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The effect of differential quality and differential zealotry in the best-of-n problem

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Abstract

In Collective decision-making, individuals in a swarm have to reach consensus on an issue using local interactions without any centralized control. The best-of-n problem is a special case where the swarm has to choose the best option among a set of n discrete choices. This problem is relevant for robot swarms, who need for example to agree on the location of a site to forage from. It has been shown that, when each site can be associated to a quality, direct modulation of positive feedback can be used, together with a decision mechanism such as the voter mode, to achieve consensus to the option corresponding to the highest quality. The role of zealots, stubborn individuals that are linked to an unchangeable opinion, has been studied abundantly by physicists and recently introduced to swarm robotics.

In this paper, we study the interplay between differential opinion quality and differential proportion of zealots in a best-of-2 problem, and we identify how the equilibria change with respect to these two parameters. We systematically study this via computer simulations in an antagonistic scenario whereby one option has a higher quality but a lower proportion of zealots than the other option.

1 Introduction

Collective decision making is a process to select an option collectively based only on local perception. Originally inspired by the behavior of natural swarm such as ants and bees [4, 2], collective decision making is considered an important building block to achieve more elaborated collective behaviors in swarms of robots [22], such as deciding to which site or in which direction to move collectively [3].

The best-of-n problem is a special case in collective decision-making [22] whereby individuals in a swarm need to achieve consensus to one option in a discrete set of n alternatives. In general, the n options can be assumed to be available to the members of the swarms [23], or may need to be discovered [19]. An option in the best-of-n can be *best* according to different criteria. One criterion can be the minimization of some cost, for example the cost to access the option (e.g. the distance of a location from the base), while another criterion is some intrinsic option quality [22]. In the latter case, the option quality can be used as an input of the decision-making algorithm. For example, after sampling an option quality, a robot can advertise the option for a duration that is proportional to the option's quality, a mechanism called modulation of positive feedback [6, 24, 25, 23]. With such a mechanism, the swarm is able to achieve consensus to the option corresponding to the highest quality.

A recent study [17] introduced the notion of stubborn individuals, which we here call zealots consistently with previous work done in physics (which we review in Section 2). Zealots are individuals that never change their opinion and always advertise only the option associated to them from the very beginning. The authors showed that, in a dynamic environment where the option qualities can drastically change during the experiment, a swarm is able to adapt to these changes and to select the new option with the best quality only when a small but equal number of zealots is introduced.

In this paper we use computer simulations to study the best-of-n problem with differential quality and zealot quantity. We consider the $n = 2$ case and we introduce zealots in a quantity that is not equal for the two options. At the same time, the two options are associated each to a different quality. The experimental setup is considered "antagonistic", because the number of zealots is higher for the option that has a lower quantity, hence it is not obvious which option will prevail. In this setup, we ask the following two questions: Does the swarm always achieve consensus to one of the two options, or it may sometimes converge to an intermediate undecided state? Is the consensus state or the intermediate state more biased towards the option represented by more zealots or the one represented by the highest quality? Concerning the second question, we are particularly interested in identifying the "indifference curve", which is the curve separating the region where the consensus is more likely to be to one opinion to the region where consensus is more likely to be to the other opinion.

The remaining of the paper is organized as follows. In Section 2, we discuss the related work. In Section 3, we describe the collective decision-making model utilized in this study. In Section 4, we explain the experimental setup needed to replicate our experiments. In Section 5 we discuss the results obtained. In Section 6, we conclude the paper and discuss future developments.

2 Related Works

The best-of-n problem is inspired by the biological behavior of swarms of ants and bees [7, 12, 20].

As extensively discussed in [22], quality and cost of the options can be used to further describe the nature of the best-of-n decision-making problem. For example, in a foraging scenario, where food availability is the option quality and time to reach the food patch is the option cost, the problem can be symmetric/asymmetric for quality (all food patches have/have not the same amount of food), and/or symmetric/asymmetric for cost (all food patches require/do not require the same amount of time to be reached). When both costs and quality are asymmetric, we can have scenarios in which the option cost and quality are synergistic (e.g., the best option has maximum quality and minimum cost) and scenario in which they are antagonistic (e.g., the best option has maximum quality and highest costs).

