September the 19th, 2020, Rome, Italy

TopoNets 2020

Networks beyond pairwise interactions

Satellite @ **NetSci** 2020 Rome - September 18, 19

ONLINE

Timoteo Carletti

Dynamical systems on Hypergraphs

naxys
Namur Institute for Complex Systems

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Acknowledgements

D. Fanelli
F. Battiston
G. Cencetti
S. Nicoletti

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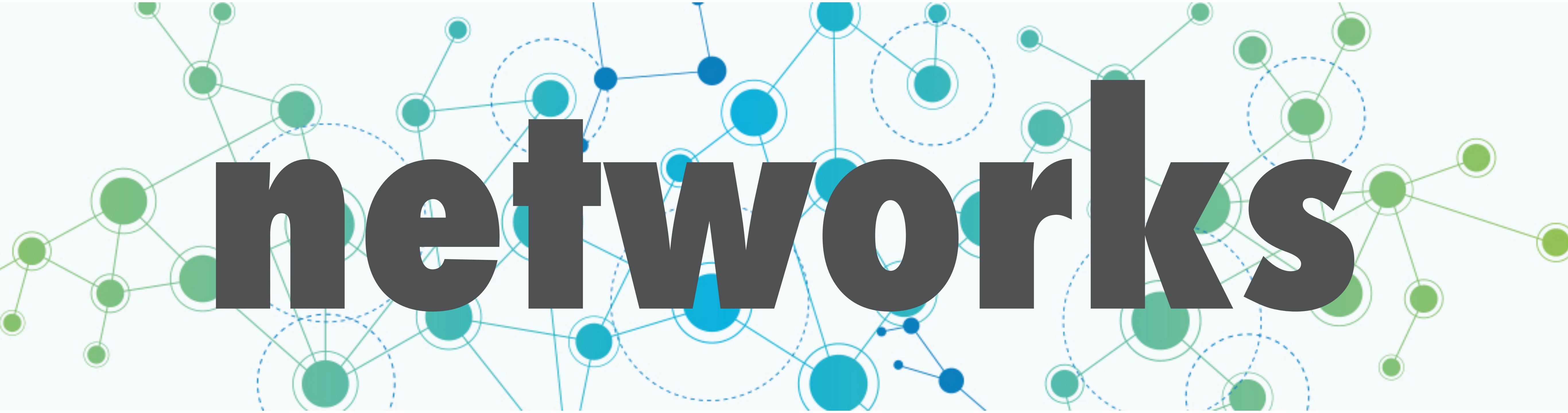

networks

Hypergraphs

timoteo.carletti@unamur.be

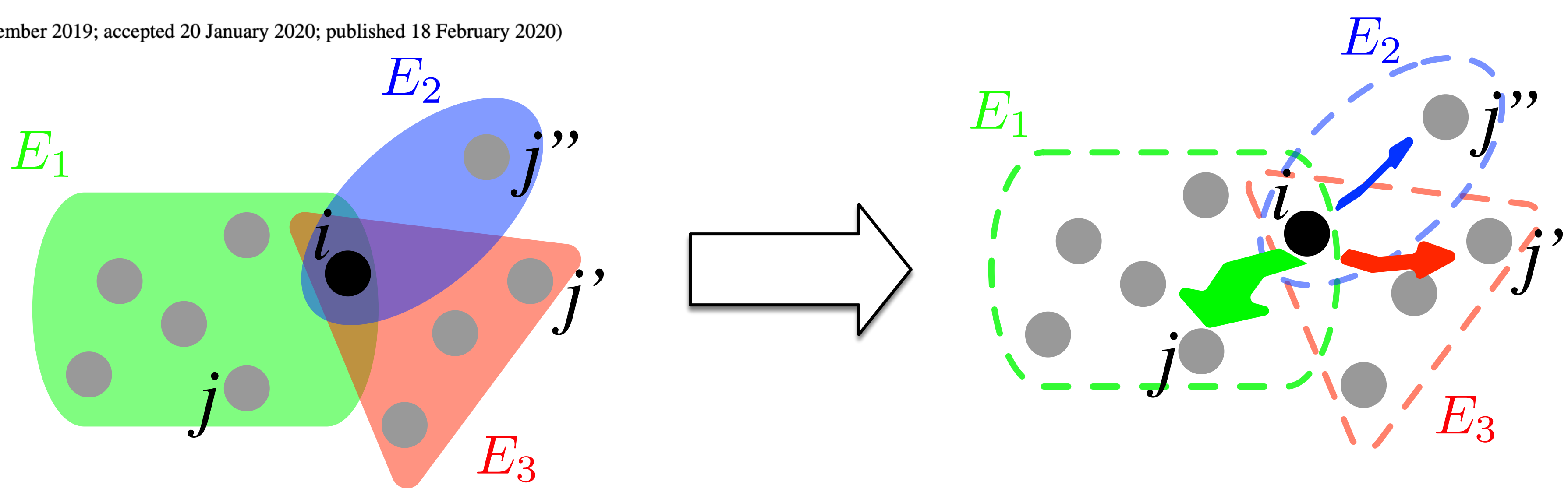
Random walk on hypergraphs

PHYSICAL REVIEW E **101**, 022308 (2020)

Random walks on hypergraphs

Timoteo Carletti,[1] Federico Battiston,[2] Giulia Cencetti,[3] and Duccio Fanelli[4]
[1]*Namur Institute for Complex Systems, University of Namur, 5000 Namur, Belgium*
[2]*Department of Network and Data Science, Central European University, Budapest 1051, Hungary*
[3]*Mobile and Social Computing Lab, Fondazione Bruno Kessler, Via Sommarive 18, 38123 Povo, Trento, Italy*
[4]*Dipartimento di Fisica e Astronomia, Università di Firenze, INFN, and CSDC, Via Sansone 1, 50019 Sesto Fiorentino, Firenze, Italy*

