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Patterns of Non-Normality in Networked Systems

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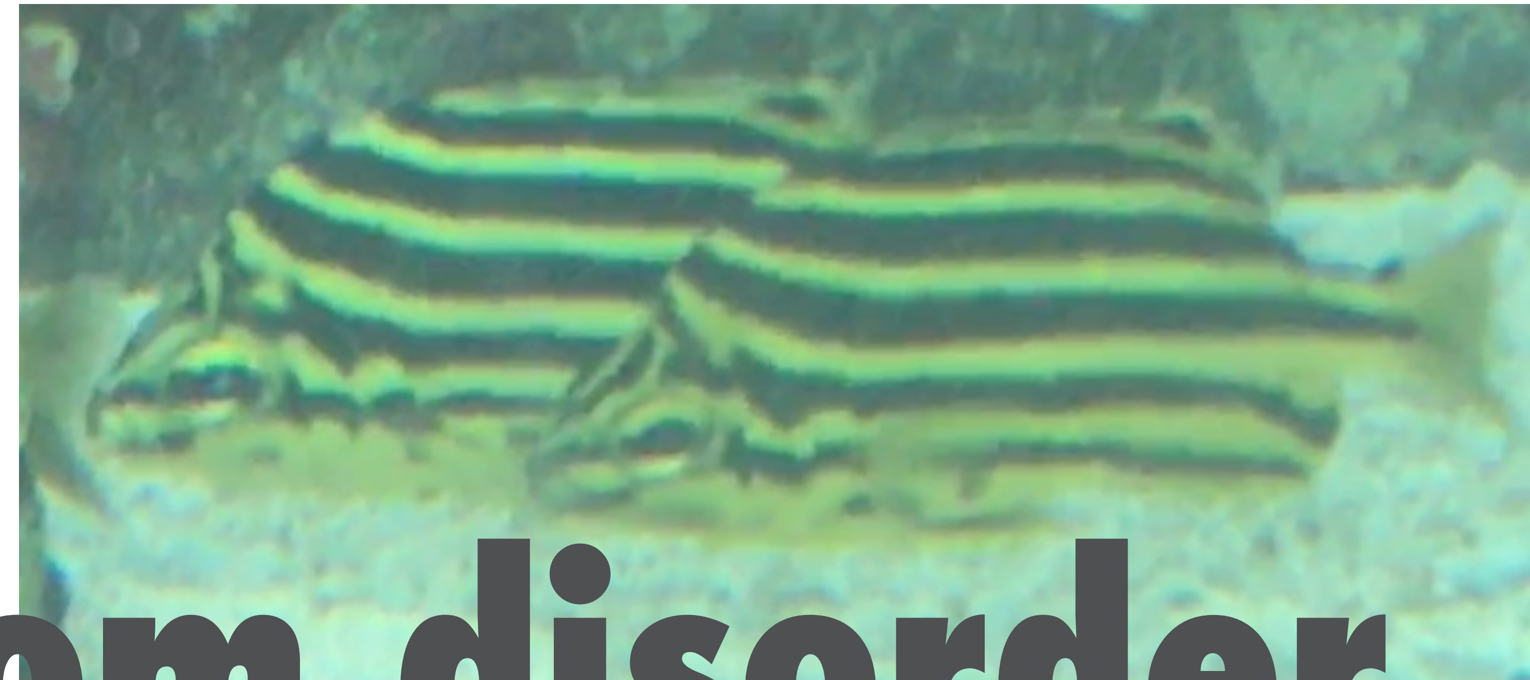
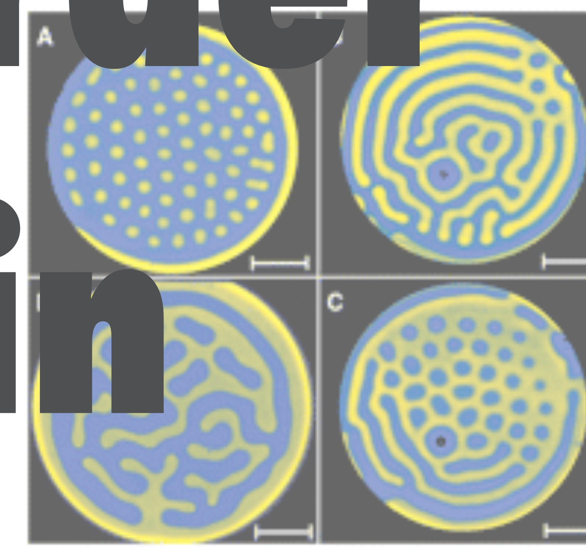
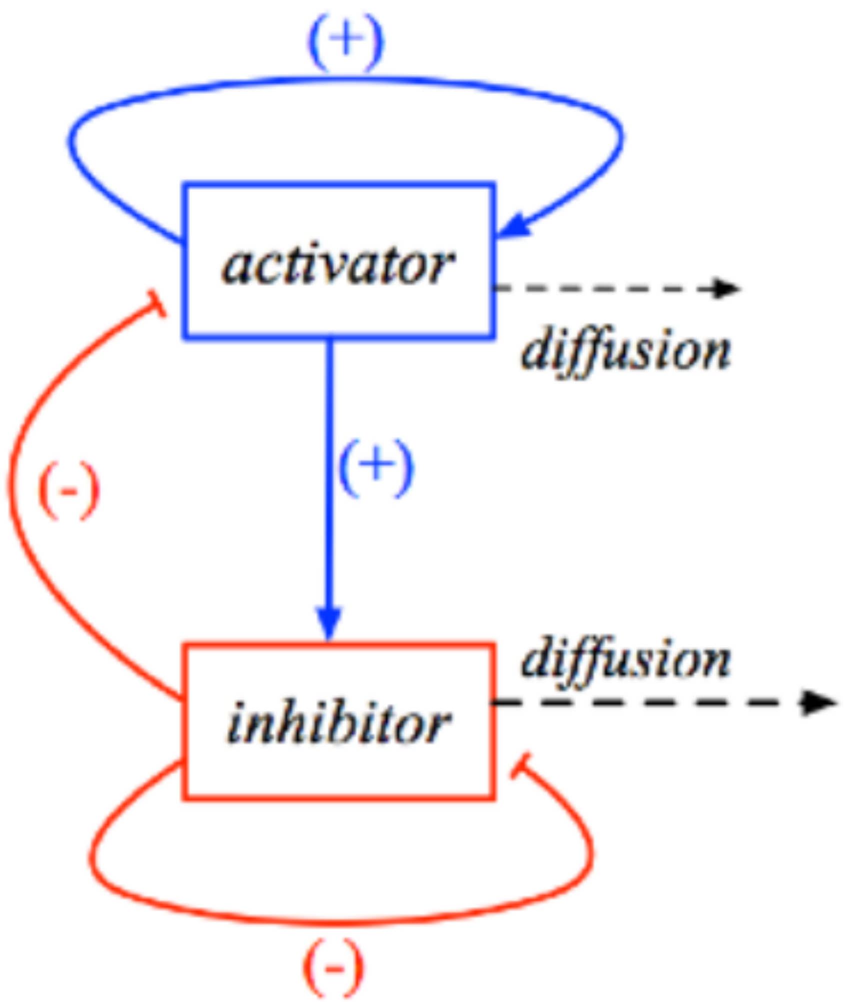


