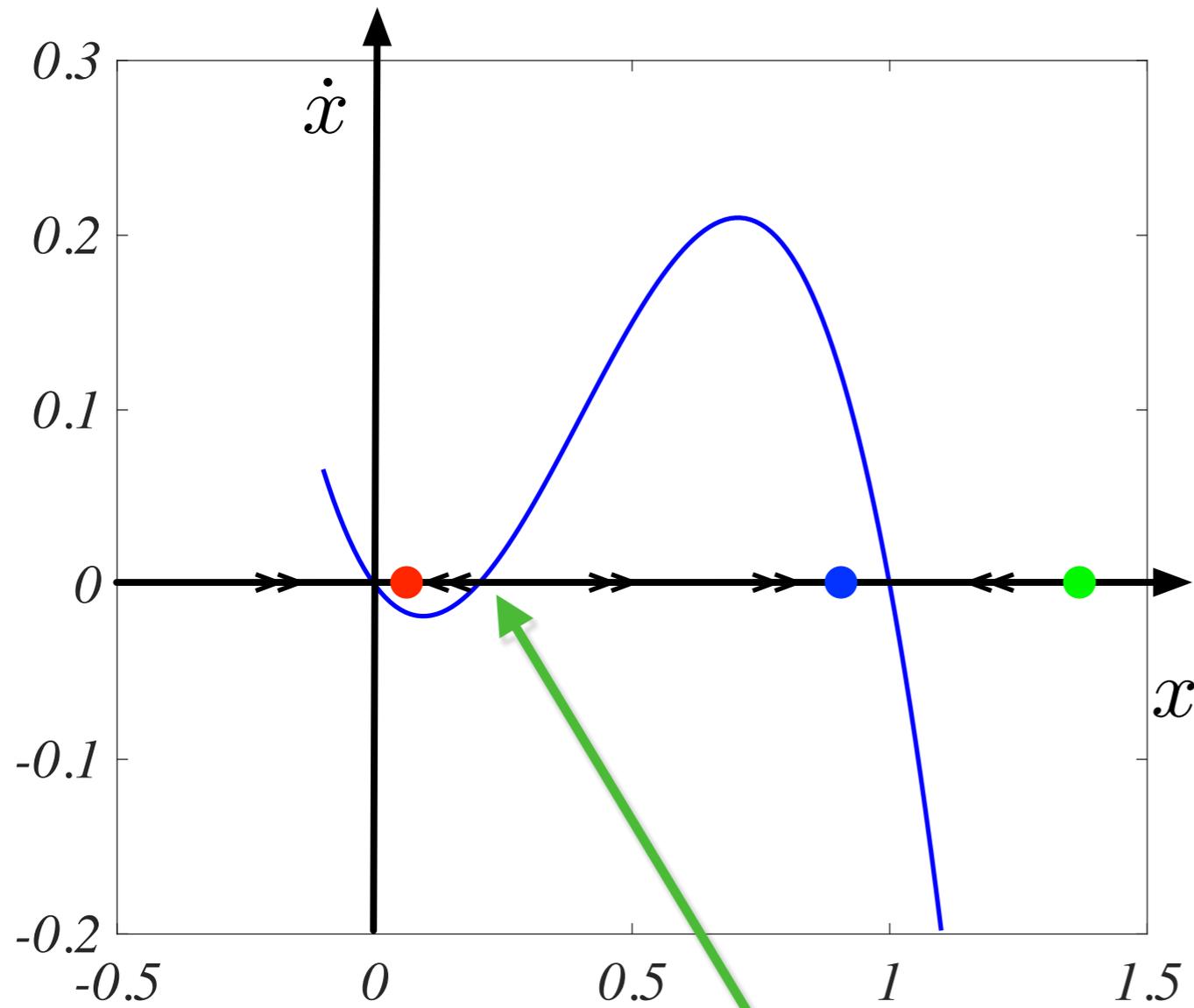
Logistic model $\dot{x} = rx(1-x)$



$$r = 2$$

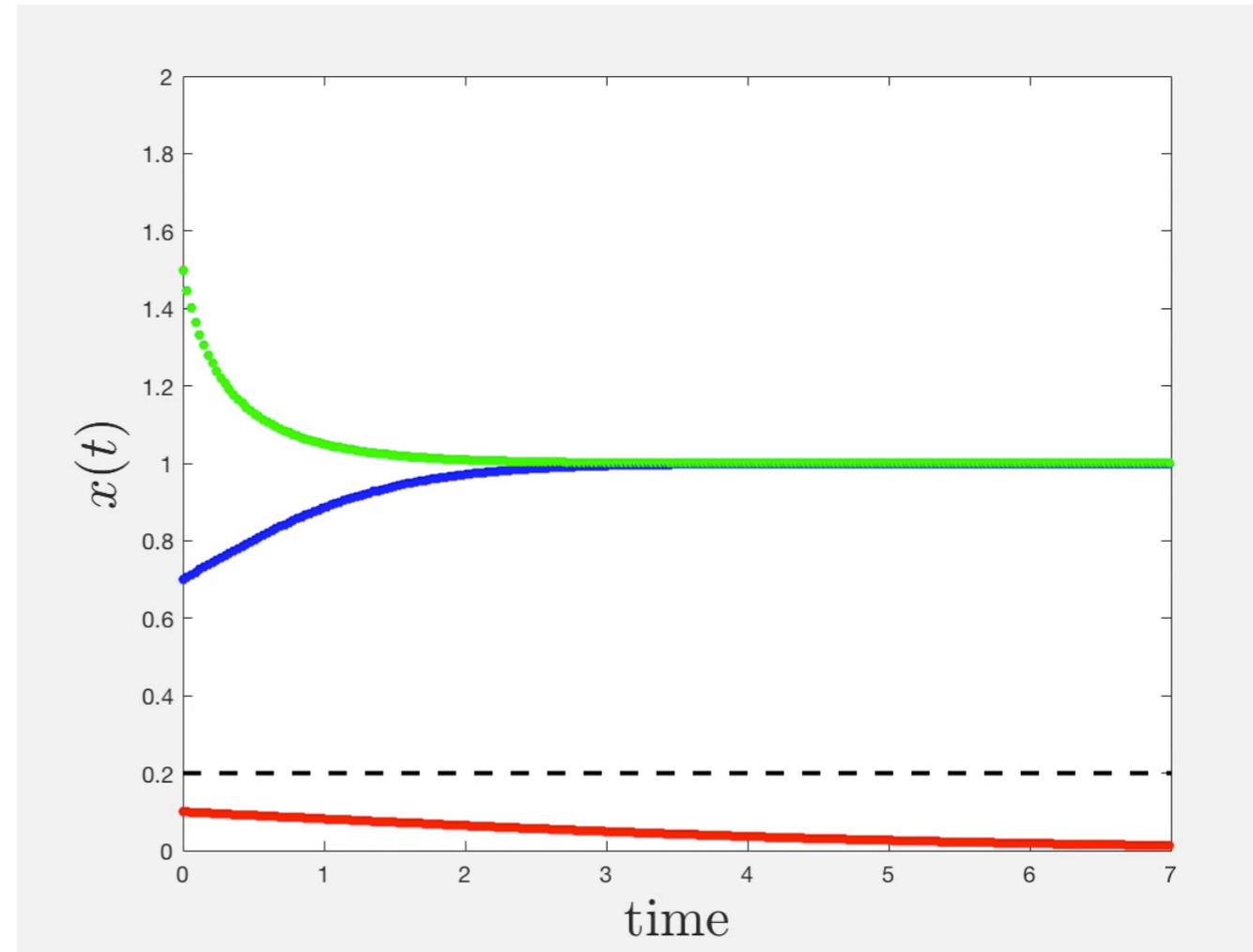


Allee effect $\dot{x} = rx(1-x)(x-a)$



