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"Too Young to Die": Deprivation Measures Combining Poverty and Premature Mortality†

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Most measures of deprivation concentrate on deprivation among the living population and, thus, ignore premature mortality. This omission leads to a severe bias in the evaluation of deprivation. We propose two different measures that combine information on poverty and premature mortality of a population. These measures are consistent and satisfy a number of desirable properties unmet by all other measures combining early mortality and poverty. Moreover, one measure is readily computable with available data and easily interpretable. We show that omitting premature mortality leads to an underestimation of total deprivation in 2015 of at least 36 percent at the world level. (*JEL* C43, I12, I32, N33, N34, O15)

No winning words about death to me, shining Odysseus!
By god, I'd rather slave on earth for an other man—
some dirt-poor tenant farmer who scrapes to keep alive—
than rule down here over all the breathless dead.
—Achilles's ghost to Odysseus, Homer, *The Odyssey*

Consider the evolution of Botswana at the end of the last century. In 1990, life expectancy in Botswana was 63.6 years while 33.6 percent of its population was considered as extremely poor. In 2000, life expectancy was 45.6 years, while the proportion of extremely poor people had dropped to 29.5 percent.[1] Over a decade in Botswana, extreme (income) poverty decreased, but people also live a shorter life. The question we raise in this paper is how to evaluate, in a simple, meaningful, and unambiguous manner, the evolution of total deprivation in Botswana between 1990 and 2000.

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[1] We present our databases below.

Deprivation is a multidimensional phenomenon (Alkire and Foster 2011). The dimensions typically considered, such as income, education, or health, only affect individuals when they are alive. In this paper, we consider instead *premature* mortality as an important source of deprivation (Sen 1998, Deaton 2013). Of course, dying is not per se a form of deprivation: everyone is mortal, and being deprived means falling short of a minimal standard in a welfare-relevant resource. However, an individual dying *too young* is deprived in the sense that she will not live a number of years considered as minimally acceptable. Under this approach, the resource of interest is the number of years spent alive, that is, the life-span.[2]

We propose two measures of total deprivation that explicitly take life-span deprivation into account. The measures proposed so far in the literature are unsatisfactory either because, as most poverty indexes, they simply ignore life-span deprivation or because, as most composite indexes, they account for it in a questionable way. More precisely, simple composite indexes are not "consistent": they do not hold constant the trade-off between alive deprivation and life-span deprivation.

To illustrate this point, consider the example given in Table 1, which compares three societies. In all societies, two individuals are born every year, and no individual lives for more than two years. In society A, the 2 newborns are nondeprived, and the 2 one-year-olds are (income) poor. As we assume the age threshold defining life-span deprivation to be two years, no individual is life-span deprived in society A. Society B is identical to society A, except for the status of a one-year-old individual: she is prematurely dead instead of being poor. Similarly, society C is identical to society B, except for a one-year-old individual who is prematurely dead instead of poor.

Income poverty is measured by the head-count ratio (HC), i.e., the fraction of alive individuals who are poor, which is 0.5, 0.33, and 0 in societies A, B, and C, respectively. In this simple example, we can measure life-span deprivation by the fraction of individuals born in the last two years who are already dead (LD). This fraction is equal to 0, 0.25, and 0.5, in societies A, B, and C, respectively. A typical composite index of total deprivation simply aggregates the two dimensions by weighing them:

$$P_w = wHC + (1 - w)LD,$$

where $w \in [0,1]$ is the weight parameter w. Assuming $w = 0.5$, total deprivation as measured by $P_{0.5}$ is *smaller* in society A than in society B but *larger* in society B than in society C. Yet comparing society B to A, or C to B, the only difference between those societies is that a single individual changed status, from being poor to being dead. We call these judgments "inconsistent," as they do not satisfy a basic separability property. They arbitrarily imply that being poor is worse than prematurely dead in some situations but better in other situations. This inconsistency arises because the two measures that compose the index, HC and LD, are based on

[2] This way of accounting for premature mortality is different from the missing poor approach followed by Lefebvre, Pestieau, and Ponthiere (2013) and from the missing women approach (Anderson and Ray 2010), where individuals dying in excess to a death rate are considered missing (see Section III). We take an absolute deprivation approach to mortality, while the missing poor and missing women approaches take a counterfactual approach based on reference mortality rates.

Table 1—Composite Indexes Are Not Consistent

	0-year-old	1-year-old	HC	LD	$P_{0.5}$
Society A	Nonpoor, Nonpoor	Poor, Poor	0.5	0	0.25
Society B	Nonpoor, Nonpoor	Poor, Dead	0.33	0.25	0.29
Society C	Nonpoor, Nonpoor	Dead, Dead	0	0.5	0.25

Note: The age threshold defining life-span deprivation is two years.

different reference populations: the living population for HC and the "total" population for LD. We discuss these inconsistencies in more detail in Section IIIA and show that they affect commonly used indexes such as the Human Poverty Index (Watkins 2006).

The two indexes of total deprivation we propose, Inherited Deprivation (ID) and Generated Deprivation (GD), explicitly combine alive deprivation and life-span deprivation in a consistent and straightforward manner. These indexes also satisfy a number of desirable properties unmet by all other measures combining alive deprivation and premature mortality. Our theoretical approach provides the foundations for a particular aggregation of alive deprivation and life-span deprivation based on time units. More precisely, our indexes aggregate person-years in alive deprivation (PYADs) with person-years prematurely lost (PYPLs), given an age threshold below which dying is considered as premature. These indexes therefore measure the incidence, and not the intensity, of alive and life-span deprivation.

ID is based on past mortality and records the number of individuals who died prematurely in the past but should have been alive today. GD is based on current mortality, as measured by the number of years prematurely lost by individuals dying in the current period. Compared to ID, GD measures how much deprivation has been generated in the year considered and better corresponds to a flow measure of deprivation. This difference also makes GD more sensitive to contemporaneous changes in the society. Moreover, GD is in practice easier to compute than ID, given the type of data available. As we discuss later, the construction of ID requires a history of mortality rates, while GD only relies on current mortality rates.

Three main lessons can be drawn from this exercise. First, when aggregating different dimensions of deprivation, life-span deprivation should be treated separately. The fundamental reason lies in the *exclusive* nature of this dimension: individuals, once dead, cannot be considered as deprived along another dimension. This also implies that, to measure total deprivation, a life-span deprivation component can be added to an alive deprivation component. Second, when measuring total deprivation in a given year, the life-span deprivation component should be measured in *time units*, i.e., the number of years prematurely lost due to early death. Fundamentally, alive deprivation is also measured in time units since it records the number of (alive) individuals who are poor *in a given year*, which corresponds to the number of years spent in poverty by a population in a given year. Third, a familiar critique of composite indexes is that they typically rely on arbitrary weights, which weakens their relevance when the dimensions considered vary in opposite directions. Our analysis instead provides a normative support for a lower bound on the relative weight of premature mortality, based on the idea that one year prematurely lost is at least as bad as one year spent in alive poverty.

Using datasets on income deprivation (PovCalNet) and on mortality (Global Burden of Disease [GBD]), we show that, for the 1990–2015 period in the developing world, life-span deprivation is not negligible as compared to income poverty. The omission of life-span deprivation leads to an underestimation of global total deprivation of at least 27 to 36 percent during the whole period. In 2015, there were 705 million income poor individuals (PYADs), and premature mortality in the same year caused the loss of 402 million person-years (PYPLs). Moreover, the relative importance of life-span deprivation in total deprivation has been increasing over time: the omission of premature mortality from deprivation measures therefore leads to an increasing bias.

At the country level, important differences arise between alive deprivation and total deprivation, and the evolution of total deprivation sometimes contradicts the evolution of income poverty for several countries and periods. Thus, for 8 percent of the country-periods considered, total deprivation evolves in the opposite direction to income deprivation. Deprivation assessments ignoring premature mortality at the country level are therefore seriously biased and may lead to flawed policy evaluations.

The remainder of the paper is organized as follows. We first present the two indexes and discuss some of their properties in Section I. We also investigate their dynamic behavior. A complete characterization of the two indexes is given in Section II. We then compare the fundamental differences between our indexes and the alternative approaches proposed so far in the literature in Section III. An empirical description of the evolution of total deprivation at the world and at country level is presented in Section IV, and Section V concludes. All proofs are relegated to the online Appendix.

I. Two Families of Total Deprivation Measures

A. *Basic Framework*

In this section, we define our two measures of total deprivation, combining in a single index alive deprivation and life-span deprivation. In period t, each individual i is characterized by a **bundle** $\mathbf{x}_i = (b_i, s_i)$, where $b_i \in \mathbb{Z}$ is her birth year with $b_i \leq t$ and s_i is a categorical variable capturing individual status in period t, which can be either alive and nonpoor (NP), alive and poor (AP), or dead (D), i.e., $s_i \in \mathcal{S} = \{NP, AP, D\}$. In the following, we often refer to individuals whose status is AP as "poor." We consider here that births occur at the beginning, while deaths occur at the end of a period.[3] As a result, an individual whose status in period t is D died before period t.[4]

Let $a_i = t - b_i$ be the age that individual i would have in period t given her birth year b_i. We define a life-span threshold $\hat{a} \in \mathbb{N}$, below which a life-span is

[3] This assumption implies that life-spans have no decimals and is made for expositional reasons. We could alternatively assume that all deaths take place at the beginning of the period, which is the assumption made in the empirical section for practical reasons.

[4] All newborns have age zero during period t, and some among these newborns may die at the end of period t. This implies that $b_i = t \Rightarrow s_i \neq D$.

normatively considered too short. This threshold, which does not depend on the life-span distribution in the population, corresponds to an "absolute" approach of life-span deprivation.[5] An individual "dies prematurely" if she dies before reaching the minimal life-span. Formally, period t is "prematurely lost" by any individual i with $s_i = D$ and $a_i < \hat{a}$. A **distribution** $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_{n(\mathbf{x})})$ specifies the birth year and the status in period t of all $n(\mathbf{x})$ individuals. The set of distributions, denoted by $\mathcal{X}$, is formally defined in Section II.