Order from disorder is a leitmotif in Nature



Turing mechanism: diffusion driven instability (on symmetric networks)



$u_i(t)$ Amount of activator in node i at time t

$v_i(t)$ Amount of inhibitor in node i at time t

$$\begin{cases} \dot{u}_i &= f(u_i, v_i) + D_u \sum_j L_{ij} u_j \\ \dot{v}_i &= g(u_i, v_i) + D_v \sum_j L_{ij} v_j \ , \end{cases}$$

Local reaction term

Diffusion term (Fick's law)

A.M.Turing, The chemical basis of morphogenesis, Phil. Trans. R Soc London B, 237, (1952), pp.37

Nakao H. and Mikhailov A. S., Turing patterns in network-organized activator–inhibitor systems, Nature Physics, 6, pp. 544 (2010)

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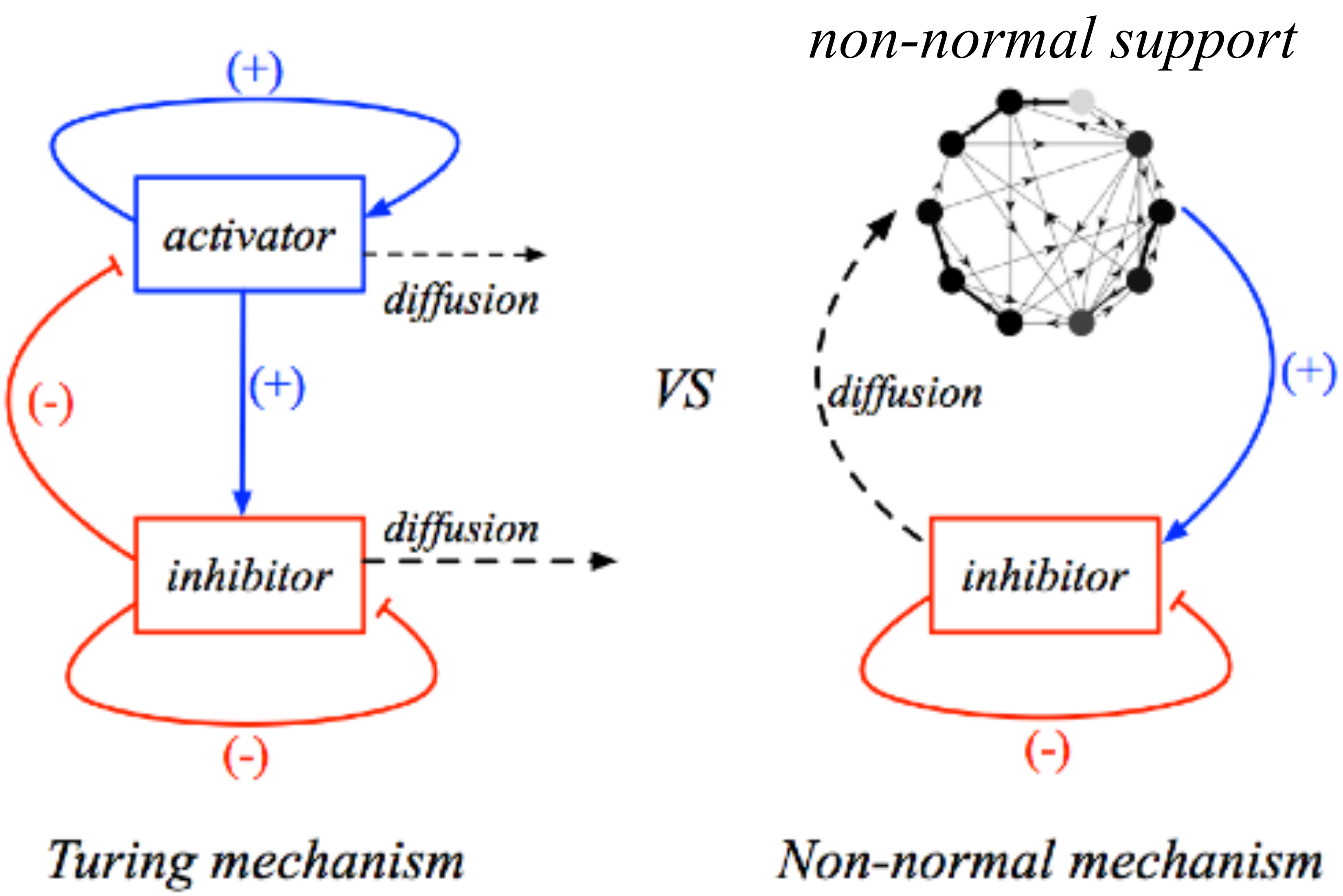
Turing mechanism:
elegant and simple, but unable to describe patterns onset in many real scenarios.