$$r = 2$$

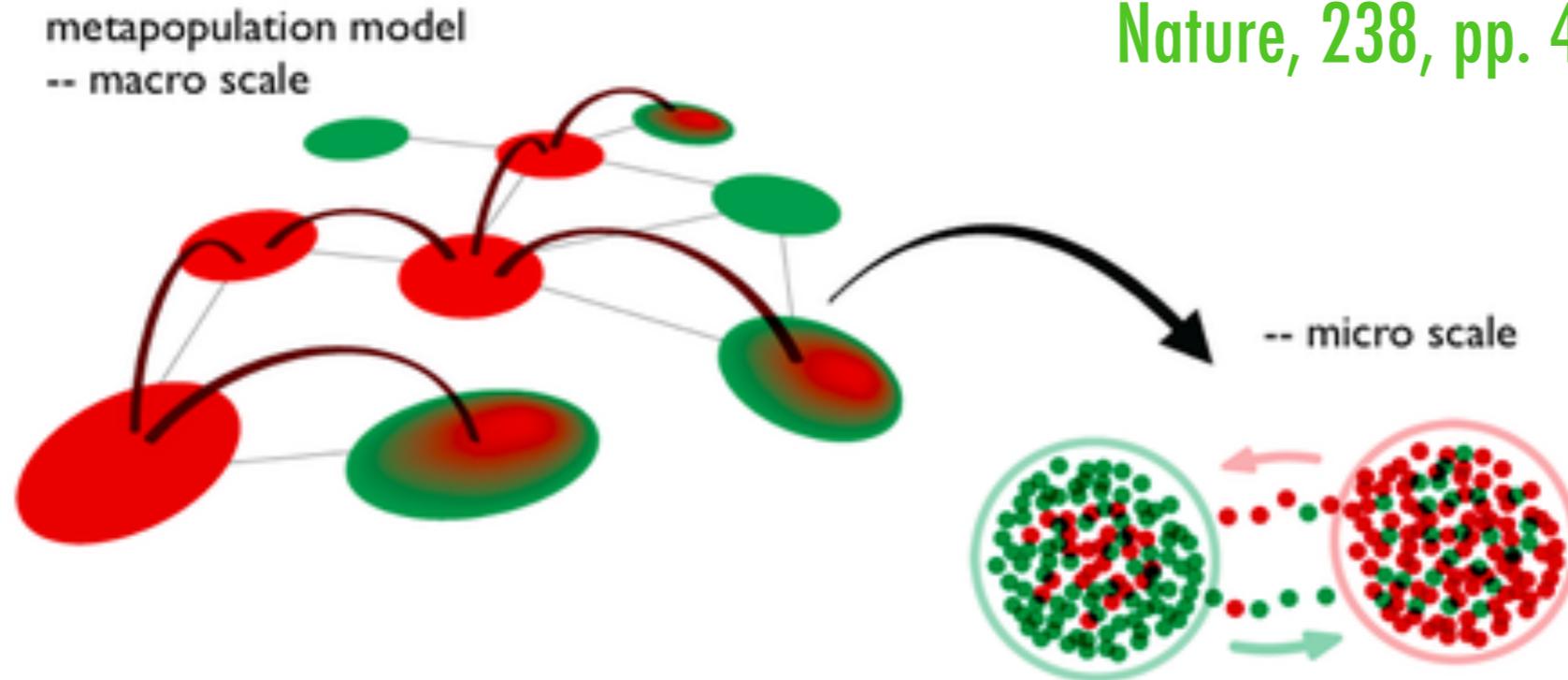
$$a = 0.2$$



Metapopulation models

e.g. in the framework of ecology:

May R., Will a large complex system be stable?
Nature, 238, pp. 413, (1972)



Species interact inside each node and diffuse among nodes across available edges in both directions: symmetric network.