This framework extends the traditional approach used in poverty measurement in two ways: to all individuals is attached a birth year, and some individuals may be dead. A total deprivation index ranks all distributions in the set $\mathcal{X}$ as a function of the deprivation that they contain. Formally, it is a function $P : \mathcal{X} \to \mathbb{R}_+$, where $P(\mathbf{x}) \geq P(\mathbf{x}')$ means that $\mathbf{x}$ has weakly more deprivation than $\mathbf{x}'$ and strictly more if $P(\mathbf{x}) > P(\mathbf{x}')$. (Age threshold $\hat{a}$ is assumed fixed.) Note also that the three status framework $\{NP, AP, D\}$ is intentionally restrictive in order to focus on the aggregation of life-span deprivation with other forms of deprivation. Our results can easily be extended to richer structures where individual achievements while alive are measured in multiple dimensions.[6] These achievements could also be measured using continuous rather than categorical variables, thereby accounting for the intensity of deprivation, but we stick here to the incidence.

By construction, classical deprivation indexes do not measure life-span deprivation. Consider the following distribution in period t with three individuals:

$$\mathbf{x} = \big((young, NP), (young, D), (old, D)\big),$$

where a birth year that is at least $\hat{a}$ years distant from year t is noted as *old*, and *young* otherwise. Because she is young and dead, individual 2 has prematurely lost period t.

We contrast distribution $\mathbf{x}$ with two alternative distributions $\mathbf{x}'$ and $\mathbf{x}''$ in period t that are both obtained from $\mathbf{x}$ by changing the status of individual 2. In $\mathbf{x}'$, individual 2 is alive and nonpoor, while in distribution $\mathbf{x}''$, individual 2 is alive and poor, i.e.,

$$\mathbf{x}' = \big((young, NP), (young, NP), (old, D)\big),$$

$$\mathbf{x}'' = \big((young, NP), (young, AP), (old, D)\big).$$

These three distributions are compared in Table 2.

In these three distributions, no individual is alive and poor, except individual 2 in distribution $\mathbf{x}''$. As a result, the head-count ratios (HC) of distributions $\mathbf{x}$ ($HC = 0/1$) and $\mathbf{x}'$ ($HC = 0/2$) are identical and equal to zero, while the head-count ratio of $\mathbf{x}''$

[5] The introduction of an age threshold is in line with the methodology used in the literature on multidimensional poverty measurement, which assumes dimension-specific thresholds in order to define dimension-specific deprivation status (Alkire and Foster 2011, Pattanaik and Xu 2018).

[6] In this richer framework, we would need to define dimension-specific deprivation thresholds, impose a series of classical axioms that would constrain how to aggregate the continuous achievements in these multiple dimensions, and ultimately obtain a classification of individuals into those who are multidimensionally deprived and those who are not multidimensionally deprived. The first category could be described by a continuous multidimensional poverty score, in the vein of Alkire and Foster (2011). In order to simplify the exposition, we directly assume this score to be zero or one.

TABLE 2—COMPARING DISTRIBUTIONS BY CHANGING THE STATUS OF ONE INDIVIDUAL

	$(young, AP)$	$(young, NP)$	$(young, D)$	(old, D)
Distribution $\mathbf{x}$	0	1	1	1
Distribution $\mathbf{x}'$	0	2	0	1
Distribution $\mathbf{x}''$	1	1	0	1

is equal to $1/2$.[7] However, distribution $\mathbf{x}'$ is arguably better than distribution $\mathbf{x}$ since individual 2 is not prematurely dead in $\mathbf{x}'$. Moreover, it is not clear that distribution $\mathbf{x}''$ is worse than distribution $\mathbf{x}$: individual 2 is poor in $\mathbf{x}''$ but prematurely dead in $\mathbf{x}$. Whether distribution $\mathbf{x}$ is preferable to distribution $\mathbf{x}''$ is a judgment based on how one compares spending period t in poverty to prematurely losing period t. In our epigraph, for example, Achilles clearly states that spending a year in poverty is much preferable to spending a year in life-span deprivation: Achilles would consider that distribution $\mathbf{x}$ is much worse than distribution $\mathbf{x}''$.

B. *The Inherited Deprivation Index*

We first introduce an index based on mortality inherited from the past and refer to this index as the Inherited Deprivation Index (ID). Let $d(\mathbf{x})$ denote the number of *prematurely dead* individuals in distribution $\mathbf{x}$, which is the number of individuals i for whom $s_i = D$ and $\hat{a} > t - b_i$; $p(\mathbf{x})$ the number of individuals who are poor; and $f(\mathbf{x})$ the number of alive and nonpoor individuals. ID is defined as

$$(1) \qquad ID_\gamma(\mathbf{x}) = \underbrace{\frac{p(\mathbf{x})}{f(\mathbf{x}) + p(\mathbf{x}) + d(\mathbf{x})}}_{\text{alive deprivation}} + \gamma \underbrace{\frac{d(\mathbf{x})}{f(\mathbf{x}) + p(\mathbf{x}) + d(\mathbf{x})}}_{\text{life span deprivation}},$$

where $\gamma > 0$ is a parameter weighing the relative importance of alive deprivation and life-span deprivation. An individual losing prematurely period t matters γ times as much as an individual spending period t in alive deprivation.

Index ID_γ has an alive deprivation component (poverty) and a life-span deprivation component (premature mortality). The alive deprivation component records the number of persons who are poor in period t, and the life-span deprivation component records the number of persons who were born less than $\hat{a}$ years before t but have already died. The denominator of both components is identical and equal to the reference population. This reference population includes all individuals, whether dead or alive, born less than $\hat{a}$ years before t as well as all older individuals still alive in t.

Comparing distributions $\mathbf{x}$ and $\mathbf{x}'$ given above, the inherited deprivation index considers distribution $\mathbf{x}$ as unambiguously more deprived than distribution $\mathbf{x}'$. By contrast, classical deprivation indexes, such as HC, are not able to capture a

[7] The comparison of distribution $\mathbf{x}''$ to distribution $\mathbf{x}$ is an example of the "mortality paradox": the reason why the HC of $\mathbf{x}''$ is higher than that of $\mathbf{x}$ is because the poor individual of distribution $\mathbf{x}''$ is dead in distribution $\mathbf{x}$. We discuss this question in more detail in Section III.

difference between these two distributions.[8] This is because these indexes satisfy an Independence of Dead property, according to which the presence of an additional dead individual (all properties are formally defined in Section II) leaves them unaffected. As a result, they ignore prematurely dead individuals.

By contrast, the inherited deprivation index captures premature mortality. A priori, a distribution contains all individuals who ever lived in a particular society. We impose a Weak Independence of Dead property, implying the index is not affected by the presence of an additional dead individual, if this individual is born at least $\hat{a}$ years before period t. This last property defines the relevant population in period t by excluding two types of individuals: those who died after reaching the age threshold and those who died below the age threshold but too far away in the past. Among the dead individuals, only those who died prematurely and whose birth year is less than $\hat{a}$ years before t are considered as part of the reference population.

When comparing distributions $\mathbf{x}$, $\mathbf{x}'$, and $\mathbf{x}''$ given above, ID focuses on old individuals who are alive and all young individuals, whether alive or not (as required by the Weak Independence of Dead property). In all three distributions, the reference population is composed of two individuals. As individual 2 is prematurely dead in distribution $\mathbf{x}$, whereas she is alive and nonpoor in distribution $\mathbf{x}'$, $ID_\gamma(\mathbf{x}) > ID_\gamma(\mathbf{x}')$. In addition, as individual 2 is prematurely dead in distribution $\mathbf{x}$, whereas she is alive and poor in $\mathbf{x}''$, $ID_\gamma(\mathbf{x}) \geq ID_\gamma(\mathbf{x}'')$ when $\gamma \geq 1$. The larger premature mortality in $\mathbf{x}$ more than compensates for the larger alive deprivation in $\mathbf{x}''$, and ID contradicts HC.

The implementation of ID involves two important normative choices: (i) the choice of $\hat{a}$, the age threshold below which the death of an individual is considered as premature and contributes to total deprivation, and (ii) the value of γ, the parameter weighing the relative importance of poverty and premature mortality. We believe that $\gamma \geq 1$ is a meaningful constraint, as one year "not lived" in a young age can be considered as at least as undesirable as one year spent in poverty.[9] A revealed preference argument supports $\gamma \geq 1$ given that committing suicide is an outside option (plausibly) available. Of course, some people do commit suicide. In particular, Bantjes et al. (2016) document that poverty is associated with mental illnesses leading to suicide. However, the constraint $\gamma \geq 1$ is relevant as long as the fraction of "young" individuals who prefer to be dead instead of poor is quantitatively negligible.

In Section II, we show that ID is (completely) characterized by a small number of desirable properties. In particular, our characterization implies that alive and lifespan deprivation enter the index in an additive way, so that computing ID amounts to a very basic accounting exercise. The fundamental intuition underlying this additive separability is that an individual cannot simultaneously be "prematurely dead" and "poor": these two statuses are mutually exclusive, which allows us to sum the

[8] Prematurely dead individuals are not *intrinsically* valued by the HC. In practice, the HC is *instrumentally* affected by premature mortality when premature mortality is selective, i.e., when it affects poor individuals more than nonpoor individuals. See Ravallion (2005) for an assessment of the contribution of both selective mortality and selective fertility to measures of poverty.

[9] Assuming $\gamma < 1$ would imply that a policy whose sole impact is to delay the premature death of a poor individual by one year increases total deprivation, an arguably dubious judgment.

number of prematurely dead individuals and the number of individuals affected by alive deprivation. In contrast, nonexclusive dimensions of alive deprivation, such as income and health deprivation, would not necessarily be additively separable, as the same individual can be simultaneously income and health deprived.

Relatedly, our definition of a distribution does not simultaneously contain information about an individual deprivation status and on her chances of survival. This particular assumption, which we discuss more carefully at the end of Section III, is motivated by the near absence of comparable datasets that simultaneously contain at the individual level information about lifetime duration and deprivation status. Our measures are therefore indifferent to the joint distribution across individuals of periods spent in alive deprivation and periods prematurely lost.