Another important element that bears upon the decision-making dynamics is the presence within the swarm of zealots. Their contribution to the decision-making process can largely influence the behaviour of the entire swarm, as discussed in various papers. The topic of zealot has been abundantly studied in physics, but introduced within swarms only recently. In the following, we will first review the few contributions focusing on zealots within swarms, and then review some of the works done within phyics.

In the context of swarms, the study illustrated in [17] looks at the role of zealots in a particular type of best-of-n scenario characterised by a dynamic environment (i.e. the quality of options changes over time). The results of this study shows that without zealots, a swarm of simulated agents can converge to the best option but is not able to adapt and to change its consensus state in case that option is no longer the best. The study shows that introducing zealots allows the swarm to re-evaluate the options quality at all times, including when a near-consensus state is reached and the option quality changes afterwards.

Canciani et al. [5] introduces three types of malicious agents that can affect resilience of a swarm: contrarians, wishy-washy, and zealots. This study focuses on comparing four mechanisms: voter, majority, cross inhibition, and k-unanimity (q-voter). The study illustrates which of those types of agents makes the swarm more resilient to attacks by different types of malicious agents above. In [18], the authors also look at the effects of malicious adversarial zealots in a data communication manipulation scenario. In particular, this study illustrates an alternative probabilistic decision-making rule, and evaluates it in different contexts varying in the number of adversarial and legitimate zealots with swarms of simulated agents. The study shows that the probabilistic decision-making rule can limit the disrupting effects of adversarial zealots even in conditions in which the difference between the number of adversarial and legitimate zealots within the swarm is just one agent.

In the context of physics, Hunter and Zaman [11] studied the best placement of stubborn agents in order to maximize the impact on the opinion of the whole population, in social network systems, like Instagram or Twitter. They also analyzed the communication over two real social networks in Twitter, concerning issues such as the Brexit, and the demonstrations in France managed by the Gilets Jaunes. The study shows that a small number of zealots can significantly influence the overall opinion dynamics and induce the entire population to reach a large consensus over disputed issues. In [15], the authors study the role of zealots as committed minorities in a social system modeled with a different collective decision making mechanism called naming game. Interestingly they introduce the role of heterogeneous activities of different nodes. They show that even a very small minority can drive the opinion of a large population, if committed agents are more active than the others. On the other hand, this effect can be hindered if nodes with same opinion are more connected with each other than with nodes with different opinion, producing a polarization inside the network.

The work of [9, 16] study the impact of zealots in a social network. But unlike in [11], these studies introduces zealots with different degree of zealotry. The focus of [9] is studying the effect of zealotry on the convergence time of the system. In [16], despite a different model (which also included the majority rule instesd than the voter model), the authors were able to find similar dynamics to those found by [17, 13], in which the existence of equal number of zealots on both option sides prevents the network from reaching a consensus state. Similarly, in [27], the presence of zealots is proven to prevent the formation of consensus, introducing instabilities and fluctuations in a binary voter model of a small-world network. A recent study illustrated in [1] aims at studying the influence of zealots on "politically polarized" state vs consensus state and found that higher "influence of zealots" produces more polarization, shorter time to polarization, and conversely less consensus and longer to impossible time to consensus. Differently from our method, zealots are modeled as single nodes that represent "competing news sources". Networks are assumed either fully connected, or 2-clique, with each clique closer to one of the two big zealots. Voter model is "biased" because there is p (probability to pick zealot) vs $(1-p)$ probability to pick neighbor.

In [26], the author shows that there exists a tipping point at which a consistent and zealot minority of zealots is able to swing the initial majority opinion of a network. The study described in [14] focuses on zealots and voter model to perform peer-to-peer opinion influence like in our work. Contrary to us, in [14], the authors try to control opinion dynamic by introducing zealots as nodes on a complex network. In[8], the authors studied the case of two opinions with zealots, which they call "inflexible minorities", promoting their respective opinions using the Majority rule. The results of the study shows that the decision-making process is equally likely to reach a consensus for any of the two option when the number of zealots on each side are exactly the same. A very minimal unbalance in the number of zealots for the two options is sufficient to drive the consensus in favour of the opinion supported by the largest numbers of zealots.