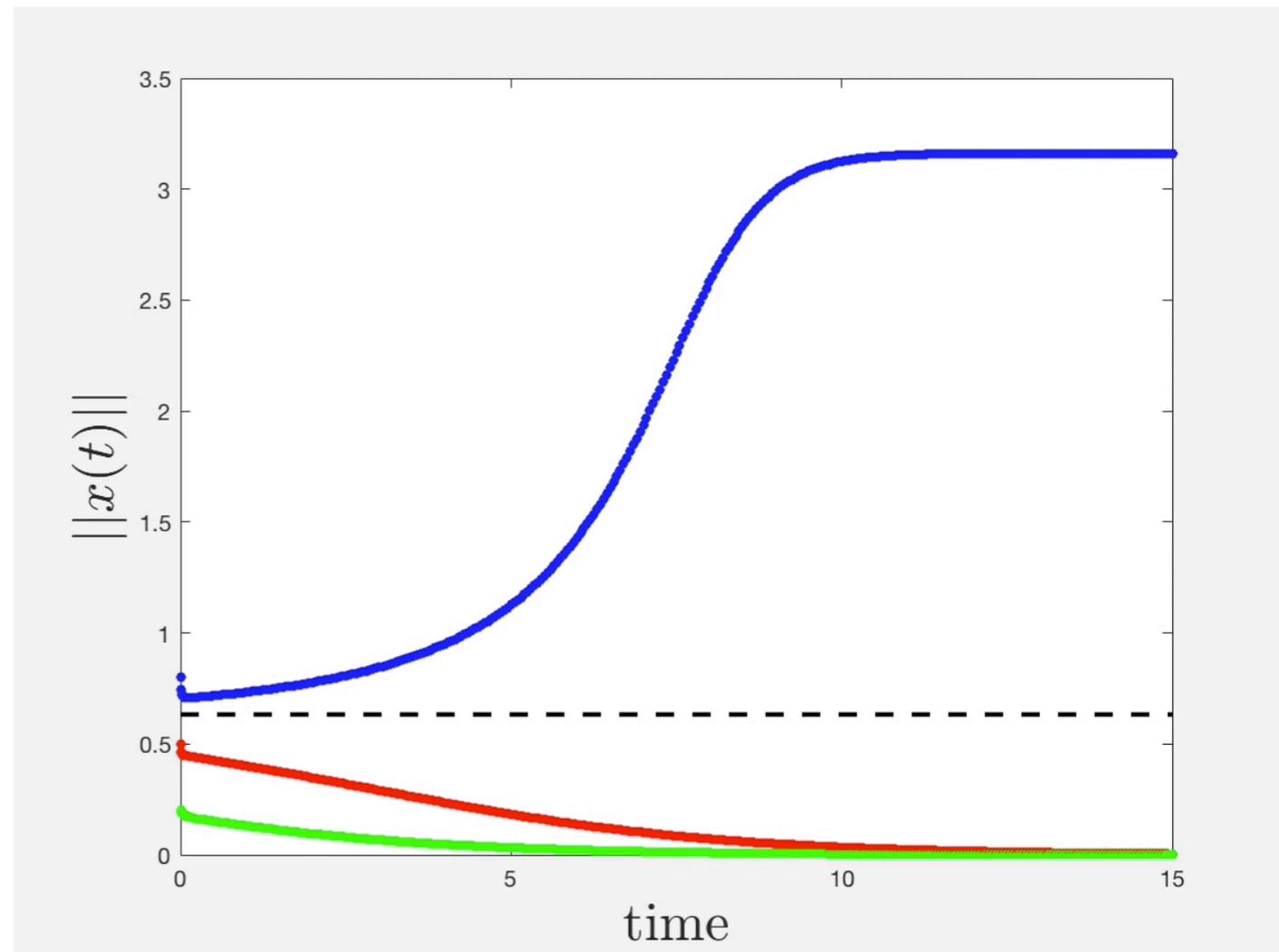
Allee effect on a symmetric network

System (bio)mass