- At least two diffusing species are needed;
- The inhibitor must diffuse much faster than the activator;

$$D_v \gg D_u$$

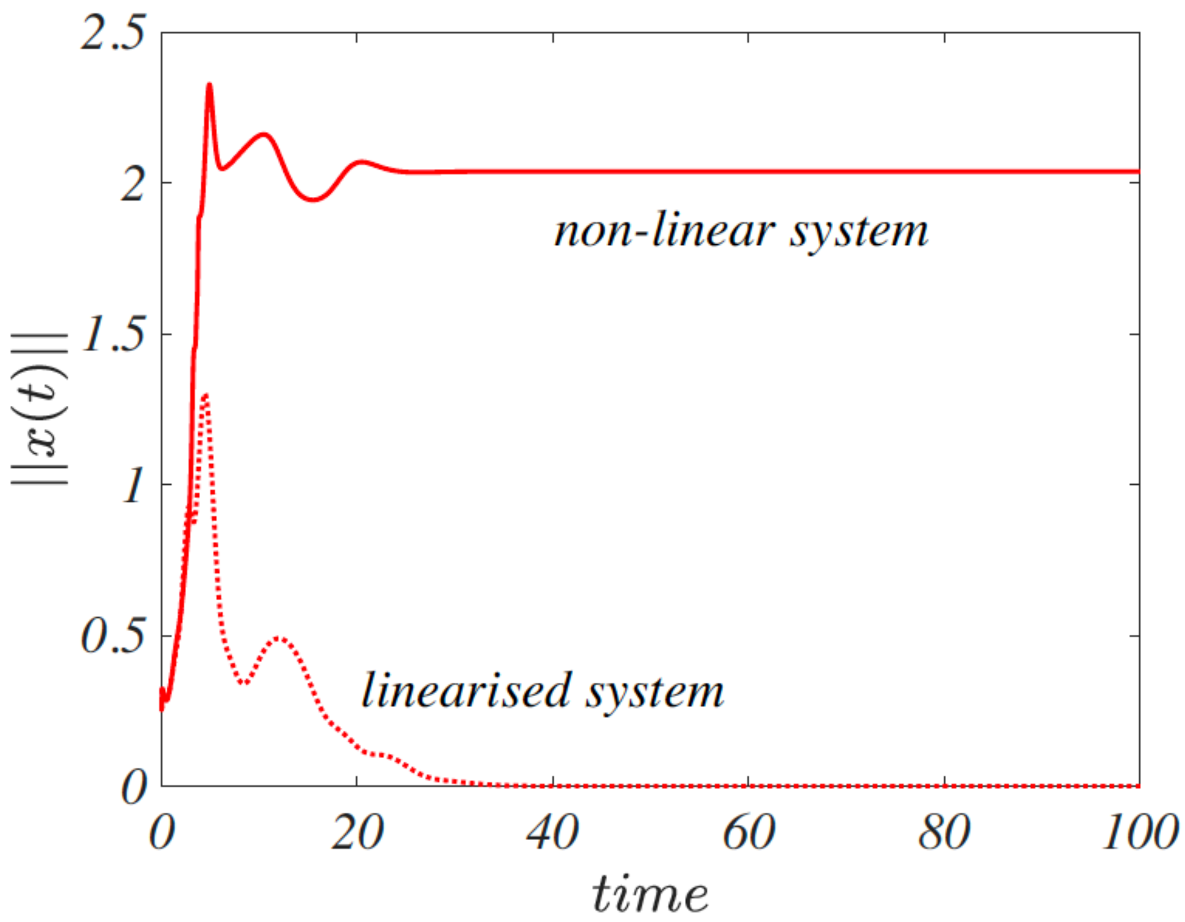
- Activator and inhibitor are both necessary.