C. *The Generated Deprivation Index*

ID is an intuitive and straightforward manner to include premature mortality in deprivation measures. By definition, it measures current life-span deprivation resulting from past mortality. This makes its empirical implementation difficult, as its computation requires detailed information on mortality of each age cohort for all $\hat{a}$ years preceding t. Also, the impact of a mortality shock, whether permanent or temporary, takes decades to be fully accounted for. This implies that ID exhibits inertia, which may be undesirable when used to evaluate the impact of contemporary public policies. For instance, today's ID for Rwanda still accounts for children who died during the genocide of 1994: this is probably an accurate picture of total deprivation in Rwanda but of little use to evaluate its current policies. The alternative index we propose, called the Generated Deprivation Index (GD), does not suffer from these limitations. It shares closely related properties and is based on the same intuition as ID. However, it is defined on current, instead of past, mortality rates.

Consider the population pyramid in period t, and let $n_a(\mathbf{x})$ be the number of *alive* individuals of age a in distribution $\mathbf{x}$, i.e., the number of individuals i for whom $a_i = a$ and $s_i \neq D$. Letting $d_a(\mathbf{x})$ be the number of *dead* individuals born a years before t in distribution $\mathbf{x}$, the total number of individuals born a years before t is then equal to $n_a(\mathbf{x}) + d_a(\mathbf{x})$. The age-specific mortality rate $\mu_a \in [0,1]$ denotes the fraction of alive individuals of age a dying at the end of period t: the number of a-year-old individuals dying at the end of period t is $n_a(\mathbf{x}) \times \mu_a$. Letting $a^* \in \mathbb{N}$ stand for the maximal life-span (which implies $\mu_{a^*} = 1$), the **vector of age-specific mortality rates** in period t is given by $\mu = (\mu_0, \ldots, \mu_{a^*})$. Vector μ summarizes mortality in period t, while distribution $\mathbf{x}$ summarizes alive deprivation in period t as well as mortality before period t.[10]

The generated deprivation index is defined as follows:

$$GD_\gamma(\mathbf{x}, \mu) = \underbrace{\frac{p(\mathbf{x})}{f(\mathbf{x}) + p(\mathbf{x}) + d^{GD}(\mathbf{x}, \mu)}}_{\text{alive deprivation}} + \gamma \underbrace{\frac{d^{GD}(\mathbf{x}, \mu)}{f(\mathbf{x}) + p(\mathbf{x}) + d^{GD}(\mathbf{x}, \mu)}}_{\text{life span deprivation}}, \tag{2}$$

[10] Observe again that this framework is consistent with our data constraint. A pair $(\mathbf{x}, \mu)$ does not simultaneously contain information on an individual's deprivation and her chances of survival.

where d^{GD} measures the number of person-years prematurely lost due to deaths occurring in period t:

$$d^{GD}(\mathbf{x}, \mu) = \sum_{a=0}^{\hat{a}-1} n_a(\mathbf{x}) \times \mu_a \times (\hat{a} - (a+1)). \tag{3}$$

Like ID, GD is the sum of an alive deprivation component, recording the number of person-years in alive deprivation, and a life-span deprivation component. The life-span deprivation component of GD differs from that of ID, as it records the number of person-years prematurely lost generated by deaths occurring in period t. When an individual dies at age $a < \hat{a}$, she prematurely loses $\hat{a} - (a+1)$ periods of life. GD records these $\hat{a} - (a+1)$ PYPLs and assigns them to the year during which the death occurs. By contrast, ID records all the PYPLs in period t that were generated by deaths occurring before period t. The denominator of GD is analogous to that of ID, as it simply adds the number of alive individuals in period t to the number of PYPLs.

GD and ID are similar in many ways. In particular, a person-year lost due to a "mature" death does not enter the reference population, all PYPLs have the same weight, and the weight γ of a PYPL relative to a year in alive deprivation is constant. Moreover, as discussed in Section II, they are both *additively* decomposable. The main difference between the two is that GD relies on *current* mortality, while ID relies on *past* mortality.

While, in general, *current* mortality is a priori unrelated to *past* mortality, the two coincide in *stationary populations*. In a stationary population, the number of newborns and the mortality vectors are constant over time, so that the population pyramid in period $t+1$ replicates the population pyramid in period t. Formally, the pair $(\mathbf{x}, \mu)$ is **stationary** if, for some $n^* \in \mathcal{N}$ and all $a \in \{0, \ldots, a^*\}$, we have

- $n_a(\mathbf{x}) + d_a(\mathbf{x}) = n^* \in \mathcal{N}$ (constant natality),
- $n_{a+1}(\mathbf{x}) = n_a(\mathbf{x}) \times (1 - \mu_a)$ (identical population pyramid in $t+1$).

In a stationary pair, the population pyramid is such that the size of each cohort can be obtained by applying to the preceding cohort the current mortality rate.[11] In such a case, past and current mortality coincide, and the mortality vector μ does not convey any information that cannot be inferred from the population pyramid associated to distribution $\mathbf{x}$. As vector μ is redundant, a deprivation index can be computed from the distribution $\mathbf{x}$ only.

Our characterization of ID (see Section II) shows that, when measuring deprivation on distribution $\mathbf{x}$ only, one should use ID. We therefore impose an ID Equivalence property, requiring that, for stationary pairs, total deprivation indexes correspond to ID. This property implies that deprivation indexes agree with ID on the *long-run* consequences of permanent changes in mortality or natality rates. GD satisfies this ID Equivalence property.

[11] Such population pyramids correspond to the ones prevailing in the long run if current mortality and natality rates remain constant over time (see, for instance, Preston, Heuveline, and Guillot 2000).

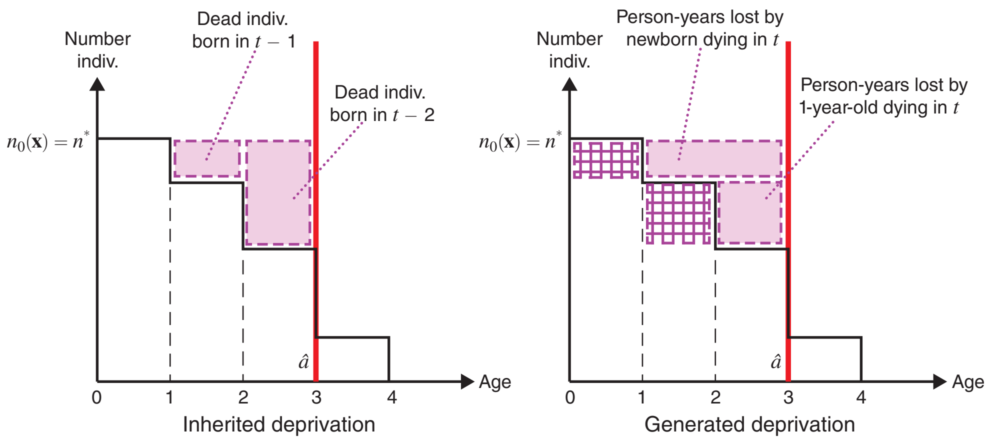

Figure 1

Notes: Left panel: The shaded area above the population pyramid represents $d(\mathbf{x})$. Right panel: The shaded area above the population pyramid represents $d^{GD}(\mathbf{x}, \mu)$, and the hatched areas represent the age-specific number of deaths in the period.

As formally proven in Section II, the GD_γ index and ID are identical in *stationary populations* because d^{GD} coincides with d in that case. In stationary populations, counting the number of individuals who prematurely miss period t due to past mortality is equivalent to counting the number of person-years lost due to premature mortality in period t. The fundamental intuition for this equivalence is illustrated in Figure 1. The left panel shows that d counts "vertically" the number of individuals who are younger than $\hat{a}$ years and died before period t. The right panel shows that d^{GD} counts "horizontally," for each age group below $\hat{a}$, the number of person-years prematurely lost by individuals in that age group who die in period t. When the mortality rates of the young correspond to the population pyramid, the two shaded areas coincide.

D. *Dynamic Behavior of the Two Indexes*

GD is equivalent to ID for stationary populations. Actual populations, however, are typically nonstationary. Permanent and transitory mortality shocks regularly affect population pyramids, which take decades to adjust to these shocks. In this section, we compare the behavior of our two indexes in nonstationary populations and investigate their reactions to different types of mortality shocks.

Transitory Mortality Shocks.—We first investigate responses to a transitory mortality shock in a simple example. We consider a population with a fixed natality $n_0(\mathbf{x}) = n^* = 1$ for all periods t. At each period, all alive individuals are nonpoor, implying that $HC(\mathbf{x}) = 0$. For all $t \neq 0$, we assume a constant mortality vector $\mu = \mu^* = (0, 0, 1)$ so that each individual lives exactly three periods. Let us fix the normative parameters at one for γ and three for $\hat{a}$, so that an individual dies prematurely if she dies before her third period of life. Before period $t = 0$, the

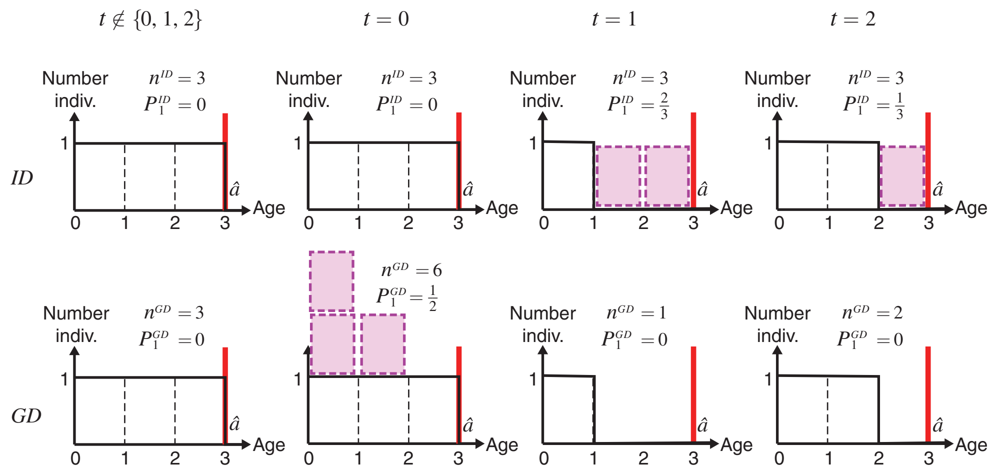

Figure 2. Response of ID and GD to the Transitory Mortality Shock in $t^* = 0$

Note: The person-years prematurely lost are shaded.

population is stationary, and the two indexes are equal to zero since there are no poor and no premature deaths. Let us now consider a one-period shock at period 0, such that all individuals die: $\mu^0 = (1, 1, 1)$. After the shock, mortality rates directly come back to their initial value, and the population pyramid returns to its stationary state in period 3, after a (mechanical) transition in periods 1 and 2 during which the newborns of periods 1 and 2 grow up. This example is illustrated in Figure 2, where the reference populations are denoted by n^{ID} and n^{GD}, for ID and GD, respectively.