As it is clear, zealotry has been abundantly studied in physics, whose studies typically consider fixed interaction topologies often represented as networks.

Parameter	Description	Values
N	swarm size	{100, 1000}
ρ_A	site A quality	1
ρ_B	site B quality	$\{1, 1.05, 1.10, .., 2\}$
σ_B	proportion of zealots with opinion B to N	$\{0.0125, 0.025, 0.05\}$
σ_A/σ_B	ratio of zealots with opinion A to σ_B	$\{1..10\}$

Table 1: Model parameters used in simulations

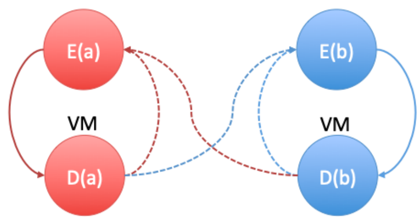

Figure 1: Probabilistic finite state machine. D_a, D_b , E_a and E_b represent the dissemination and exploration state. Solid lines represent deterministic transitions, while dotted lines stochastic transitions. The symbol VM indicates that the voter model is used at the end of the dissemination state. Note that stochastic transition may be the results of either the application of the decision rule, or of the spontaneous opinion switching mechanism, if enabled.

The concept has been only recently introduced in the context of swarms, whereby the topology is dynamic and determined by the instantaneous neighbors, like in our case. Being swarm robotics an engineering context, options are also typically associated to a quality, an aspect that tends to be not treated in the physics literature. Compared to the most recent work in swarms [17, 5, 18], to the best of our knowledge, in this paper we study for the first time the interplay between different option quality and different zealot quantity.

3 Method

In the best-of-n problem, a swarm of agents has to reach a collective decision among n possible alternatives towards the choice that has the best quality. In this paper, we restrict n to 2 options, labelled A and B, two options that have intrinsic quality value ρ_A and ρ_B. A best-of-n problem reaches the optimal solution when the collective decision of the swarm is for the option with maximum quality. That means that a large majority $M \leq N(1-\delta)$ of agents agrees on the same option, where δ is a small number chosen by the experimenter. In the case where $\delta = 0$ there is *perfect consensus*. The agents are required to choose the best option in different best-of-n experimental scenarios which vary according to the difference in quality values between the two options and for nature of the

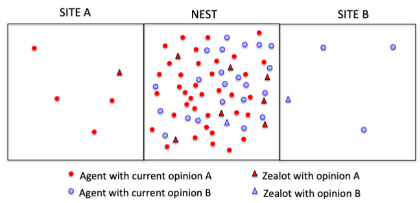

Figure 2: Visualisation of the simulation arena.

Parameter	Description	Values
N	swarm size	{100, 1000}
ρ_A	site A quality	1
ρ_B	site B quality	$\{1, 1.05, 1.10, .., 2\}$
σ_B	proportion of zealots with opinion B to N	$\{0.0125, 0.025, 0.05\}$
σ_A/σ_B	ratio of zealots with opinion A to σ_B	{1..10}

Table 2: Model parameters used in simulations

swarm. For the latter, the swarm is made of two different types of agents: the zealots, always committed to either option A or B, and characterised by the fact that they never change option; and normal agents, initially committed to either option A or B, but subject to change their commitment by applying a decision mechanism that relies on the observation of other agents in local proximity.