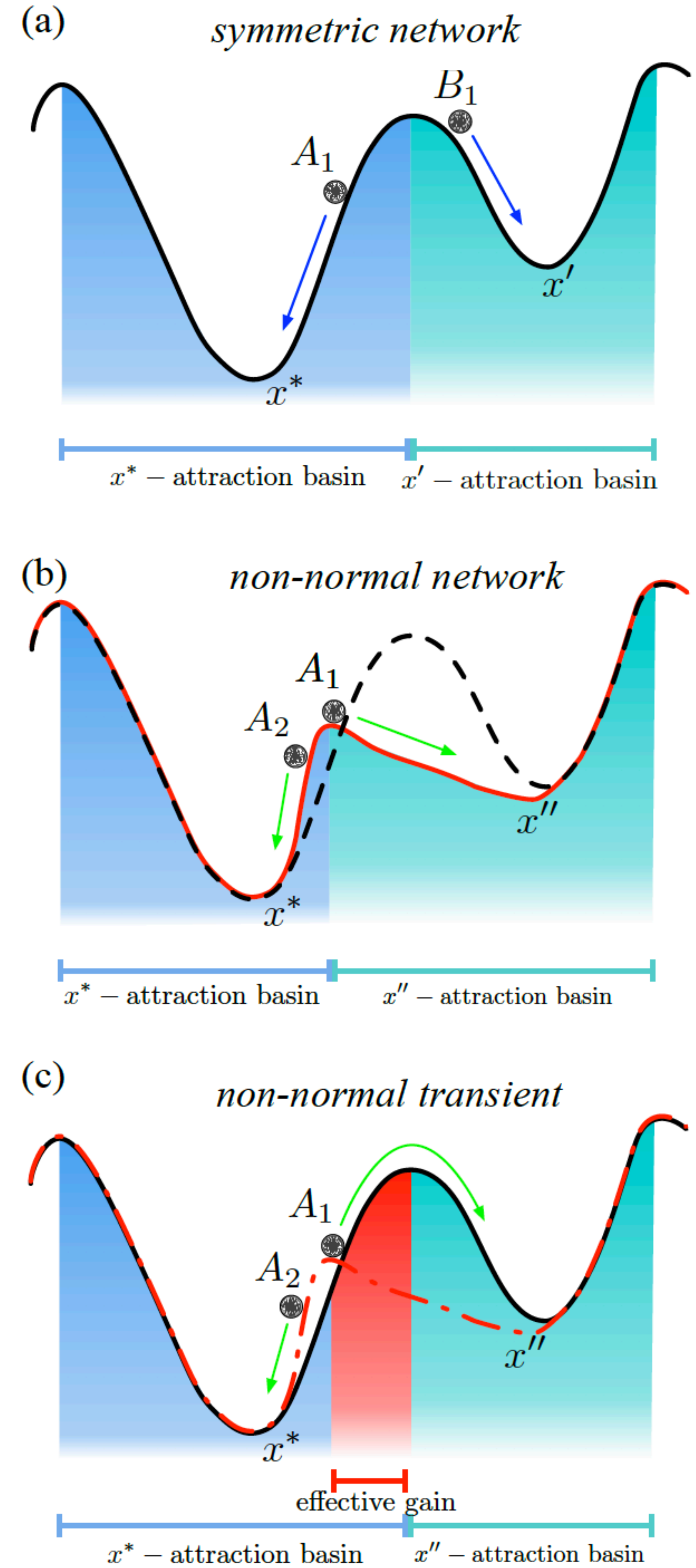
Beyond the Turing mechanism: non normal network



- The homogeneous equilibrium remains stable;
- One species is enough;
- No need for separate diffusion scales: $D_u \sim D_v$
- No need for activator/inhibitor.

Network non-normality reduces the stability of the homogeneous equilibrium.