Consider first ID. In period 0, no premature deaths are recorded since they all happen at the end of period 0. The number of person-years prematurely lost recorded by ID is equal to two in period 1, one in period 2, and zero afterward, as illustrated by the shaded areas in the first row of Figure 2. Given that one individual is born in every period and $\hat{a} = 3$, the relevant population is given by $n^{ID} = 3$ in all periods. Therefore, ID is equal to $2/3$ in period 1 and $1/3$ in period 2.

GD records the shock immediately in period 0. The newborn who dies in period 0 produces two PYPLs, and the individual aged one in that period produces one PYPL. To compute GD in period 0, we consider a total of six person-years (PYs), and GD is equal to $1/2$ in period 0. Since the newborn in period 1 does not die in period 1 and is the only individual alive, GD records one PY with no deprivation and no PYPL. It is therefore equal to zero. Similarly, for period 2, there are two individuals alive, but no deprivation, and GD is again equal to zero.[12] Note that both ID and GD record the premature loss of three person-years. This is no coincidence, as we show in the online Appendix. In a stationary population affected by transitory mortality

[12]Note that the fact that the index returns to its initial value after one period is a particularity of this simple example. If instead we had $n^* = 4$, $\mu^0 = (1/2, 1, 1)$, and $\mu^* = (0, 1/2, 1)$, the index would take longer to return to its stationary value.

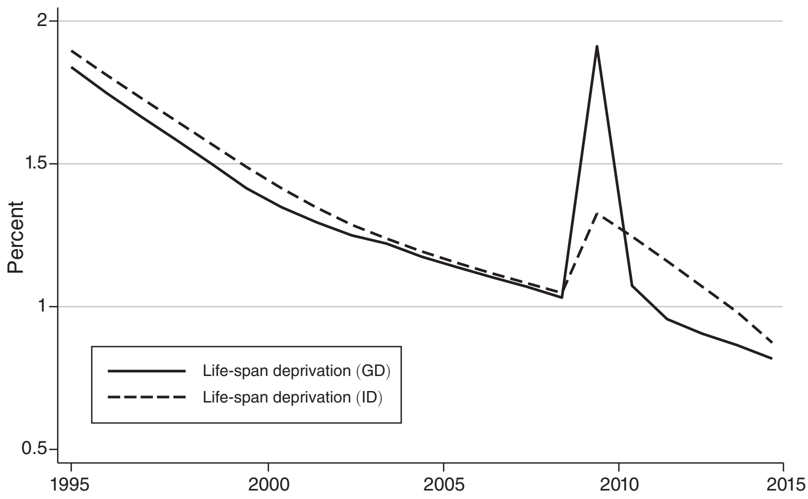

FIGURE 3. EVOLUTION OF THE LIFE-SPAN COMPONENTS OF ID AND GD IN HAITI ($\hat{a} = 5$)

Sources: Global Burden of Disease (2018). Authors' calculations.

shocks, GD and ID indexes compute the same number of PYPLs but distribute these PYPLs over different periods of time.

There are many instances of large transitory mortality shocks in history. For example, in 2010, Haiti was hit by a devastating earthquake, killing hundreds of thousands.[13] Figure 3 presents the evolution of the life-span component of ID and GD with $\hat{a} = 5$.[14] Between 1995 and 2009, the life-span components of ID and GD are quite similar. However, the person-years prematurely lost due to the earthquake are distributed differently. The GD approach attributes them all to 2010: there is a large spike in 2010 and a return to the long-term trend right afterward. The evolution of ID is different, with a smaller spike in 2010 but values that remain above the trend for the four subsequent years: it is only in 2015, i.e., $\hat{a}$ years after the earthquake, that the ID's life-span component returns to its long-term trend.

Permanent Mortality Shocks.—We now investigate the consequences of a permanent mortality shock on a stationary population. After a mortality shock, a transition phase sets in during which the population pyramid adjusts to the new mortality vector till the population reaches a new stationary equilibrium. This transition takes several decades and is particularly long for mortality shocks affecting young individuals. During this transition, the two indexes are not equivalent.

To illustrate this point, we use simulations, assuming constant natality rates and no alive deprivation. The age threshold is 50, and the maximal age is 100. Before

[13] 315,000 according to the Global Burden of Disease, our main data source described in Section IVA.

[14] We use this low threshold to illustrate the equivalence between ID and GD in the "long run." Given that our data end in 2015 and the earthquake took place in 2010, $\hat{a} = 5$ is the highest threshold we could use for this example.

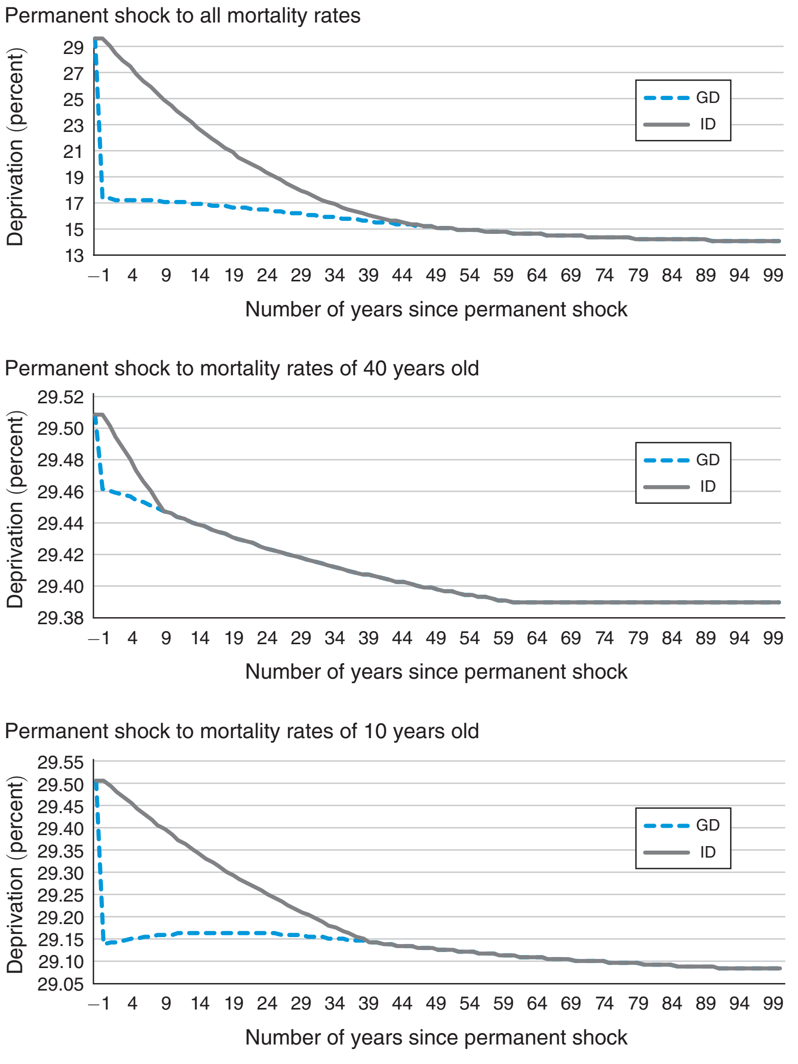

Figure 4. Simulation of Permanent Mortality Shocks on a Stationary Population ($\hat{a} = 50$)

the shock, the population pyramid is consistent with a mortality vector such that, at each age before 100, the mortality rate is equal to 2 percent. Figure 4 illustrates the relative evolution of the two indexes for three types of permanent shocks: (i) the mortality rates fall from 2 to 1 percent for all ages, (ii) the mortality rate falls from 2 to 1 percent only at age 40, and (iii) the mortality rate falls from 2 to 1 percent only at age 10.

The upper graph illustrates the consequences of the uniform mortality shock, the middle graph of the mortality shock at age 40, and the bottom graph of the mortality shock at age 10. The two indexes evolve very differently over the transition period. In all scenarios, ID remains unaffected during the period of the shock but adjusts in a smooth monotonic way afterward. GD jumps discretely in the period of

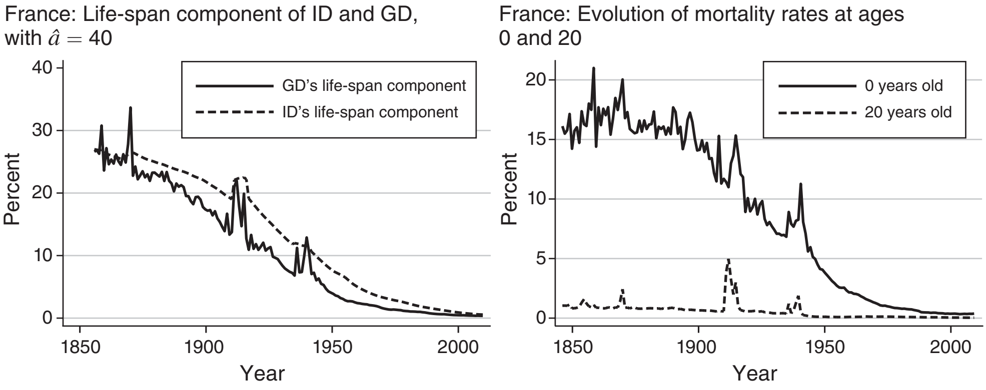

FIGURE 5. COMPARING ID AND GD: FRANCE

the shock and continues to slowly adjust to the induced changes in the population pyramid. In the long run, the two indexes are equal. Note that when mortality falls, GD is systematically lower than ID until they converge again once the shock is fully accounted for.