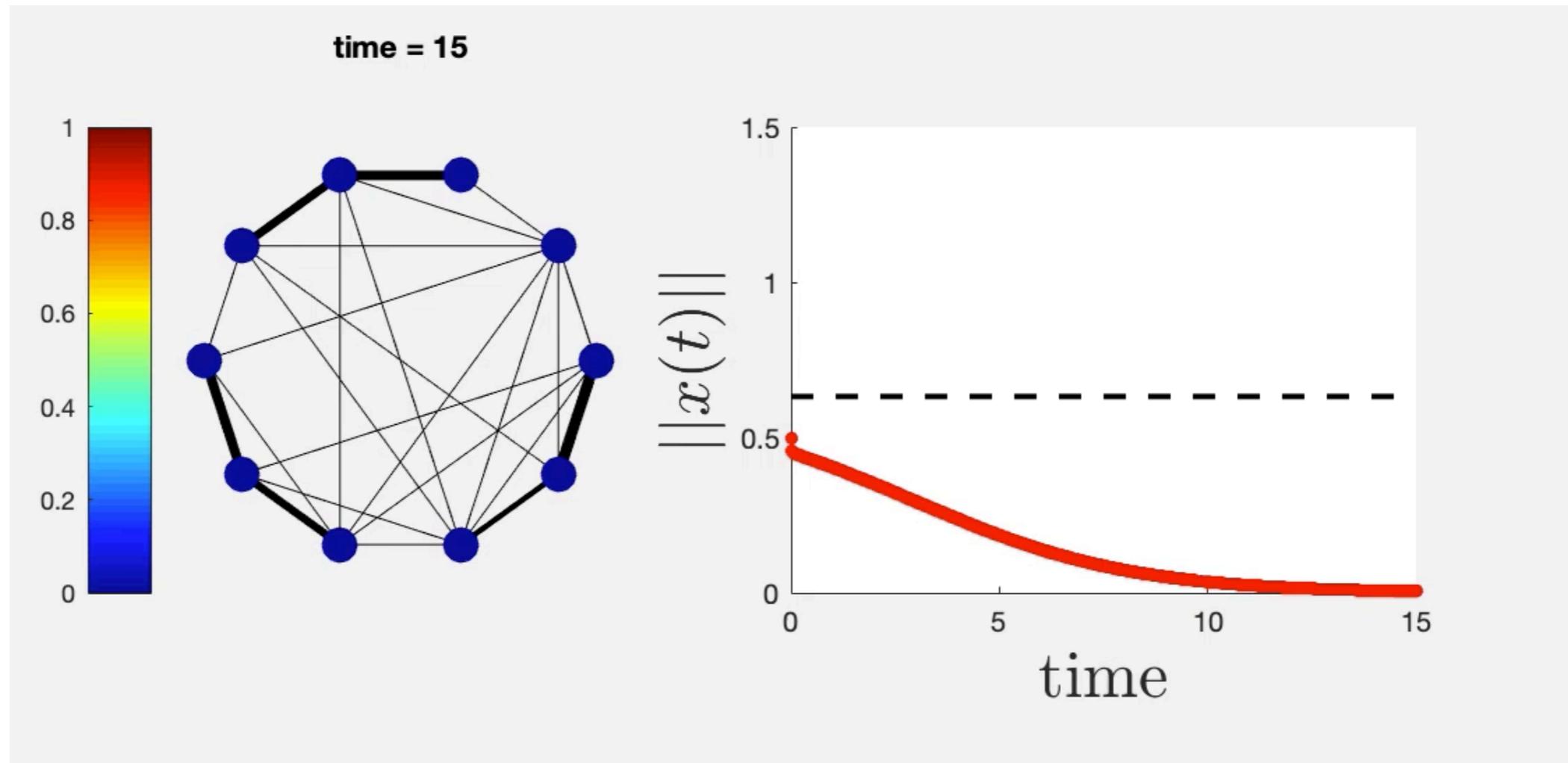
$$\|x(t)\|^2 = \sum_{i=1}^M [x_i(t)]^2$$