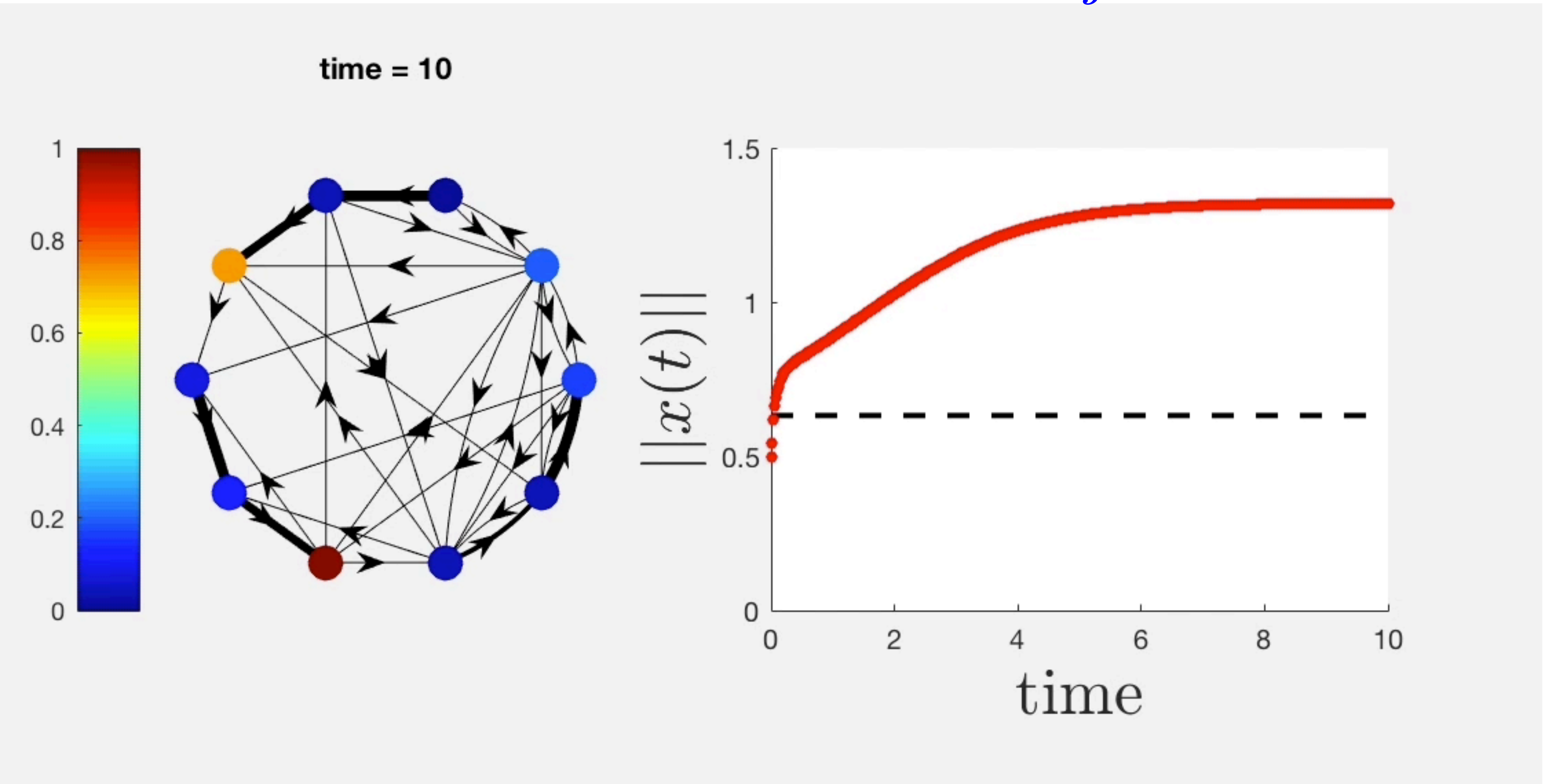




Because of a strong transient amplification

Allee effect with diffusion on a non-normal network (1 species)

$$\dot{x}_i = r x_i (1 - x_i)(x_i - a) + D \sum_{j=1}^{M} L_{ij} x_j$$



The origin remains stable but small perturbations can be macroscopically amplified and create patterns.