The behaviour of the agents is controlled by a probabilistic finite state machine controller shown in Figure 3. This controller has four possible states: dissemination state of opinion A (D_A), dissemination state of opinion B (D_B), exploration state of opinion A (E_A), and exploration state of opinion B (E_B). In our simulations, agents are located in a rectangular arena whose central part is called the *nest*, while the left and the right sides are the two sites, one associated to quality A and the other to quality B, respectively (see Figure 3). As initial conditions, agents are initialized inside the nest. Half of the agents

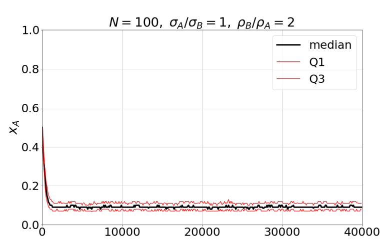

Figure 3: Consensus results over time $T = 0 - 40000$ for quality ratio $\rho_B/\rho_A = 2.00$, $N = 100$, $\sigma_b = 5$ with zealots ratio $\sigma_a/\sigma_b = 1$. Each scenario is repeated 100 times.

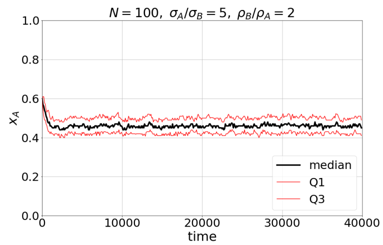

Figure 4: Consensus results over time $T = 0 - 40000$ for quality ratio $\rho_B/\rho_A = 2.00$, $N = 100$, $\sigma_b = 5$ with zealots ratio $\sigma_a/\sigma_b = 5$. Each scenario is repeated 100 times.

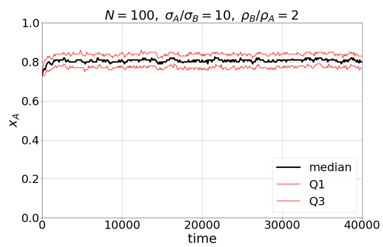

Figure 5: Consensus results over time $T = 0 - 40000$ for quality ratio $\rho_B/\rho_A = 2.00$, $N = 100$, $\sigma_b = 5$ with zealots ratio $\sigma_a/\sigma_b = 10$. Each scenario is repeated 100 times.

are initialized in state E_A, the other in state E_B, and they move toward the site associated with their opinion to explore that option. Once they reach the site, they explore it for an exponentially distributed amount of time (sampled independently per agent) with parameter $1/q$ (thus with mean time q) that does not depend on the option or option quality. During this time, agents measure the quality of that site. Subsequently, they switch to the dissemination state associated with their current opinion (D_A if they were in E_A, D_B if they were in E_B), travel back to the nest, each at a different time due to independent sampling, where they initiate opinion dissemination.

While in the nest, to meet the well-mixed this criterion as much as possible [10], agents perform a correlated random walk while disseminating and before applying the decision mechanism. In the dissemination state, each agent locally broadcasts his opinion continuously, and this message is sensed by other agents that are also in the dissemination state and situated within a limited range from the broadcasting agent. The time spent by the agent disseminating its opinion is randomly sampled from an exponential distribution characterized whose mean is proportional to site quality they have last visited $g \cdot \rho_i, i \in \{a, b\}$. As a consequence, it is more probable to meet neighbors with the best opinion than those with the worst one, because the former will disseminate longer than

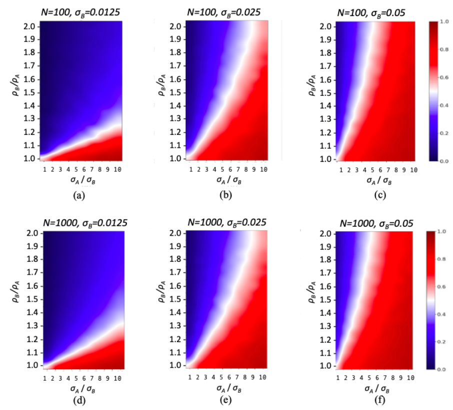

Figure 6: Consensus heatmap with $N = 100$ (Panel a, b, c) and $N = 1000$ (Panel d, e, f), $\sigma_B = 0.0125$ (Panel a, d), $\sigma_B = 0.025$ (Panel b, e),$\sigma_B = 0.05$ (Panel c,f). The colour scale represents N_A/N. Blue cells indicate perfect consensus (agreement to the best opinion, B), while red cells mean that the consensus has not been reached and most of the agents chose the worst opinion, A.

the latter. This mechanism is called modulation of positive feedback, and it is the driving mechanism to make the group converge on the option with the best quality. At the end of dissemination, each agent can change its opinion based on the opinions of other agents and using the voter model (see dotted lines in Figure 3). The result of the voter model depends on the neighbors opinion, that is, the agents within a specified spatial radius (in our simulations set to 10 units): The agent switches its opinion to the one of a random neighbors within the interaction radius.