(Received 14 November 2019; accepted 20 January 2020; published 18 February 2020)



incidence matrices

$$k_{ij}^H = \sum_\alpha (C_{\alpha\alpha} - 1) e_{i\alpha} e_{j\alpha}$$

hyperedge size

Random walk on hypergraphs … New Laplace matrix

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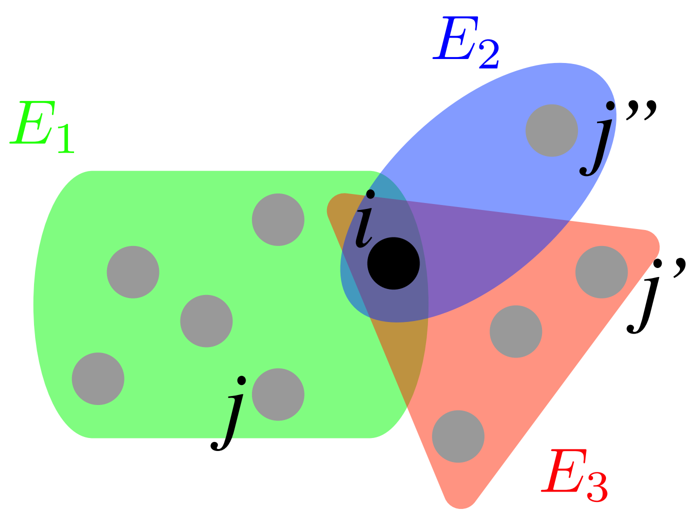

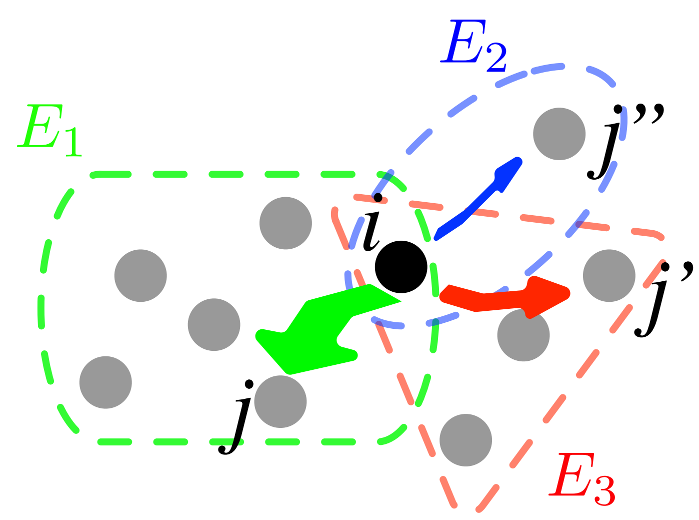

$$L^H_{ij} = \delta_{ij} - \frac{k^H_{ij}}{\sum_{\ell\neq i} k^H_{i\ell}}$$

Random walk Laplace matrix

$$L_{ij} = k^H_i \delta_{ij} - k^H_{ij}$$

$$k^H_i = \sum_{\ell\neq i} k^H_{i\ell}$$

combinatorial Laplace matrix

Random walk on hypergraphs … New Laplace matrix

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Random walks on hypergraphs

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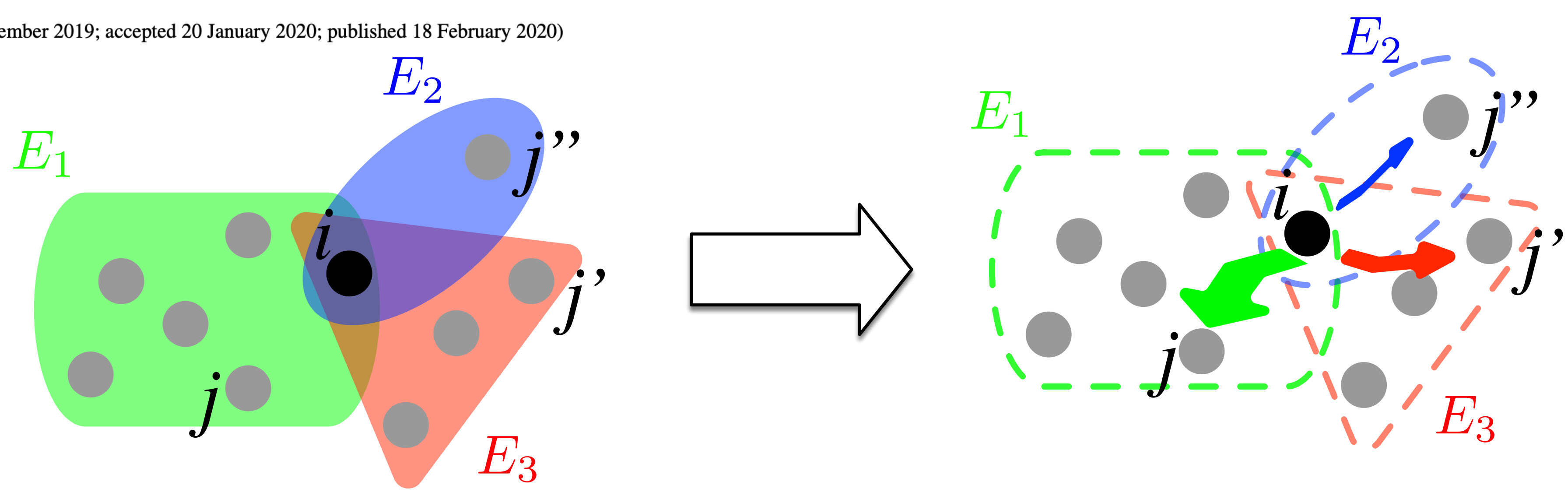

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Random walk Laplace matrix

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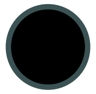
combinatorial Laplace matrix

Thu 24 : Session 12B: Dynamics II
13:00 Timoteo Carletti, Federico Battiston, Giulia Cencetti and Duccio Fanelli
Random walks on hypergraphs

Dynamical systems on hypergraphs

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}) \quad \mathbf{x} \in \mathbb{R}^d$$