These simulations indicate that GD is more reactive than ID to a permanent mortality shock. Past natality and mortality still affect GD indirectly by shaping the current population pyramid on which it is defined. GD therefore partly reflects deaths that occurred in the past, even if the magnitude of this inertia is much smaller than that of ID. Moreover, the dynamics of premature mortality is determined by the interaction between the population pyramid and the mortality vector, and the relative size of young age cohorts in the current population pyramid may not evolve monotonically. This explains why the evolution of GD is not necessarily monotonic during the transition, as shown in the bottom graph of Figure 4. We provide in the online Appendix another illustration of this property.

A Historical Application: The Case of France.—We now provide a comparison of our two indexes for the case of France. More precisely, in the absence of comparable poverty measures throughout the period, we focus on the life-span component of the two indexes and compare their evolution over time. The data used come from the Human Mortality Database (University of California, Berkeley and Max Planck Institute for Demographic Research 2019). The French age-specific mortality database begins in 1816 and is the longest series available in the Human Mortality Database. In order to discuss the period prior to 1870, we use a 40 years threshold (instead of 50 years used elsewhere in the paper).

The left panel of Figure 5 presents the evolution of the life-span component of ID and GD for France between 1856 and 2010, while the right panel reports the mortality rates of individuals aged 0 and 20 years for the same period.

Prior to the Franco-Prussian war in 1870, the life-span components of ID and GD are very similar, suggesting that the French population in those days was close to stationary. The casualties of the 1870 Franco-Prussian war lead to a large increase in the number of PYPLs, which is reflected by the spike in GD, which attributes them all to the year 1870. By contrast, ID exhibits a discrete jump, which is much less pronounced but of a much longer duration. As discussed above, this is due to the fact that ID allocates the PYPLs across the next $\hat{a}$ years. The consequences of the two World Wars in the twentieth century generate essentially similar patterns, with large spikes in the life-span component of GD and a much more sluggish reaction in ID.

Note also that, had France returned to a stationary population after 1870, we would expect ID and GD to converge in 1910, 40 years after the end of the war. ID, however, remains well above GD, a direct consequence of the dramatic fall in mortality rates, particularly among infants, that started at the end of the nineteenth century (see the right panel of Figure 5). This fall corresponds to a succession of (negative) permanent shocks in mortality rates, which GD mirrors instantaneously, while ID takes decades to reflect.

Finally, while we would expect ID in 1919 to remain at the levels of the 1914–1918 period due to its inertia, we observe a discrete fall. This is due to the changing status of the Alsace-Lorraine region. After 1918, Alsace-Lorraine, which was German since 1870, became French again. This implies that all the deaths that occurred in that region between 1870 and 1918 were not recorded by the French administration and are therefore not reported in the PYPLs we measure. (These deaths were simply not observed and were attributed to Germany.) This explains the discrete drop in ID in 1919: a large living population is added to France in 1919, but the death history of this population is not.[15] This example illustrates the difficulty in implementing ID in practice. To be consistent, ID requires stable geographical units over long enough periods, while countries' borders often change (think of the USSR in the 1990s).

II. Characterization of the Two Indexes

In this section, we provide a formal characterization of ID and GD. Excluding trivial distributions for which no individual is alive or prematurely dead, the set of distributions in period t is denoted as

$$\mathcal{X} = \Big\{ \mathbf{x} \in \bigcup_{n \in \mathcal{N}} (\mathbb{Z} \times \mathcal{S})^n \mid \text{ there is } i \text{ for whom either } s_i \neq D \text{ or } s_i = D \text{ and } \hat{a} > t - b_i \Big\}.$$

A. *Inherited Deprivation*

We first show that ID is characterized by a small number of desirable properties. First, indexes based exclusively on alive deprivation satisfy a property that requires that the presence of an additional dead individual, whether prematurely or not, does not affect them. We refer to this property as follows.

[15] Note that the annexation of Alsace-Lorraine by Germany in 1870 also affected ID during the 1870–1910 period: all deaths occurring prior to 1870 in that region are still attributed to France afterward, while the living population is not. During the 1870–1910 period, ID is systematically overestimated. The same is true when Alsace-Lorraine is annexed again by Germany during World War II.

DEPRIVATION AXIOM 1 (Independence of Dead): *For all* $\mathbf{x} \in \mathcal{X}$ *and* $i \leq n(\mathbf{x})$, *if* $s_i = D$, *then* $P(\mathbf{x}_i, \mathbf{x}_{-i}) = P(\mathbf{x}_{-i})$.

By contrast, we require that the presence of an additional dead individual does not affect ID only if this individual is born at least $\hat{a}$ years before period t.

DEPRIVATION AXIOM 2 (Weak Independence of Dead): *For all* $\mathbf{x} \in \mathcal{X}$ *and* $i \leq n(\mathbf{x})$, *if* $s_i = D$ *and* $\hat{a} \leq t - b_i$, *then* $P(\mathbf{x}_i, \mathbf{x}_{-i}) = P(\mathbf{x}_{-i})$.

The second property, Least Deprivation, requires that being nonpoor is better than being either poor or prematurely dead. This weak axiom compares distributions with a unique individual, i.e., individual 1, in which, if the individual is dead, she is prematurely dead.

DEPRIVATION AXIOM 3 (Least Deprivation): $P(b_1, NP) < P(b_1, AP)$ *and* $P(b_1, NP) < P(b_1, D)$.

The third property, Weak Independence of Birth Year, requires that birth years are only relevant in order to distinguish prematurely dead from other dead individuals.[16] Hence, if an individual belongs to the reference population, only her status matters.

DEPRIVATION AXIOM 4 (Weak Independence of Birth Year): *For all* $\mathbf{x} \in \mathcal{X}$ *and* $i \leq n(\mathbf{x})$, *if* $s_i = s_i'$ *and* $d(\mathbf{x}_i, \mathbf{x}_{-i}) = d(\mathbf{x}_i', \mathbf{x}_{-i})$, *then* $P(\mathbf{x}_i, \mathbf{x}_{-i}) = P(\mathbf{x}_i', \mathbf{x}_{-i})$.

Weak Independence of Birth Year requires that one person-year prematurely lost matters equally in the index, independently of the particular age of the individual who died. Thus, if $\hat{a}$ is equal to 50, the death of a newborn in $t-1$ is equivalent to the death of a 48-year-old in $t-1$ in the computation of ID at period t. Of course, the death of the younger individual will be recorded in the ID indexes for several periods following her death, while the death of the 48-year-old individual will be accounted for only once (in the period t following her death). In that sense, the death of the younger individual matters proportionally more.

Then, we impose a standard separability property, Subgroup Consistency. This axiom requires that, if deprivation decreases in a subgroup while remaining unchanged in the rest of the distribution, overall deprivation must decline.[17]

[16] This is why Weak Independence of Birth Year has the precondition $d(\mathbf{x}_i, \mathbf{x}_{-i}) = d(\mathbf{x}_i', \mathbf{x}_{-i})$, which holds the number of prematurely dead constant: the birth year b_i' can be different from b_i, but if $s_i = D$, then individual i is either prematurely dead in both $\mathbf{x}_i$ and $\mathbf{x}_i'$ or in none of these two bundles.

[17] The precondition $f(\mathbf{x}') + p(\mathbf{x}') + d(\mathbf{x}') = f(\mathbf{x}'') + p(\mathbf{x}'') + d(\mathbf{x}'')$ ensures that distributions $\mathbf{x}'$ and $\mathbf{x}''$ have a reference population with the same size. The additive separability result of Foster and Shorrocks (1991), which rationalizes the use of additive indexes, is based on a stronger version of Subgroup Consistency with the additional precondition $f(\mathbf{x}') + p(\mathbf{x}') = f(\mathbf{x}'') + p(\mathbf{x}'')$.

DEPRIVATION AXIOM 5 (Subgroup Consistency): *For all* $(\mathbf{x},\mathbf{x}'),(\mathbf{x},\mathbf{x}'') \in \mathcal{X}$, *if* $P(\mathbf{x}') > P(\mathbf{x}'')$ *and* $f(\mathbf{x}')+p(\mathbf{x}')+d(\mathbf{x}') = f(\mathbf{x}'')+p(\mathbf{x}'')+d(\mathbf{x}'')$, *then* $P((\mathbf{x},\mathbf{x}')) > P((\mathbf{x},\mathbf{x}''))$.

To be complete, three auxiliary properties are also needed: Anonymity, Replication Invariance, and Young Continuity. First, the name of individuals should not influence the deprivation index.

DEPRIVATION AXIOM 6 (Anonymity): *For all* $\mathbf{x} \in \mathcal{X}$, *if* $n(\mathbf{x}') = n(\mathbf{x})$ *and* $\mathbf{x}'$ *is obtained from* $\mathbf{x}$ *by a permutation of the index set* $\{1,\ldots,n(\mathbf{x})\}$, *then* $P(\mathbf{x}) = P(\mathbf{x}')$.

Second, if a distribution is obtained by replicating another distribution several times, they both have the same deprivation. For any $k \in \mathbb{N}$, we denote by $\mathbf{x}^k$ the k-**replication** of $\mathbf{x}$, which is the distribution such that $n(\mathbf{x}^k) = kn(\mathbf{x})$ and $\mathbf{x}^k = (\mathbf{x},\mathbf{x},\ldots,\mathbf{x})$.

DEPRIVATION AXIOM 7 (Replication Invariance): *For all* $\mathbf{x} \in \mathcal{X}$ *and* $k \in \mathcal{N}$, $P(\mathbf{x}^k) = P(\mathbf{x})$.

Finally, the deprivation index evolves "continuously" on its domain. Given that this domain is discrete, the index should satisfy a particular continuity property as proposed by Young (1975).