- $\|x(0)\| = 0.2$ and $x_i(0) \ll a$
- $\|x(0)\| = 0.5$ and $x_i(0) \lesssim a$
- $\|x(0)\| = 0.8$ and $\exists j : x_j(0) \geq a$

Symmetric network



Allee effect on a symmetric network



The origin is stable, no heterogeneity will emerge

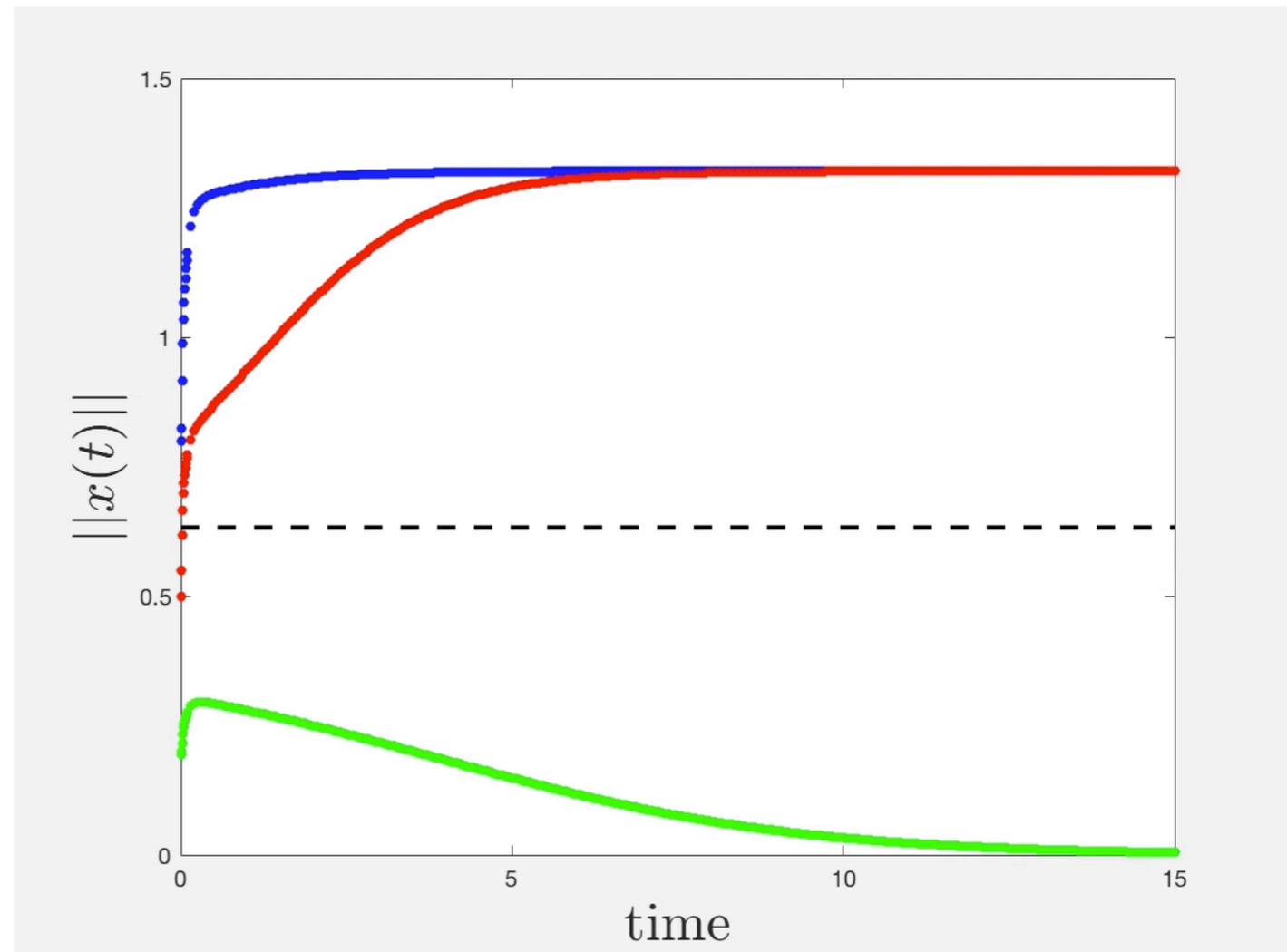
Allee effect on a non-normal network

System (bio)mass

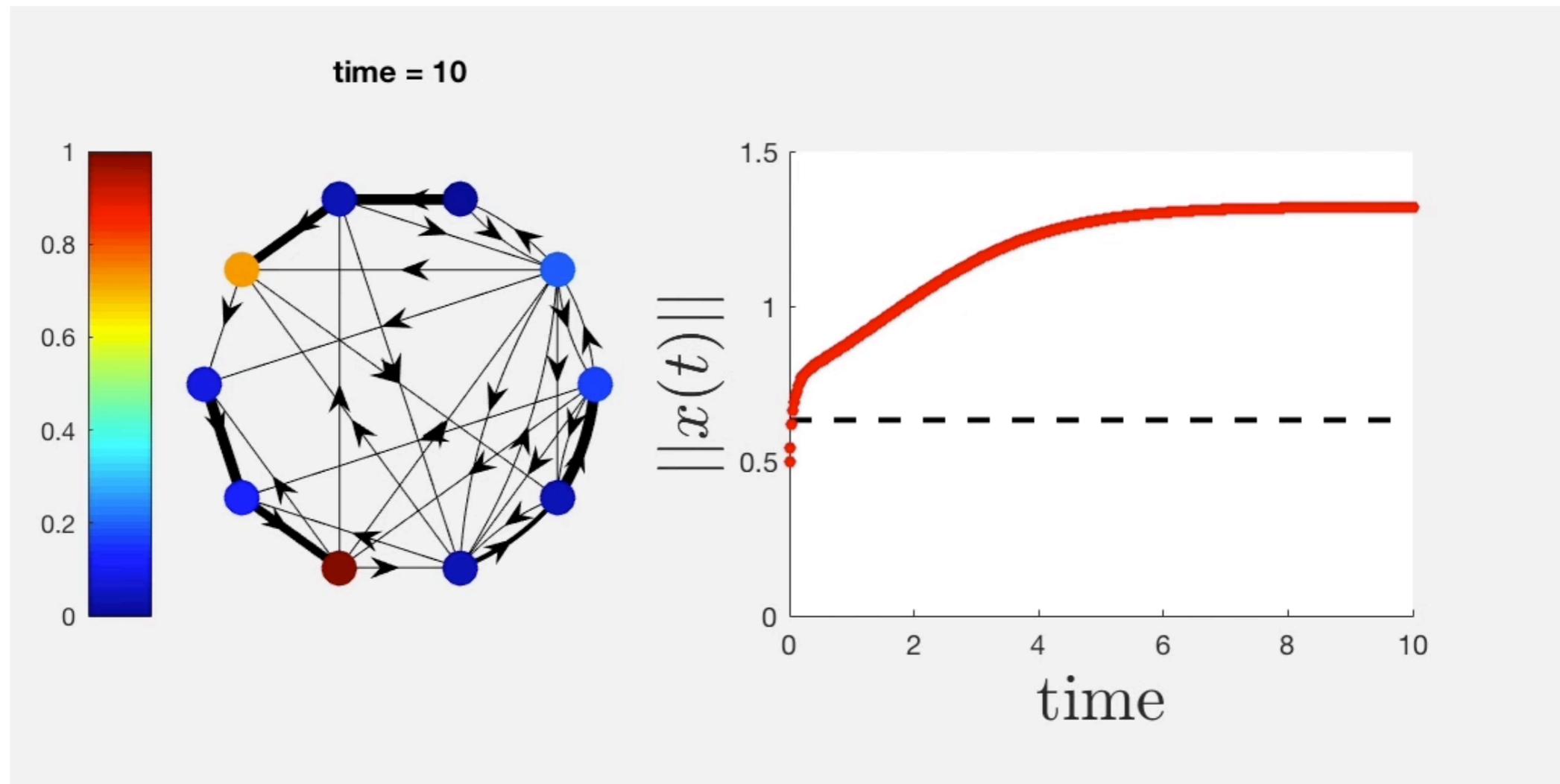
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non-normal network
(directed links)