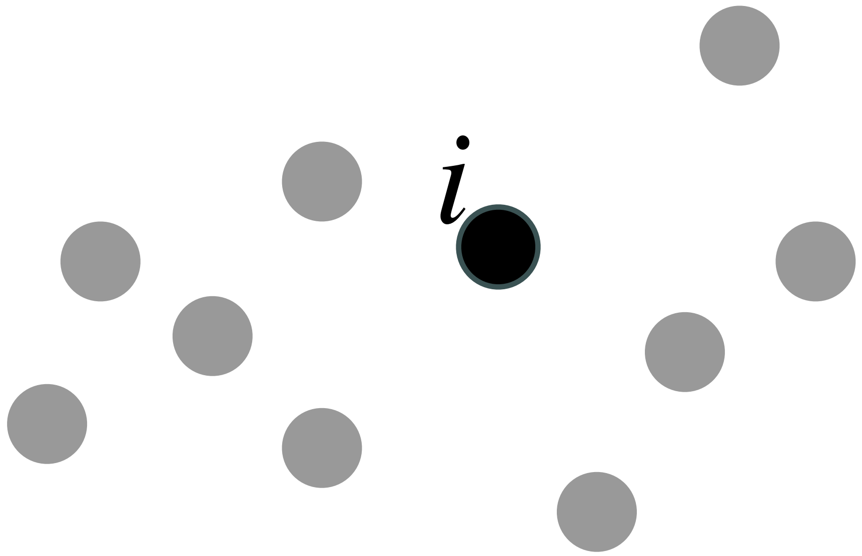


Dynamical systems on hypergraphs

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}) \quad \mathbf{x} \in \mathbb{R}^d$$

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) \quad \mathbf{x}_i \in \mathbb{R}^d$$

$$i = 1, \dots, n$$



Dynamical systems on hypergraphs

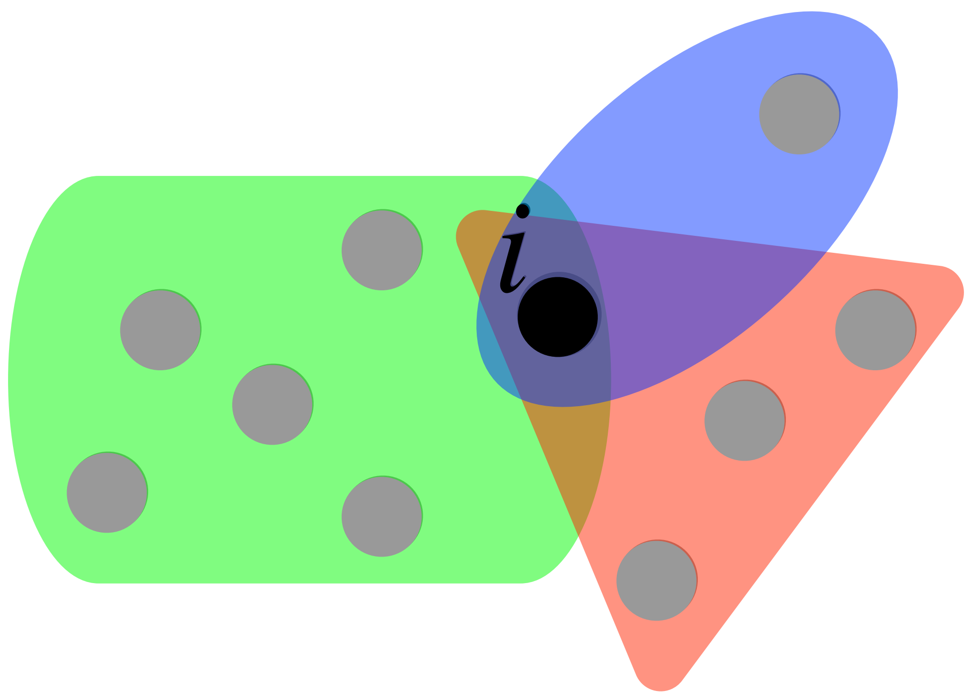
$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}) \quad \mathbf{x} \in \mathbb{R}^d$$

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) \quad \mathbf{x}_i \in \mathbb{R}^d$$

$$i = 1, \dots, n$$

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) - \varepsilon \sum_\alpha (C_{\alpha\alpha} - 1)\, e_{i\alpha} e_{j\alpha} \left[\mathbf{G}(\mathbf{x}_i) - \mathbf{G}(\mathbf{x}_j)\right]$$

nonlinear coupling function



$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) - \varepsilon \sum_j L_{ij} \mathbf{G}(\mathbf{x}_j)$$

Master Stability Function

$$\frac{d\mathbf{s}(t)}{dt} = \mathbf{F}(\mathbf{s}(t))$$

Homogeneous solution of the interconnected system

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) - \varepsilon \sum_j L_{ij}\mathbf{G}(\mathbf{x}_j)$$

Master Stability Function

$$\frac{d\mathbf{s}(t)}{dt} = \mathbf{F}(\mathbf{s}(t))$$

Homogeneous solution of the interconnected system

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) - \varepsilon \sum_j L_{ij}\mathbf{G}(\mathbf{x}_j)$$

$$\delta\mathbf{x}_i = \mathbf{x}_i - \mathbf{s}$$

$$\frac{d\delta\mathbf{x}_i}{dt} = D\mathbf{F}(\mathbf{s}(t))\delta\mathbf{x}_i - \varepsilon \sum_j L_{ij} D\mathbf{G}(\mathbf{s}(t))\delta\mathbf{x}_j$$

Master Stability Function

$$\frac{d\mathbf{s}(t)}{dt} = \mathbf{F}(\mathbf{s}(t))$$

Homogeneous solution of the interconnected system

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{F}(\mathbf{x}_i) - \varepsilon \sum_j L_{ij} \mathbf{G}(\mathbf{x}_j)$$