DEPRIVATION AXIOM 8 (Young Continuity): *For all* $\mathbf{x},\mathbf{y},\mathbf{z} \in \mathcal{X}$, *if* $P(\mathbf{x}) > P(\mathbf{y})$ *and* $n(\mathbf{z}) = 1$, *then for* $k \in \mathbb{N}$ *sufficiently large we have* $P(\mathbf{x}^k,\mathbf{z}) > P(\mathbf{y})$ *and* $P(\mathbf{x}) > P(\mathbf{y}^k,\mathbf{z})$.

Proposition 1 fully characterizes ID, which implies that any deprivation index satisfying our properties ranks distributions in exactly the same way as ID.

PROPOSITION 1 (Characterization of ID): *P is ordinally equivalent to* ID_γ *for some* $\gamma > 0$ *if and only if P satisfies Weak Independence of Dead*, *Least Deprivation*, *Weak Independence of Birth Year*, *Subgroup Consistency*, *Anonymity*, *Replication Invariance*, *and Young Continuity.*

PROOF:

See online Appendix. ∎

Proposition 1 provides a necessary step in the characterization of GD. Observe that our definition of the individual status is agnostic to the particular definition of alive deprivation and could as well capture income deprivation, as in our empirical application, or multidimensional poverty (Alkire and Foster 2011). Proposition 1 can be extended to a framework in which alive deprivation is measured as a continuous variable such as an income deprivation score or a multidimensional poverty score, provided that the axioms are duly adapted (see Foster and Shorrocks 1991).

B. *Generated Deprivation*

We focus here on current mortality and define the set of mortality vectors as

$$\mathcal{M} = \left\{\mu \in [0,1]^{a^*+1} \middle| \mu_{a^*} = 1\right\}.$$

We consider pairs $(\mathbf{x}, \mu)$ for which the distribution $\mathbf{x}$ is a priori unrelated to vector μ. We assume that the age-specific mortality rates μ_a must be feasible given the number of alive individuals $n_a(\mathbf{x})$. Given that distributions have finite numbers of individuals, mortality rates cannot take irrational values, i.e., $\mu_a \in [0,1] \cap \mathbb{Q}$, where $\mathbb{Q}$ is the set of rational numbers. The set of pairs considered is[18]

$$\mathcal{O} = \Big\{ (\mathbf{x}, \mu) \in \mathcal{X} \times \mathcal{M} \,|\, \text{for all } a \in \{0, \ldots, a^*\} \text{ with } n_a(\mathbf{x}) > 0 \quad \text{we have } \mu_a = \frac{c_a}{n_a(\mathbf{x})} \text{ for some } c_a \in \mathbb{N} \Big\}.$$

An index is a function $\mathbf{P} : \mathcal{O} \to \mathbb{R}_+$.

Our characterization above argues that, when measuring deprivation using past mortality, ID is the appropriate measure. As current mortality is always the same as past mortality in stationary populations, we therefore require that any index defined on current mortality rates is equivalent to ID in the case of stationary populations.[19] Let $\mathcal{O}^*$ denote the subset of all pairs in $\mathcal{O}$ that are stationary.

DEPRIVATION AXIOM 9 (ID Equivalence): *There exists some $\gamma > 0$ such that for all $(\mathbf{x}, \mu) \in \mathcal{O}^*$, we have $\mathbf{P}(\mathbf{x}, \mu) = ID_\gamma(\mathbf{x})$.*

Besides ID Equivalence, GD is characterized by two desirable properties. First, GD satisfies Additive Decomposibility, a strengthening of Subgroup Consistency. This property implies that, if deprivation decreases in a subgroup while remaining unchanged in the rest of the population, overall deprivation declines.

DEPRIVATION AXIOM 10 (Additive Decomposibility): *For all $(\mathbf{x}', \mu'), (\mathbf{x}'', \mu'') \in \mathcal{O}$, if $\mathbf{x} = (\mathbf{x}', \mathbf{x}'')$ and $\mu_a = \left(n_a(\mathbf{x}') \times \mu'_a + n_a(\mathbf{x}'') \times \mu''_a\right) / \left(n_a(\mathbf{x}') + n_a(\mathbf{x}'')\right)$ for all $a \in \{0, \ldots, a^*\}$, then*

$$\mathbf{P}(\mathbf{x}, \mu) = \frac{\eta(\mathbf{x}', \mu') \times \mathbf{P}(\mathbf{x}', \mu') + \eta(\mathbf{x}'', \mu'') \times \mathbf{P}(\mathbf{x}'', \mu'')}{\eta(\mathbf{x}', \mu') + \eta(\mathbf{x}'', \mu'')}, \tag{4}$$

where the "size" function $\eta : \mathcal{O} \to \mathbb{N}_0$ is such that $\eta(\mathbf{x}, \mu) = \eta(\mathbf{x}', \mu') + \eta(\mathbf{x}'', \mu'')$.

[18] To be complete, the definition of $\mathcal{O}$ is such that for all $a \in \{0, \ldots, a^*\}$ with $n_a(\mathbf{x}) = 0$ we have $\mu_a = 0$ when $a < a^*$ and $\mu_{a^*} = 1$.

[19] Recall that past mortality is recorded in distribution $\mathbf{x}$, while current mortality is recorded in vector μ. As vector μ is redundant in stationary pairs, the index can be computed on distribution $\mathbf{x}$ only. Proposition 1 then argues that the index should be equal to ID_γ.

Additive Decomposibility implies that the index is decomposable in subgroups. A decomposable index measured on a set of individuals can always be calculated as the weighted sum of the same index measured on any partition of this set, where the weight attributed to a subset is the fraction of its reference population divided by the total reference population. This property matters if one wishes to compare the relative deprivation of different groups in a society, such as men and women or Black and White individuals.

Second, GD does not directly depend on mortality prior to period t and satisfies instead Independence of Dead* (where the asterisk denotes that Independence of Dead is adapted to domain $\mathcal{O}$).

DEPRIVATION AXIOM 11 (Independence of Dead*): *For all* $(\mathbf{x}, \mu) \in \mathcal{O}$ *and* $i \leq n(\mathbf{x})$, *if* $s_i = D$, *then* $\mathbf{P}\big((\mathbf{x}_i, \mathbf{x}_{-i}), \mu\big) = \mathbf{P}(\mathbf{x}_{-i}, \mu)$.

Proposition 2 shows that these three properties jointly characterize GD.

PROPOSITION 2 (Characterization of GD): $\mathbf{P} = GD_\gamma$ *for some* $\gamma > 0$ *if and only if* $\mathbf{P}$ *satisfies ID Equivalence*, *Independence of Dead**, *and Additive Decomposibility.*

PROOF:

See online Appendix. ∎

We conclude this section by discussing a particular feature of GD. By definition, the reference population of GD depends on premature mortality. The larger the premature mortality associated with vector μ, the larger GD's reference population. Such is not the case for ID since (past) premature mortality changes the *status* of individuals in ID's reference population but not its size. Given that its reference population depends on premature mortality, GD may violate the following natural requirement: for a fixed alive deprivation, if life-span deprivation increases, total deprivation should not decrease. We therefore require GD to satisfy Monotonicity in Current Mortality.

DEPRIVATION AXIOM 12 (Monotonicity in Current Mortality): *For all* $(\mathbf{x}, \mu), (\mathbf{x}, \mu') \in \mathcal{O}$, *if* $\mu_a \geq \mu'_a$ *for all* $a \in \{0, \ldots, \hat{a} - 2\}$, *then* $\mathbf{P}(\mathbf{x}, \mu) \geq \mathbf{P}(\mathbf{x}, \mu')$.

An implication of Monotonicity in Current Mortality is that GD cannot attribute a lower weight to one PYPL than to one PYAD.

PROPOSITION 3: GD_γ *satisfies Monotonicity in Current Mortality if and only if* $\gamma \geq 1$.

PROOF:

See online Appendix. ∎

Note that, as shown in the Proof for Proposition 3, the condition under which GD is monotonic in mortality rates for a given society requires the weight γ to be at least as large as the society's head-count ratio.[20] As a result, when alive deprivation is moderate, the constraint on γ is less restrictive.

III. Comparison with Alternative Approaches

Several measures have been proposed in the literature to combine basic welfare with mortality indicators into a single index. In this section, we compare our deprivation indexes to these alternative measures. This allows us to discuss some important assumptions underlying the construction of our indexes.

A. *Composite Indexes*

The first approach is to use *composite* indexes such as the Human Development Index. This simple indicator of well-being aggregates mortality with income information as a weighted sum of its mortality and income components, typically using equal weights. As discussed in Ravallion (2012b), this type of aggregation hides underlying trade-offs between the dimensions being aggregated. More fundamentally, as shown in the introduction, a composite deprivation index $\mathbf{P}_w$ is inconsistent with a basic separability property, as it does not assign a fixed relative weight to one PYPL compared to one PYAD. In particular, when allowing some individuals to live longer in alive deprivation instead of dying prematurely, the index $\mathbf{P}_w$ may increase or decrease, depending on the fraction of the living population that is initially income poor.[21]

Closely related to the indexes proposed in this paper, the Human Poverty Index (HPI) is a composite index that aggregates both premature mortality and alive deprivation (Watkins 2006). Its premature mortality component $HC_{\hat{a}}$ measures the probability that a newborn dies before turning $\hat{a}$ years. The HPI is defined as a weighted average of alive deprivation, HC, and life-span deprivation thus defined, $HC_{\hat{a}}$:

$$HPI_w(\mathbf{x},\mu) = w \times HC(\mathbf{x}) + (1-w) \times HC_{\hat{a}}(\mu),$$

with $w \in [0,1]$. One can easily adapt the example given in the introduction to show that the HPI suffers from the same inconsistency as index $\mathbf{P}_w$, even though HPI does not measure premature mortality in time units.[22]

The inconsistency affects all composite indexes as soon as their components have different reference populations. In the case of $\mathbf{P}_w$, its alive deprivation component

[20] This is not in contradiction with the stronger necessary condition stated in Proposition 3. This proposition provides the condition under which GD is monotonic *for all* pairs $(\mathbf{x},\mu)$ in $\mathcal{O}$, some of which have an HC equal to one.