We consider two kinds of agents: *normal* and *zealots*. Each agent has an initial opinion, which consists in one of the two options A or B. Normal agents are able to change their opinion by applying a decision mechanism that relies on the observation of other agents in local proximity. Zealots instead never change their opinion and keep the one they have at the very beginning, either A or B. Differently from previous work, were here consider a differential number of zealots, meaning they are not equal between the two options A and B. Zealots are indicated by proportion expressed with respect to the swarm size N. The proportion of zealots for A (resp. B) will be denoted by σ_A (σ_B), where $0 \leq \sigma_A \leq 1$ (resp. $0 \leq \sigma_B \leq 1$). We are interested in the antagonistic case where there are more agents for the option that is associated with the lowers quality. Without loss of generality, in Section 4 and Section 5 we will only consider cases where $\sigma_A \geq \sigma_B$ and $\rho_A \leq \rho_B$.

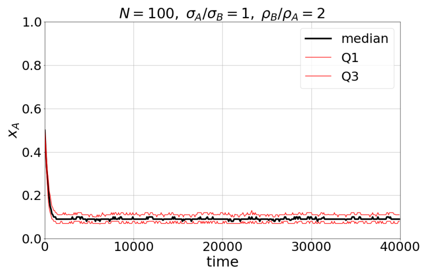

Figure 7: Consensus results over time $T = 0 - 40000$ for quality ratio $\rho_B/\rho_A = 2.00$, $N = 100$, $\sigma_b = 5$ with zealots ratio $\sigma_a/\sigma_b = 1$. Each scenario is repeated 100 times.

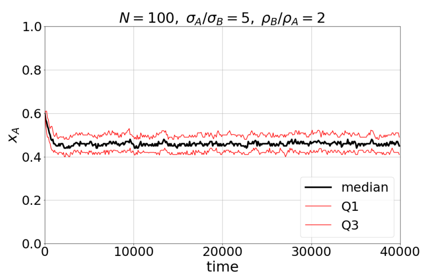

Figure 8: Consensus results over time $T = 0 - 40000$ for quality ratio $\rho_B/\rho_A = 2.00$, $N = 100$, $\sigma_b = 5$ with zealots ratio $\sigma_a/\sigma_b = 5$. Each scenario is repeated 100 times.

4 Experimental Setup

The experiments were conducted using a simulation tool originally developed by Valentini et al. [24]. The simulated arena is a rectangular, two-dimensional space. The collision of the agents is not modeled, however, previous results show that this type of simulation is sufficient to approximate well the result of real robot experiments. [21]

We study two swarm sizes in order to determine if this factor plays a role, $N = 100$ and $N = 1000$. In order to make sure any effect of the swarm size is not due to density, we keep the density constant by setting the nest size to 100×100 square units for $N = 100$ and to 316×316 for $N = 1000$. The size of the two sites is identical to the size of the nest in both cases. In this way, the area in the second setup is 10 times the area of the first one. In each experiment, there are σ_A zealots committed to A and σ_B zealots committed to B. In every run, we first initialize the zealots according to σ_A and σ_B. Afterwards, we set 50% of the remaining (normal) agents to opinion A and the remaining (normal) agents to opinion B.

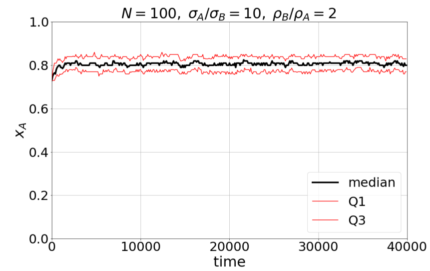

Figure 9: Consensus results over time $T = 0 - 40000$ for quality ratio $\rho_B/\rho_A = 2.00$, $N = 100$, $\sigma_b = 5$ with zealots ratio $\sigma_a/\sigma_b = 10$. Each scenario is repeated 100 times.