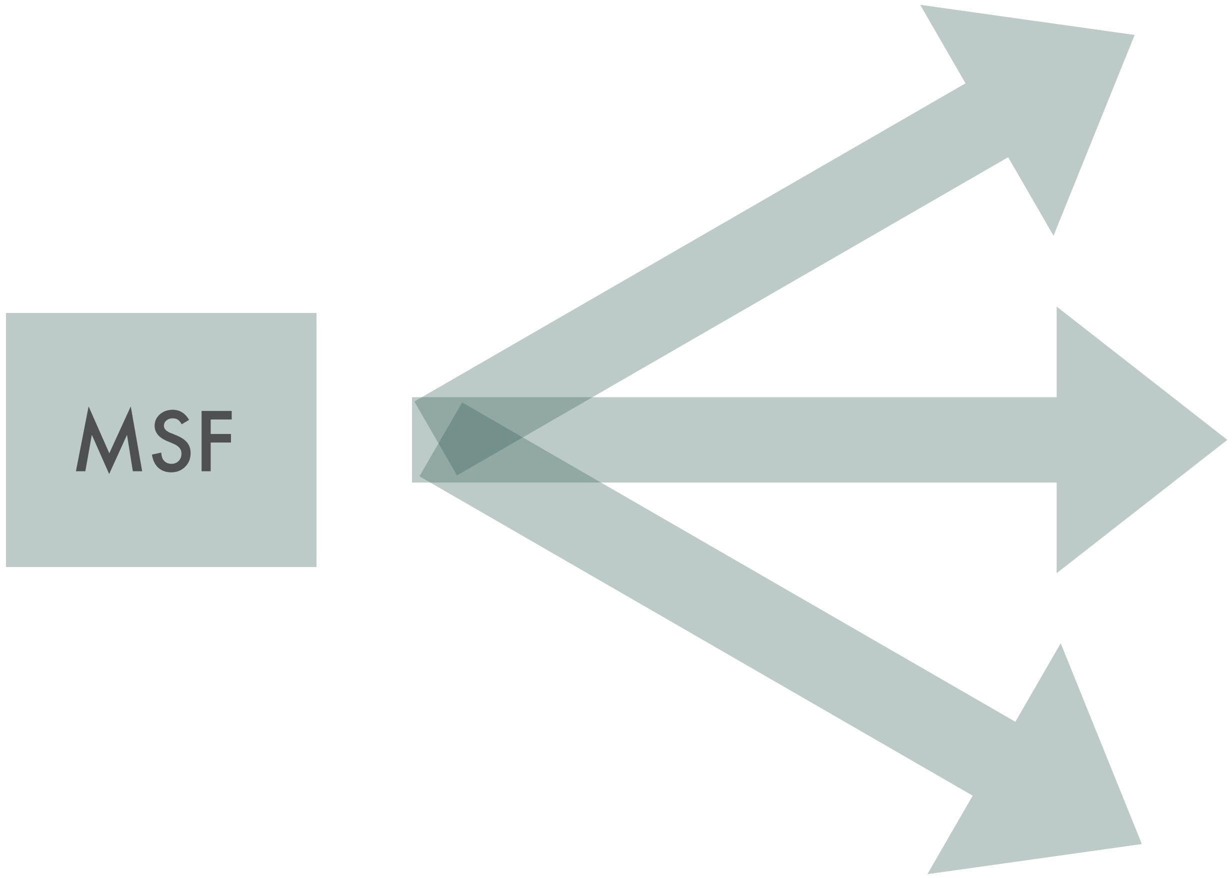
$$\delta \mathbf{x}_i = \mathbf{x}_i - \mathbf{s}$$

$$\frac{d\delta \mathbf{x}_i}{dt} = D\mathbf{F}(\mathbf{s}(t))\delta \mathbf{x}_i - \varepsilon \sum_j L_{ij} D\mathbf{G}(\mathbf{s}(t))\delta \mathbf{x}_j$$

$$\mathbf{L}\boldsymbol{\phi}^\alpha = \Lambda_H^\alpha \boldsymbol{\phi}^\alpha$$

$$\frac{d\delta \mathbf{y}_\alpha}{dt} = \left[D\mathbf{F}(\mathbf{s}(t)) - \varepsilon \Lambda_H^\alpha \mathbf{G}(\mathbf{s}(t))\right] \delta \mathbf{y}_\alpha$$

Master Stability Function on hypergraphs



Synchronisation of nonlinear oscillators

Synchronisation of chaotic orbits

Turing patterns

J.Phys.Complex. 1 (2020) 035006 (16pp) https://doi.org/10.1088/2632-072X/aba8e1

Journal of Physics: Complexity

PAPER

Dynamical systems on hypergraphs

Timoteo Carletti[1,4], **Duccio Fanelli**[2] and **Sara Nicoletti**[2,3]