Allee effect on a non-normal network



The origin remains stable but small perturbations can now be macroscopically amplified and create a patchy distribution.

✔ Network non-normality can strongly modify the system behavior and create new patchy solutions (diversity).

✔ Many real networks exhibit strong non-normality and thus challenge our comprehension of their dynamics.

✔ M. Asllani and T. Carletti, Topological resilience in non-normal networked systems, *Physical Review E*, **97**, (2018), 042302

✔ M. Asllani, R. Lambiotte and T. Carletti, Structure and dynamical behavior of non-normal networks, *Science Adv.*, **4**, (2018), eaau9403

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Non-normal networks are “the rule”

Network name	nodes	links	ω	$\omega - \alpha$	α_ϵ	Δ	\hat{d}_F
Foodwebs							
Cypress wetlands South Florida (wet)	128	2016	296.71	132.11	167.46	0.83	1.00
Cypress wetlands South Florida (dry)	128	2137	217.60	152.50	82.20	0.89	1.00
Little Rock Lake (Wisconsin, US)	183	2494	21.69	14.69	10.02	0.95	0.93
Biological							
Transcriptional regulation network (<i>E. coli</i>)	423	578	5.11	4.11	2.52	0.81	0.93
Metabolic network (<i>C. Elegans</i>)	453	4596	13.44	12.44	6.89	0.98	1.00
Pairwise proteins interaction (<i>Homo sapiens</i>)	2239	6452	15.79	13.02	4.01	0.99	0.99
Transport							
US airport 2010	1574	28236	$1.19 \cdot 10^7$	79.30	$1.19 \cdot 10^7$	0.01	1.00
Road transportation network (Rome)	3353	8870	$2.40 \cdot 10^4$	120.05	$2.39 \cdot 10^4$	0.08	0.28
Road transportation network (Chicago)	12982	39018	4.23	$4.29 \cdot 10^{-4}$	4.54	0.04	0.19
Communication							
e-mails network DNC	2029	39264	28.00	2.00	26.37	0.53	0.89
Enron email network (1999-2003)	87273	1148072	85.14	14.54	71.05	0.30	0.99(*)
e-mails network European institution	265214	420045	76.02	6.09	70.30	0.30	0.84(*)
Citation							
Citations to Milgram’s 1967 paper (2002)	395	1988	10.48	10.48	4.49	1.00	1.00
Articles from Scientometrics (1978-2000)	3084	10416	10.32	8.32	5.28	0.98	1.00
Citation network DBLP	12591	49743	21.50	16.82	8.45	0.87	1.00
Social							
Hyper-network of 2004 US election blogs	1224	19025	45.37	10.95	34.95	0.72	0.98
Reply network of the news website Digg	30398	87627	15.92	6.56	10.18	0.61	0.97
Trust network from the website Epinions	75879	508837	123.00	16.47	106.96	0.13	0.80(*)