[21] This problem does not depend on the value of the parameter w. For all possible values of the $w \in (0,1)$, one can always find situations under which the composite index $\mathbf{P}_w$ is not consistent.

[22] Consider, for instance, three stationary societies: A, B, and C. Four individuals are born every year. In society A, all individuals live until they reach old age. Two individuals live their whole life in poverty, and the other two are never poor. $HC(A) = 0.5$, $HC_{\hat{a}}(A) = 0$, and $HPI_{0.5}(A) = 0.25$. Society B is the same as society A, except that one poor individual dies in early childhood. $HC(B) = 0.33$, $HC_{\hat{a}}(B) = 0.25$, and $HPI_{0.5}(A) = 0.29$. Society C is the same as society A, except that the two poor individuals die in early childhood. $HC(C) = 0$, $HC_{\hat{a}}(C) = 0.5$, and $HPI_{0.5}(C) = 0.25$.

divides the number of PYADs by the number of PYs spent *alive,* while its mortality component divides the number of PYs spent alive by the normative life-span. The implicit weight that this index attaches to one PYAD over one PYPL therefore depends on the relative levels of alive deprivation and life expectancy. The root of the problem is that $\mathbf{P}_w$ first normalizes each component using different reference populations, before summing them. In contrast, our total deprivation indexes add the number of PYADs to the number of PYPLs *before* normalizing by the same reference population. As a result, the relative weight attributed to one PYPL over one PYAD remains constant. Moreover, the value of this weighing parameter can be chosen normatively, in a meaningful and transparent way. Another difference between our indexes and composite indexes is that our total deprivation indexes generalize the alive deprivation index. In the absence of premature mortality, they are identical to alive deprivation, as measured by *HC*.

B. *Preference-Based Indicators and the Choice of* γ

A second approach is to use preference-based indicators that aggregate the quality and quantity of life by assuming or calibrating a particular intertemporal utility function, unique across time and space (Becker, Philipson, and Soares 2005; Grimm and Harttgen 2008; Jones and Klenow 2016). These indicators are partly based on the actual achievements of nonpoor individuals, which our deprivation indexes disregard. Moreover, these approaches implicitly attribute values to one year of extra life that vary with the living standards of the country, reflecting the higher opportunity cost of dying in richer countries. This property is shared by some composite measures of well-being, such as the Human Development Index (HDI). As shown by Ravallion (2012b, a), the implicit value of one extra year of life given by the HDI is typically larger in richer countries.

Our indicators instead aggregate alive and life-span deprivation without relying on a particular representation of the preferences. From the perspective of the practitioner, they require fewer normative assumptions and essentially rely on selecting values for two transparent normative parameters: the age threshold $\hat{a}$ and γ, the fixed weight parameter. Also, in our deprivation framework, there is at the individual level no reason to trade off differently one PYAD over one PYPL depending on the observed levels of alive deprivation and life expectancy: a fixed weight γ is a natural requirement. Fundamentally, the two dimensions we compare in computing these indexes are deprivation statuses (i.e., being poor or being prematurely dead) instead of "achievements" (such as mean income or life expectancy). Under a common utility function, a given deprivation status (AP or D) leads to a fixed level of instantaneous utility, regardless of the country.[23] As a result, at the individual level, a given deprivation status carries the same weight in all countries, and the trade-off between two deprivation statuses is the same across countries. Of course, a country could become increasingly averse to life-span deprivation (or poverty) as its level

[23] For instance, two individuals living in different countries are assumed to have the same (low) utility if they are both under the extreme poverty line of the World Bank.

increases, implying that its normative weight would depend on this level.[24] Here, such judgments are ruled out by Subgroup Consistency, which forces the deprivation index to use the same normative weight on the whole population as that used on subgroups. As an individual can constitute a subgroup, this property forces the index to use a constant weight, regardless of the level of life-span deprivation (or poverty). In other words, having a constant weight is necessary if one wishes to compare the relative deprivation of different groups in a society (Additive Decomposibility).

As the discussion above illustrates, parameter γ is conceptually different from the HDI's trade-off between mean income and life expectancy. However, they are not entirely disconnected. Even if neither mean income nor life expectancy have intrinsic value in our indexes, they indirectly affect alive deprivation and life-span deprivation. For instance, poverty typically decreases nonlinearly with increases in mean income, whereas life-span deprivation typically decreases nonlinearly with increases in life expectancy. Using an estimate of these nonlinear functions, one could in principle compute the average loss in mean income that leaves our index unaffected when life expectancy is increased by one year. Such a monetary estimate of the value of one extra year of life depends directly on parameter γ. If γ is close to zero, then the monetary estimate is close to zero, and when γ tends to infinity, so does this estimate.

C. *Deprivation Measures Improving on the Mortality Paradox*

The third approach keeps an exclusive focus on poverty but "corrects" poverty measures for the higher mortality rates affecting low-income groups. Kanbur and Mukherjee (2007) argue that such selective mortality leads to serious mismeasurement of income poverty. Indeed, higher mortality rates among the poor lead to a "mortality paradox," whereby poor who died early are ignored. A central objective of that literature is to design poverty measures that explicitly take this into account. The idea is to remove the *instrumental* impact of selective mortality on standard poverty measures by assigning fictitious incomes to the prematurely dead individuals (Kanbur and Mukherjee 2007; Lefèbvre, Pestieau, and Ponthiere 2013, 2018). The validity of these approaches relies on the assumptions made in the construction of these counterfactual, "fictitious" incomes.

In this perspective, premature mortality is not *intrinsically* valued but only taken into account when associated with poverty. Our approach is fundamentally different, as we consider premature death as an intrinsic form of deprivation. We do not think that the income an individual would have earned had she remained alive is relevant to quantify the total deprivation experienced by a society. Consider two populations, A and B, which are identical except that in A, the prematurely dead individuals would have been poor had they lived, but not in B. By construction, alive deprivation and life-span deprivation are the same in both societies. In our view, the total deprivation experienced in these two societies is identical, while the measures addressing the mortality paradox would systematically consider population A as poorer.

[24] As pointed out by a referee, in the case of GD, such society would have its parameter γ depend on $p(\mathbf{x})$ or $d^{GD}(\mathbf{x}, \mu)$.

To take another example, consider a poor population made of two subgroups of equal size, say men and women. Both women and men are poor in the first period. If they survive to the second period, women become nonpoor, but men stay poor. In scenario A, all women die at the end of the first period, while all men survive. In scenario B, all men die at the end of the first period, while all women survive. Premature mortality is the same in the two scenarios, but there is more poverty in A because men survive. While we would conclude that deprivation is higher in scenario A, poverty measures correcting for the mortality paradox would typically assess the same levels of poverty across the two scenarios. In other words, these measures allow the higher counterfactual poverty of the prematurely dead to outweigh the lower poverty levels of the living.

Clearly, poverty is certainly a cause of premature mortality (and vice versa), and policy recommendations should certainly take these causal relationships into account. However, we do not believe that these *positive* relationships matter for the *normative* comparison of deprivation outcomes. Here is an analogy. An individual derives the utility $U(b,f)$ from the number of bees (b) and the number of flowers (f) in his garden. In practice, the number of flowers affects the number of bees and vice versa. Yet the normative evaluation of the garden, i.e., the function U, does not take into account the causal links between b and f.

The literature on the mortality paradox proposes various methods to assign fictitious incomes to missing individuals. One such method assigns fictitious incomes regardless of the premortem income of missing individuals (e.g., Lefèbvre, Pestieau, and Ponthiere 2013, 2018). This idea can in principle be applied in our constrained information setup. However, the definition of a missing poor used there is conceptually very different from ours, as it is based on a reference mortality vector, corresponding to that of the most affluent societies, such as Norway or the United States: the missing population is defined as those individuals who died *in excess* with respect to this reference mortality vector. As a result, not all individuals dying prematurely are considered as missing individuals, while an 80-year-old individual dying in excess would. Our deprivation approach focuses on all premature deaths and, therefore, does not rely on a notion of excess mortality relative to a reference vector.

Alternatively, one could assign fictitious incomes that depend on the incomes earned before dying. Thus, Kanbur and Mukherjee (2007) attribute to rich individuals dying prematurely fictitious incomes that are above the deprivation threshold. In our approach, we do not distinguish between the premature mortality affecting the poor and that affecting the nonpoor. As noted in the introduction, the necessary information on the mortality rates of different income groups is often not available.

More fundamentally, a normative issue raised by the literature on multidimensional poverty (Bourguignon and Chakravarty 2003) is that there is more overall poverty if the same individuals concentrate several dimensions of deprivation. The premature mortality of poor individuals constitutes such a nondesirable concentration of deprivations. To address this question, we need to distinguish mortality rates between poor and nonpoor individuals. To make our indexes sensitive to concentration, we can, for instance, define an individual as being in *total poverty* if she spends more than k person-years in deprivation, either in the form of PYPLs or PYADs.

Our indexes can therefore be accommodated to allow for this type of approach.[25] However, to compute such concentration-sensitive indexes of total poverty, we need not only mortality rates by income groups but also information on *mobility in and out* of alive deprivation across consecutive periods, a type of information that is typically not available.[26]

IV. Measuring Deprivation

In this section, we apply our indexes of total deprivation for low- and middle-income countries for the period of 1990 to 2015. Our objective is to characterize the level of deprivation worldwide, how it differs across countries, and how it has changed over time, as well as to understand how these patterns, based on total deprivation, differ from those based on a more standard poverty measure such as the head-count ratio.

ID requires information on the number of deaths by age in the past $\hat{a}$ years. Such information exists, for example in the Human Mortality Database, but the countries available in this database are very different from those for which comparable alive deprivation data are available or for which deprivation measures would be relevant. In addition, as illustrated in Section ID, ID also requires country borders to remain stable in the last $\hat{a}$ years to be meaningful, while the twentieth century has seen considerable changes in countries' borders and in the number of countries. For these reasons, ID is ill-suited for our empirical exercise, and we focus in the following on GD, which we consider as the most relevant index in practice, given the data constraints.