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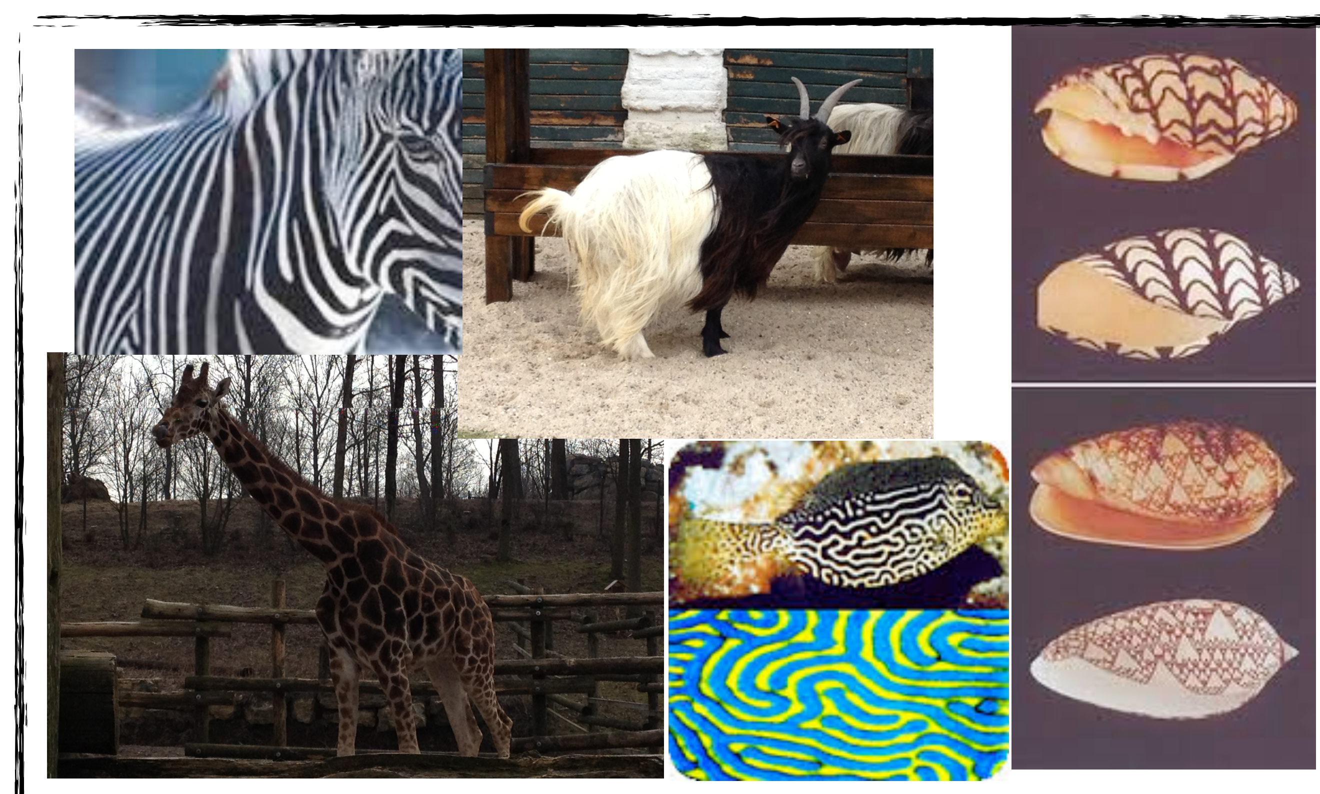
[2] Università degli Studi di Firenze, Dipartimento di Fisica e Astronomia, CSDC and INFN, via G. Sansone 1, 50019 Sesto Fiorentino, Italy
[3] Dipartimento di Ingegneria dell'Informazione, Università di Firenze, Via S. Marta 3, 50139 Florence, Italy
[4] Author to whom any correspondence should be addressed.

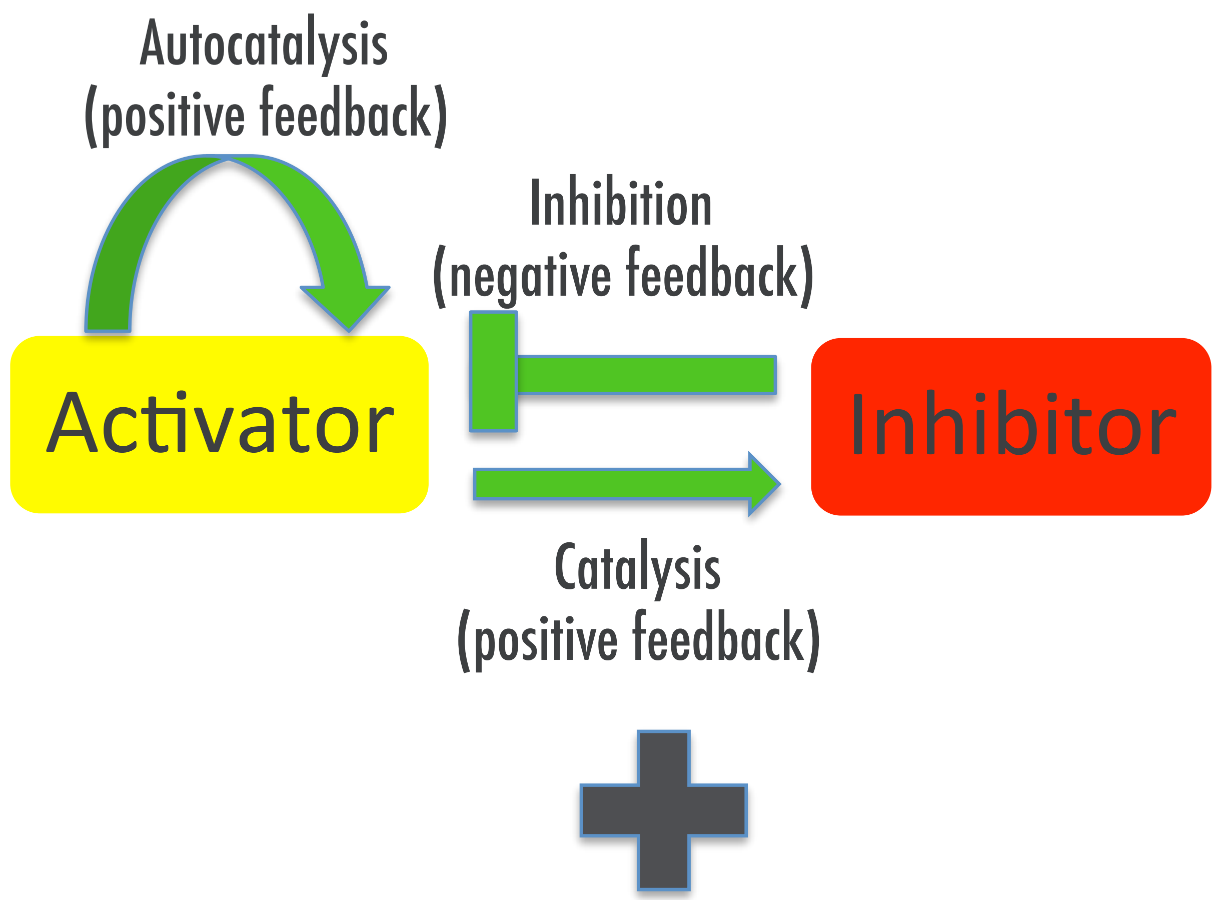
E-mail: timoteo.carletti@unamur.be

Keywords: hypergraphs, master stability function, synchronisation, Turing patterns, dynamical systems