To study the effect of quality versus zealot quantity on the swarm's decision-making, we vary the quality ratio of option A and option B, as well as the ratio of zealots on both sides. The quality value of option A (ρ_A) is fixed at 1 and the quality of option B (ρ_B) varies from 1 to 2 with steps of 0.05. We vary the ratio between the proportion of zealots for A to zealots to B $(\sigma_A)/(\sigma_B)$ from 1 to 10 with steps of 1. We further evaluate the results under various proportion of zealots committed to B: $\sigma_B = \{0.0125, 0.025, 0.05\}$. The rest of the parameters are fixed. The average exploration time is $q = 10$, average dissemination time of opinion A and B are respectively $g\rho_A$ and $g\rho_B$, with $g = 100$ for all experiment setups. The parameters and their values used in this experiment is summarized in Table 2. Simulation runs for the duration of $T = 40000$ and repeated 100 times for each parameter combination.

5 Results

To obtain an initial picture on the effect of differential number of zealots on the collective-decision making dynamics, we performed a preliminary experiment with $N = 100$, fixed quality ratio $\rho_B/\rho_A = 2.00$, and equal number of zealots for the two options $\sigma_A = \sigma_B = 0.05$, which corresponds to a ratio $\sigma_A/\sigma_B = 1$. We record the evolution of the consensus dynamics over time from $T = 0$ to $T = 40000$. The results, depicted in Figure 4, are consistent with previous research [17], with the consensus state $x_A = N_A/N$ (the number of agents with opinion A divided by the swarm size N) converging to a low non-zero value which represents almost-consensus to the option corresponding to the highest quality (B) except for the effect of the zealots committed to A.

We then introduce differential latency by repeating the same process with increased proportions of zealots for A and thus two biased zealot ratios: $\sigma_A/\sigma_B = 5$ and $\sigma_A/\sigma_B = 10$. The results in Figure 4 and 4 clearly show first a shift of the consensus state x_A to a neutral state ($x_A \approx 0.5$ in Figure 4, followed by a shift biased towards opinion A (see Figure 4). These experiments already answer the first question we asked in our work, that is, perfect consensus is no

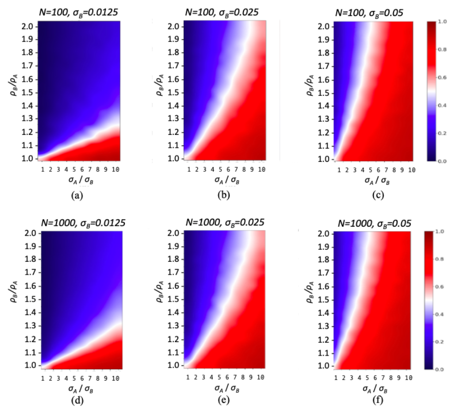

Figure 10: Consensus heatmap with $N = 100$ (Panel a, b, c) and $N = 1000$ (Panel d, e, f), $\sigma_B = 0.0125$ (Panel a, d), $\sigma_B = 0.025$ (Panel b, e),$\sigma_B = 0.05$ (Panel c,f). The colour scale represents N_A/N. Blue cells indicate perfect consensus (agreement to the best opinion, B), while red cells mean that the consensus has not been reached and most of the agents chose the worst opinion, A.

longer guaranteed in a system with the voter model with differential quality and zealots, while a possibly infinite number of intermediate states can now become attractors (as it will be clear shortly). This preliminary experiment also already shows the competition between differential quality and differential zealot number: a tipping point for some values of σ_A/σ_B and ρ_B/ρ_A exists. From one side of this tipping point, differential quality has a predominant effect, while from the other side the differential zealot number has a predominant effect.