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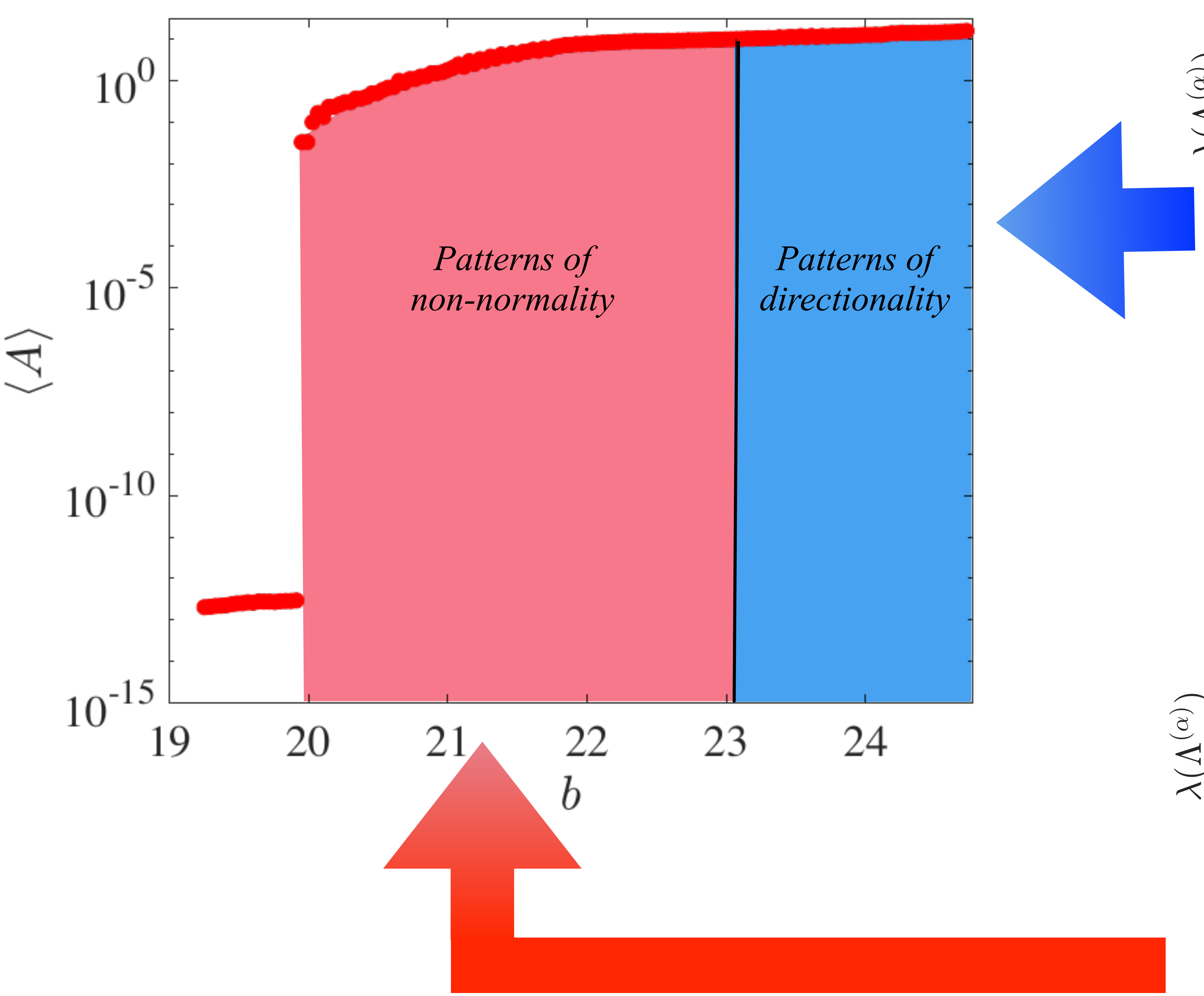
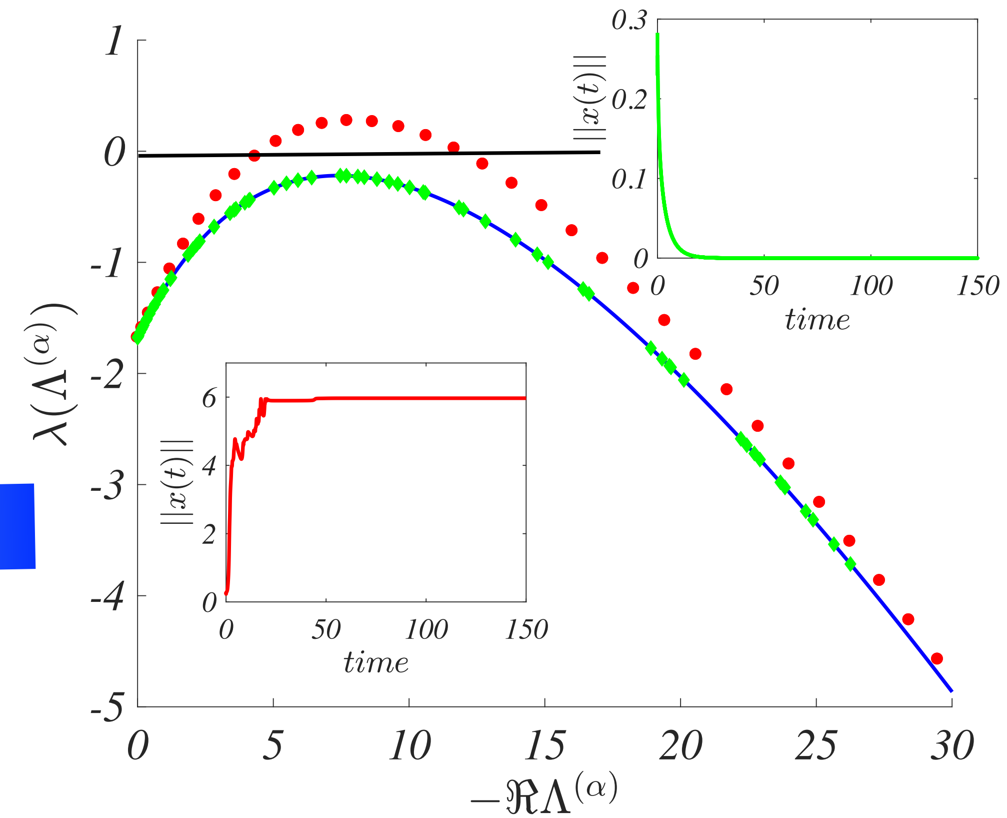
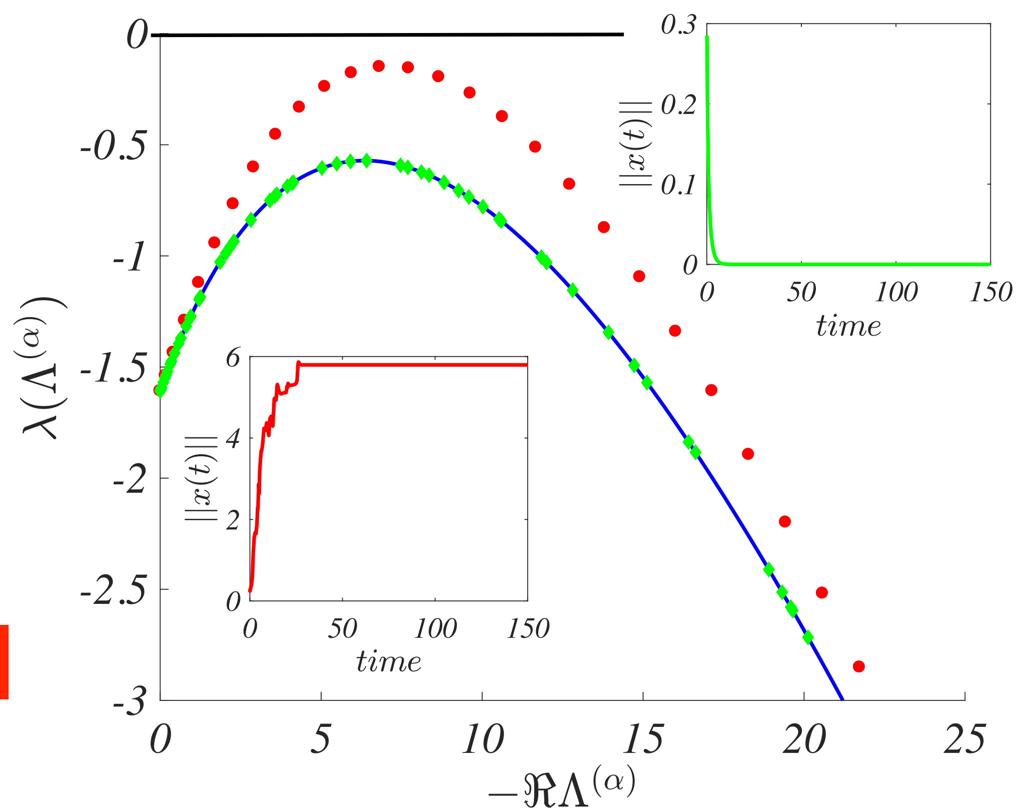
Patterns for $D_u \sim D_v$ on a non-normal network

$\langle A \rangle$ Pattern amplitude







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- Network non-normality can strongly modify the system behavior and create new patchy solutions (diversity).

- Many real networks exhibit strong non-normality and thus challenge our comprehension of their dynamics.

- M. Asllani and T. Carletti, *Topological resilience in non-normal networked systems*, Physical Review E, **97**, (2018), 042302

- M. Asllani, R. Lambiotte and T. Carletti, *Structure and dynamical behavior of non-normal networks*, Science Adv., **4**, (2018), eaau9403