A.M.Turing, The chemical basis of morphogenesis, Phil. Trans. R Soc London B, 237, (1952), pp.37

Reaction



Diffusion

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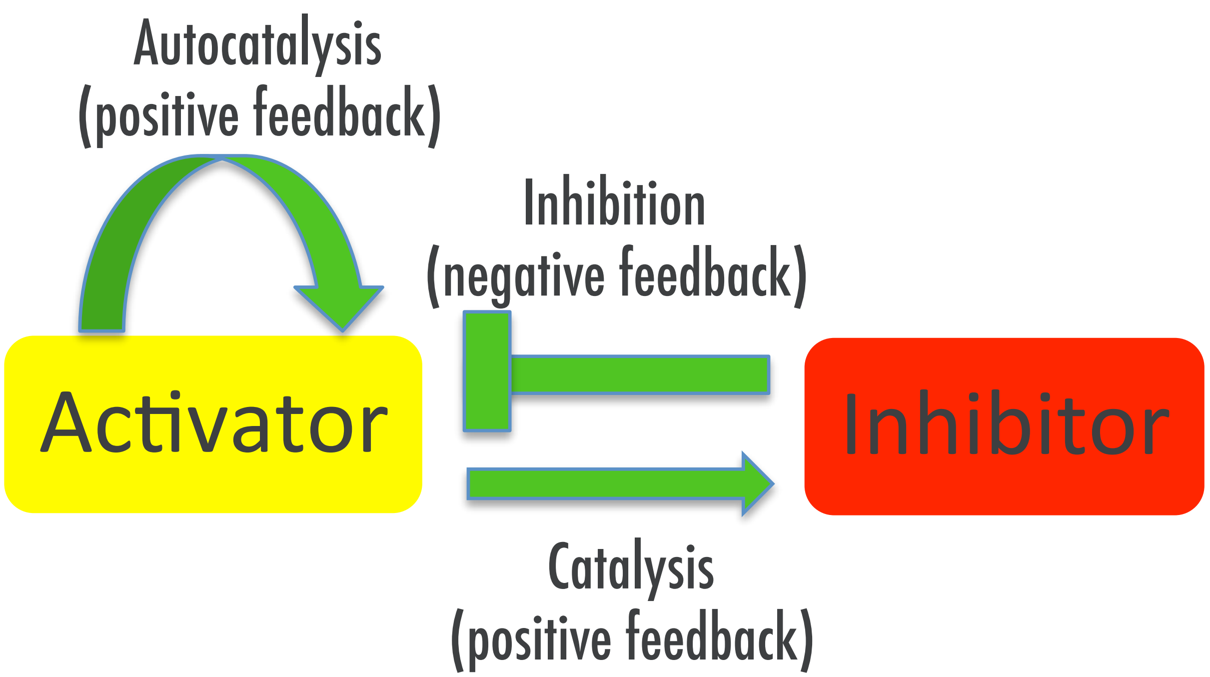