To obtain a broader picture of the tipping points, we decided to study the two-dimensional space encompassing systematic combinations of ρ_B/ρ_A and σ_A, σ_B. In this two-dimensional space, we expect the tipping points to describe a curve: we call this curve the indifference curve. This setup, described in Section 4, corresponded to 1260 parameter combinations in total. We run those scenarios 100 times each and took the median consensus value at the end of $T = 40000$ of that setup. The values then used to create heatmap diagrams as shown in Figure 10.

In the figures the colour scale represents $x_A = N_A/N$. Dark blue colored regions indicate perfect consensus (agreement to the best opinion, B), while red cells indicate perfect consensus to opinion A, which in these experiments it is the worst in terms of quality. Colors become paler as the consensus state gets weaker. The white curve is the separation line between consensus to A (red) and consensus to B (blue), the indifference curve.

The results in the heatmaps show that, when the proportion of zealots associated to the best option B is low ($\sigma_B = 0.0125$), differential quality ρ_B/ρ_A has a more relevant role compared to the differential proportions of zealots σ_A/σ_B, as clear from the presence of a wider blue area, compared to the red one. In this case, the consensus is almost always reached to the best opinion B. Only for very few cases, where the difference in quality is very small and the number of A zealots is very high, the system converges to the worst opinion A. This result is however dramatically affected by one parameter, the proportion of zealots for the best option σ_B, which in turns also affects by increasing the total proportion of zealots $\sigma_A + \sigma_B$. Increasing the number of zealots with the best opinion (e.g. $\sigma_B = 0.0250$), the tendency to agree to the worst opinion increases, as evident from the motion towards north-west of the indifference curve and from the increase in size of the red area, corresponding to cases with small quality difference and high number of zealots A. This tendency is even stronger for even higher proportions of zealots B ($\sigma_B = 0.0500$). Also in this case, only a high percentage of zealots with the worst opinion A (σ_A/σ_B larger than 6, that is σ_A larger than 30%) can drive the whole swarm to the consensus to A (red area), despite the difference in qualities. Importantly, although σ_B appears to be the most influencing parameters, from the plots it also emerges that the consensus dynamics and the behaviour of the indifference curve do not depend on the swarm size N, as it is clear when comparing the two cases with $N = 100$ and $N = 1000$ represented in the top and bottom row of Figure 10.

6 Conclusions and Future Work

In this paper, we considered the best-of-n problem in its original formulations by Valentini et al. [22], with the addition of zealots, individuals that are committed to one of the n options and cannot change their opinion. The novel contribution of this research is the study of the interplay between differential quality, that is different quality values associated to the two options, and differential zealot quantity, that is different proportions of zealot for the different options. In the 2-option case, we studied an antagonistic scenario in which the option associated to the best quality has less zealots compared to the other option. Our goal here was to study the asymptotic consensus state of the system on the differential quality and zealot quantity plane, where in one axis we vary the ratio between zealots and in the other the ratio between the qualities. In this plane, we looked at the behaviour of the indifference curve, which separates the areas where the asymptotic consensus is for one option from areas where it is for the other option. Our results show that these dynamics do not depend on the swarm size, but they do depend from the total proportion of zealots: The indifference curve cuts the considered plane in half only for intermediate values of this parameter, below this value it appears instead that quality plays a more significant role in determining the consensus equilibrium, while above this value it is the zealot quantity that seem to play a more significant role. Importantly, the total proportion of zealots is changed by changing the proportion of zealot for the right option, the option associated with the highest quality. This means that, contrarily to intuition, having individuals committed to the right option could have the surprising effect of producing dynamics that can be more severely affected by zealots of the opposite faction.

This study is can be extended in future works in many ways. First of all, a mathematical model, such as the one in [17], can be used to have a better and wider prediction of the dynamics in our setting. Second, we are interested in introducing the notion of *dissemination cost*, a parameter that adds a new dimension to the study because, from the engineering perspective, one can ask the question of how to design a parsimonious but effective modulation of positive feedback mechanism for normal agents, or how to optimize the number of zealots to inject in a system in order to bias the consensus dynamics while minimizing at the same time the cost. Finally, we would like to consider also scenarios where the zealotry level could be deteriorating over time.

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