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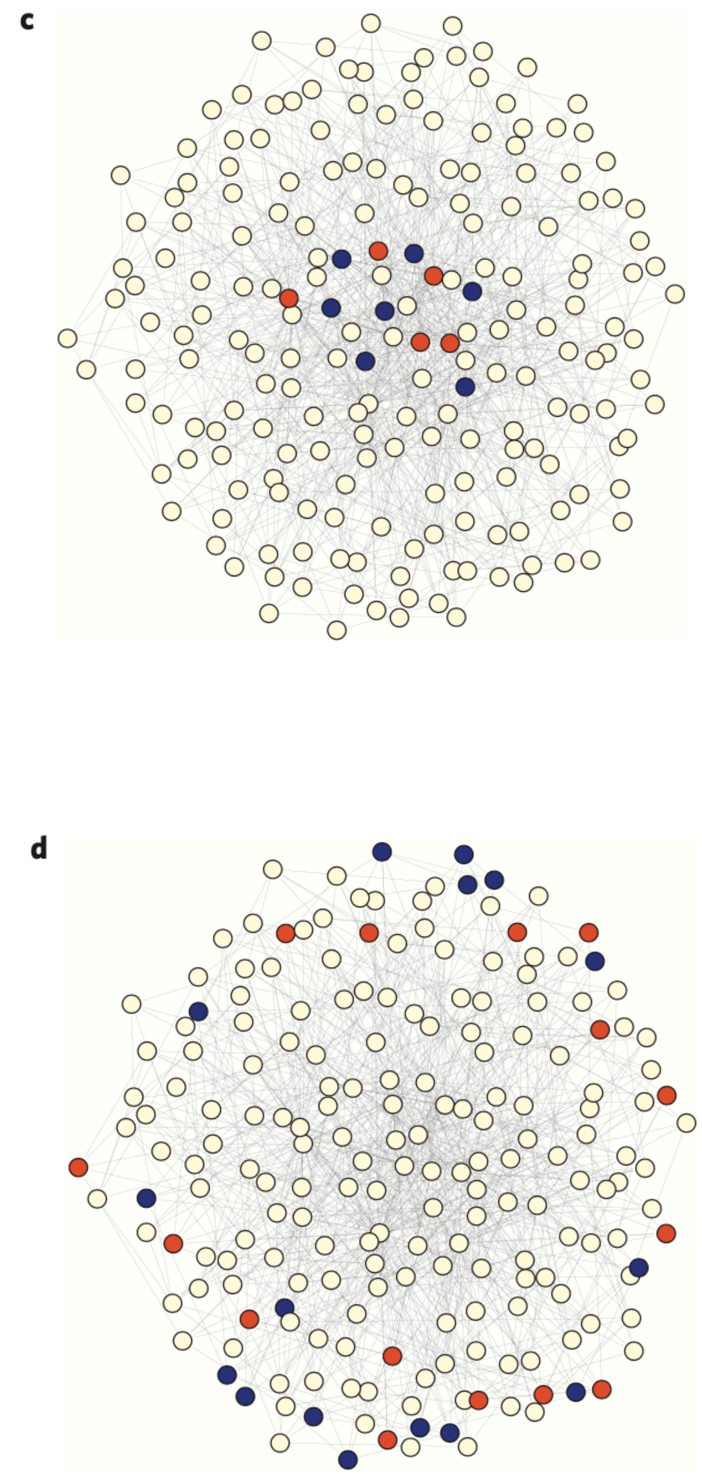
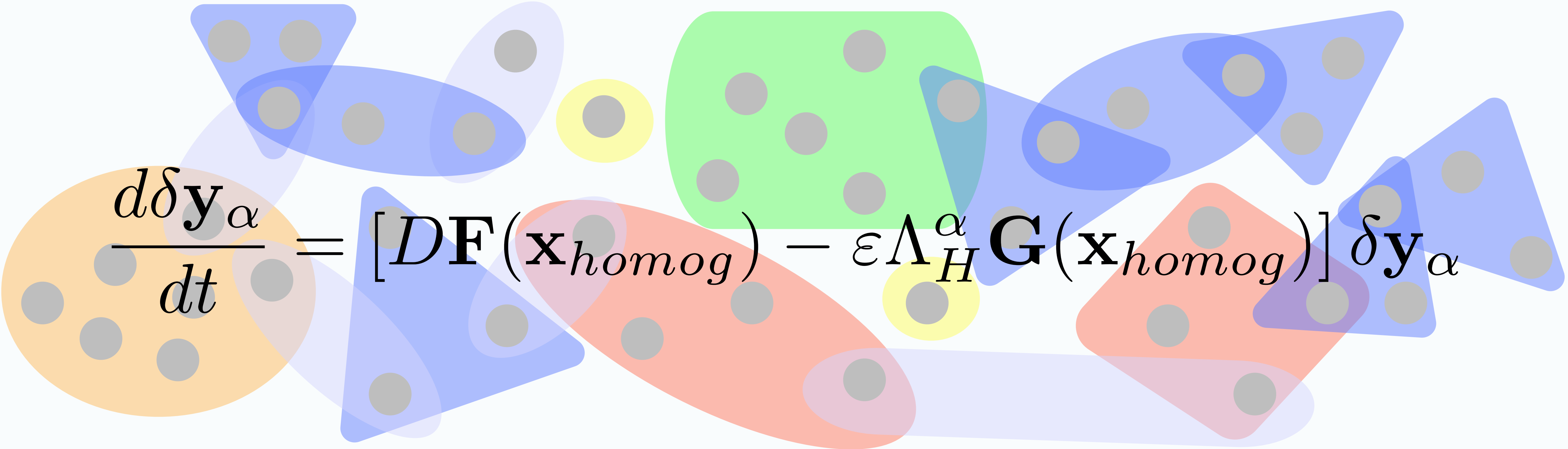
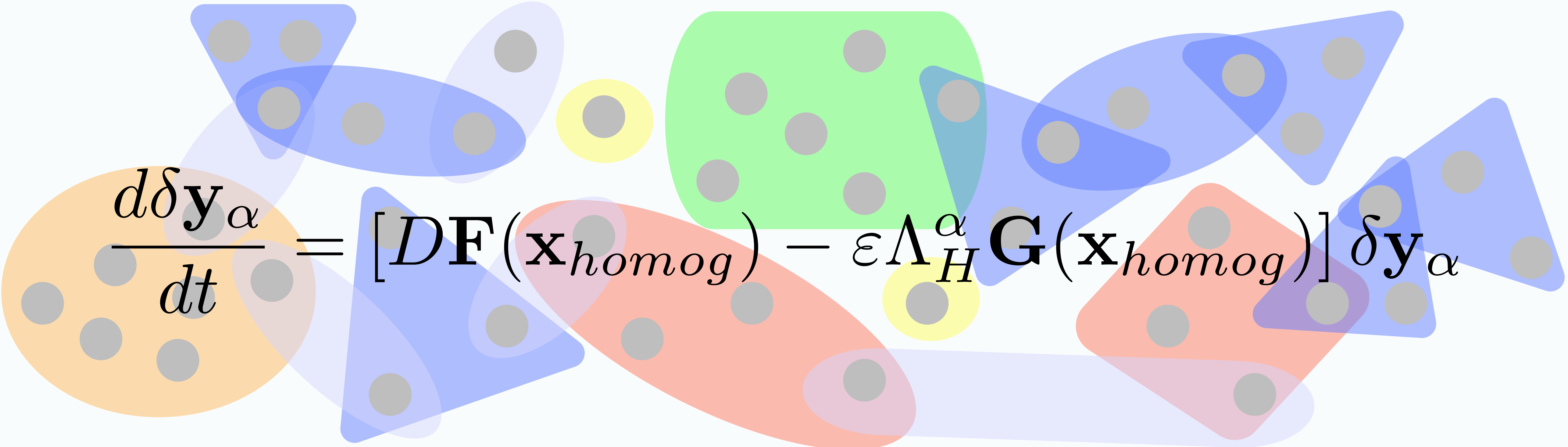
Diffusion



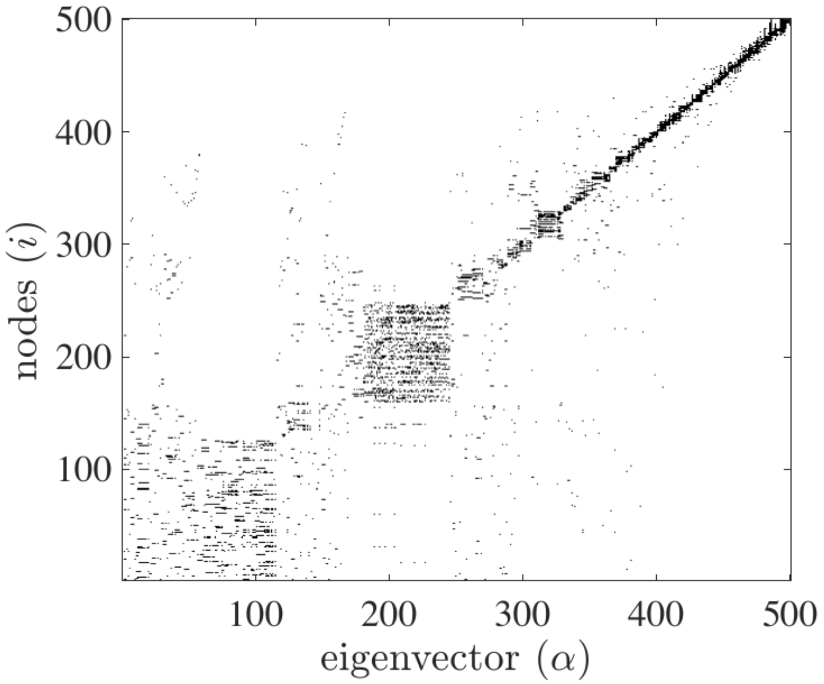
Nakao H. and Mikhailov A. S.,
Turing patterns in network-organized activator–inhibitor systems, Nature Physics, 6, (2010), pp.544

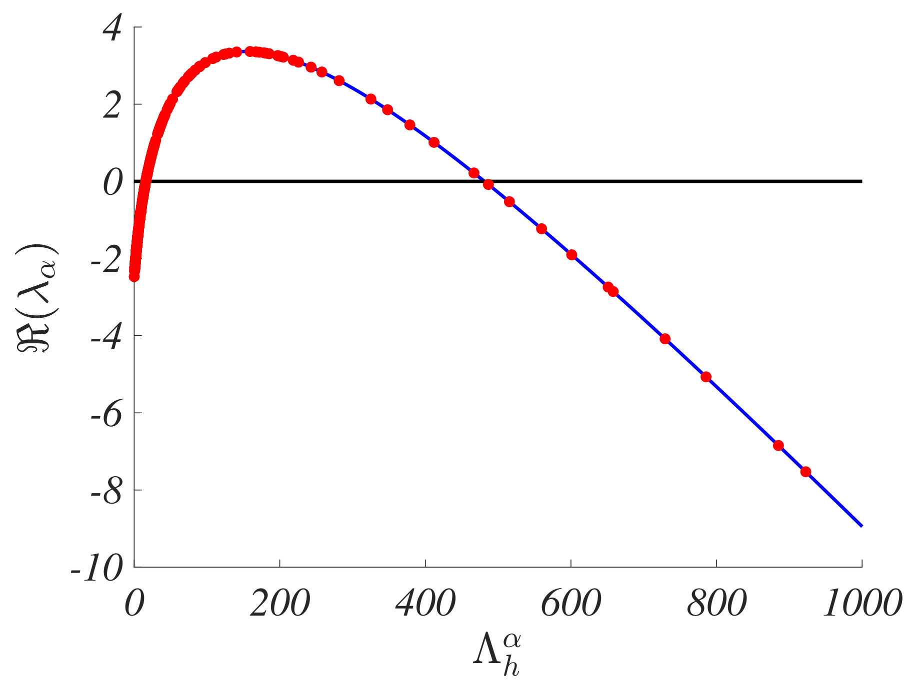


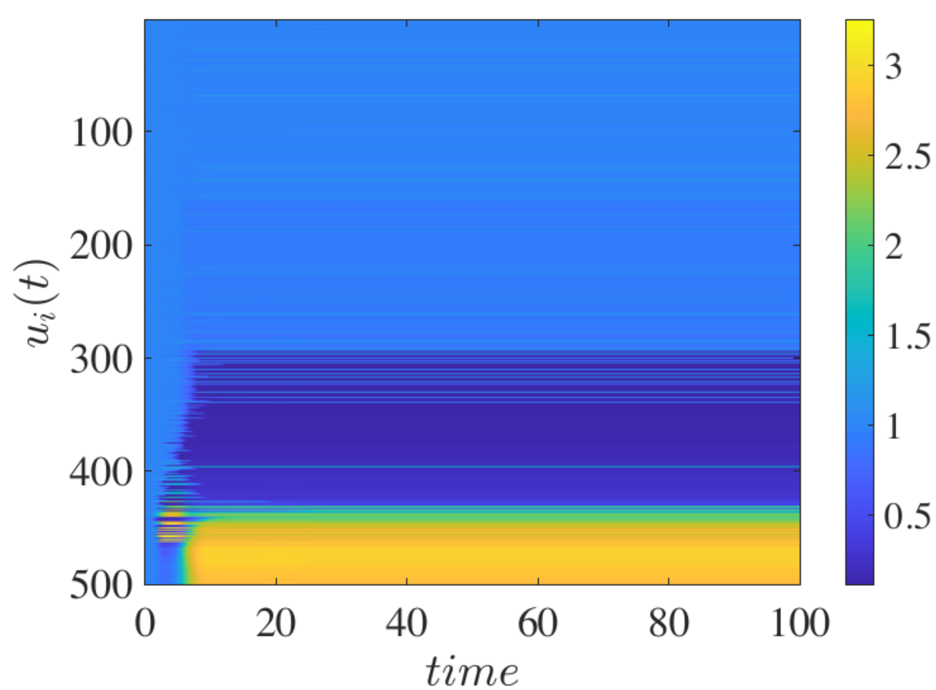
$$\frac{d\delta \mathbf{y}_\alpha}{dt} = \left[D\mathbf{F}(\mathbf{x}_{homog}) - \varepsilon \Lambda^\alpha_H \mathbf{G}(\mathbf{x}_{homog})\right] \delta \mathbf{y}_\alpha$$



$$\frac{d\delta \mathbf{y}_\alpha}{dt} = \left[D\mathbf{F}(\mathbf{x}_{homog}) - \varepsilon \Lambda_H^\alpha \mathbf{G}(\mathbf{x}_{homog})\right]\delta \mathbf{y}_\alpha$$







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Networks beyond pairwise interactions

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Thank you

Any questions?

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