A. *Data*

The definition of our indexes requires a value for the age threshold $\hat{a}$ and a weight γ. As already discussed, the latter will be set conservatively at one, so that one person-year prematurely lost is equivalent to one person-year spent in income deprivation. Choosing a higher value for γ, by increasing the weight given to the mortality component, would simply magnify the difference with respect to HC.

The choice of the age threshold is analogous to the choice of an income threshold used for income deprivation. It is ultimately a normative choice about the minimum number of years of life that a society judges essential for its members. In the following, we use a threshold $\hat{a} = 50$ years, which is much lower than the median age at death observed in our data (64 years old). Of course, a higher age threshold would inflate our indexes and their difference with income deprivation measures.[27]

The computation of GD requires information on alive deprivation as well as information on mortality by age in the period under study. In the following, we make use of two publicly available datasets to construct our measures of deprivation. The data

[25] Such a definition of total poverty is consistent with the definition of multidimensional poverty proposed by Alkire and Foster (2011): an individual is multidimensionally poor if she is deprived in at least k dimensions.

[26] Note also that, when mobility is very low and premature mortality is mostly concentrated on poor individuals, our indexes approximately count the number of person-years lost to deprivation by the poor.

[27] In the code available online, the reader can compute deprivation indexes choosing various values of $\hat{a}$ and γ as well as alternative income poverty thresholds.

on population and mortality by country, age group, and year come from the Global Burden of Disease database (2017 version of the data) (Global Burden of Disease Collaborative Network 2018). Comparable information across countries and over time is available for the 1990–2017 period and is, to our knowledge, the most comprehensive mortality data available for international comparison. To construct this database, population and mortality data are systematically recorded across countries and time from various data sources (official vital statistics data, fertility history data as well as data sources compiling deaths from catastrophic events). These primary data are then converted into data at the age group, year and country level using various interpolations and inference methods (see Dicker et al. 2018 and Murray et al. 2018 for more information on the GBD data construction).[28]

Data on alive deprivation come from the PovCalNet website (World Bank's Development Research Group 2019), which provides internationally comparable estimates of income deprivation level. This dataset is based on income and consumption data from more than 850 representative surveys carried out in 127 low- and middle-income countries between 1981 and 2015.[29] Each country's income deprivation level in PovCalNet is computed on a three-year basis, so that the yearly data used below were obtained by a linear interpolation of income deprivation estimates across years. A complete description of the dataset is given in Chen and Ravallion (2013).[30] In our empirical application, we follow the World Bank's definition of extreme income deprivation, corresponding to the \$1.90 a day threshold (Ferreira et al. 2016). We merged the two databases at the year and country level. Since the Global Burden of Disease data are only available since 1990 and the PovCalNet data until 2015, we focus on the 1990–2015 period for a total of 113 low- and middle-income countries (see online Appendix Table 2 for a list of those countries).

B. *Worldwide Deprivation*

GD is defined on the total number of years of deprivation generated in a given year. Table 3 presents this computation for the world in 1990 and 2015. In 2015, 1,105 million person-years of deprivation have been generated, 703 million from income deprivation and 402 from life-span deprivation. Relative to the reference population (that is, 6,010 million alive individuals to which we add 402 million person-years lost to life-span deprivation), this implies that 17.2 percent of the person-years in 2015 were lost to deprivation. This is much lower than the deprivation level of 52.8 percent measured in 1990.

[28] The number of deaths in each cell is an estimate and comes with a confidence interval. Following the convention in the literature, we do not use these confidence intervals and only consider the point estimate of the number of deaths (see also Høyland, Moene, and Willumsen 2012 for a critique of this approach). Moreover, the mortality information is given into five-year age brackets (except for the zero–five years group, for which the information is decomposed into zero–one and one–five). When necessary, we transform the data into age groups of one year by assuming a uniform death rate within an age category. Finally, the older age group is "95 and above." As we do not know the precise age of death of individuals in that category, we assume that 95 is the maximum age they can reach. This last assumption is of no consequence since our age threshold $\hat{a}$ is well below 95.

[29] The website address is http://iresearch.worldbank.org/PovcalNet/povOnDemand.aspx.

[30] Clearly, these transformations may matter for the empirical analysis, as they tend to smooth the evolution of income deprivation across years. In particular, in the case of catastrophic events, income deprivation appears as less reactive than life-span deprivation, which may be due to the interpolated nature of the income deprivation data.

TABLE 3—GENERATED DEPRIVATION IN THE DEVELOPING WORLD IN 1990 AND 2015, WITH $\hat{a} = 50$

	Unit	1990 value	2015 value	Computation
Living population	Person-years (million)	4,200	6,010	Source: GBD (2017)
HC	Percent	44.9	11.7	Source: PovCalNet
Alive deprivation (PYAD)	Person-years (million)	1,886	703	Living population × HC
Life-span deprivation (PYPL)	Person-years (million)	701	402	See equation (3)
Deprived population	Person-years (million)	2,587	1,105	PYAD+PYPL
Reference population	Person-years (million)	4,901	6,412	Living population + PYPL
GD_1	Percent	52.8	17.2	$\frac{\textit{Deprived Population}}{\textit{Reference Population}}$

Figure 6 presents the evolution of the world's total deprivation (GD) and its two components, alive and life-span deprivation. We also report the HC ratio for comparison purposes.[31] First, GD and HC follow a similar trend and do not offer a different diagnostic about the evolution of world deprivation in the last 25 years. World deprivation fell dramatically between 1990 and 2015.

However, life-span deprivation is far from negligible. In 1990, it represented 27 percent of total deprivation. This number is a direct measure of the underestimation of deprivation when premature mortality is not taken into account. Even though life-span deprivation declined over this period, its relative importance increased over time: its share in total deprivation increased from 27 percent in 1990 to more than 36 percent in 2015. Given our conservative choice of parameters, these estimates can be considered as a lower bound. For example, if $\hat{a}$ is set at 80 years, total deprivation in 2015 would be underestimated by more than 60 percent. The increase over time in the share of life-span deprivation indicates that more progress has been made against alive deprivation than against life-span deprivation over the past 25 years. One can only wonder if this would have been the case had premature mortality systematically been taken into account in deprivation measures.

Child mortality is a major component of life-span deprivation: the deaths of children aged 5 or below account for almost 80 percent of GD's life-span component in 1990 and close to 70 percent in 2015. This is due to the fact that (i) mortality rates of this population are high relative to other age cohorts and (ii) the number of years prematurely lost generated by deaths at younger ages is large (a newborn death generates exactly $\hat{a}$ person-years lost).

Three potential mechanisms could be at work behind the observed decline in life-span deprivation between 1990 and 2015. First, natality rates could have fallen: given that young children have a relatively larger mortality rate, this leads to a decrease in deprivation, for given levels of mortality rates. Alternatively, mortality rates could have decreased, holding natality constant. Finally, the changes in life-span deprivation could be due to changes in the relative size of each cohort resulting from the changes in natality and mortality rates that occurred before 1990.

[31] Strictly speaking, HC and GD cannot be directly compared since they are based on different reference populations.

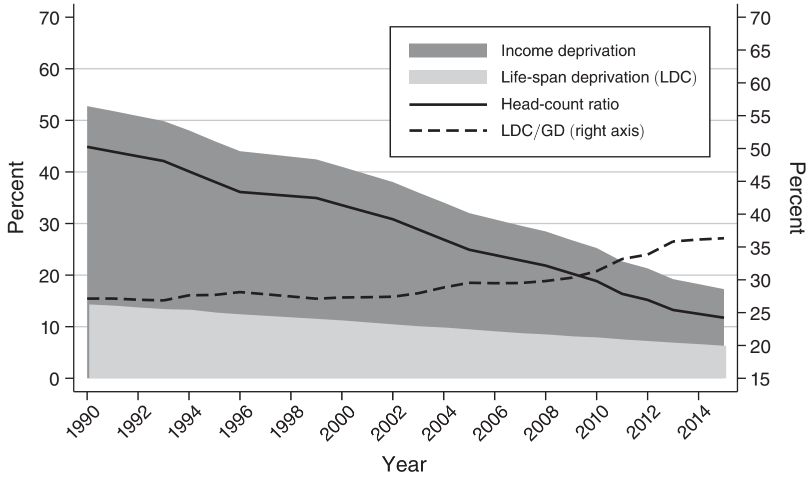

Figure 6. Decomposition of Generated Deprivation with $\hat{a} = 50$, World Level

Figure 7 illustrates the relative importance of these mechanisms using three different counterfactuals. In the first scenario, we maintain constant the number of births to its 1990 level, which neutralizes the effects of a change in natality rate. In the second scenario, we keep the age-specific mortality rates at their 1990 level, and in the third, we keep both the 1990 natality and age-specific mortality rates to concentrate on the changes in the population pyramid. Comparing the World curve (in solid line) to its various counterfactuals, the main drivers of the evolution in life-span deprivation are both the fall in mortality rates and the changes in the composition of the population pyramid. Mortality rates, particularly among newborns, fell dramatically from 6.6 percent in 1990 to 3.4 percent in 2015.[32] Changes in the population pyramid is the other important contributor, indicating that changes in mortality and natality prior to 1990 led to a long-run decline in the share of the cohorts with larger mortality rates. By contrast, natality rates remained essentially stable over the period (they increased by 2.8 percent between 1990 and 2015) and do not contribute to the observed changes in life-span deprivation.

C. *Country-Level Deprivation*

The Level of Deprivation.—We now examine deprivation at the level of individual countries. The parallel evolution of GD and HC at the world level may indeed hide large differences at less aggregated levels. Figure 8 provides an example that compares alive deprivation (HC) to total deprivation (GD) in Morocco and Gabon. According to HC, throughout the 1990s, Gabon and Morocco are virtually at the same level of alive deprivation. However, total deprivation is much higher in Gabon once life-span deprivation is taken into account.

[32] Note that since this amounts to a succession of permanent negative mortality shocks, we expect ID to be larger than GD between 1990 and 2015, as discussed in Section ID.

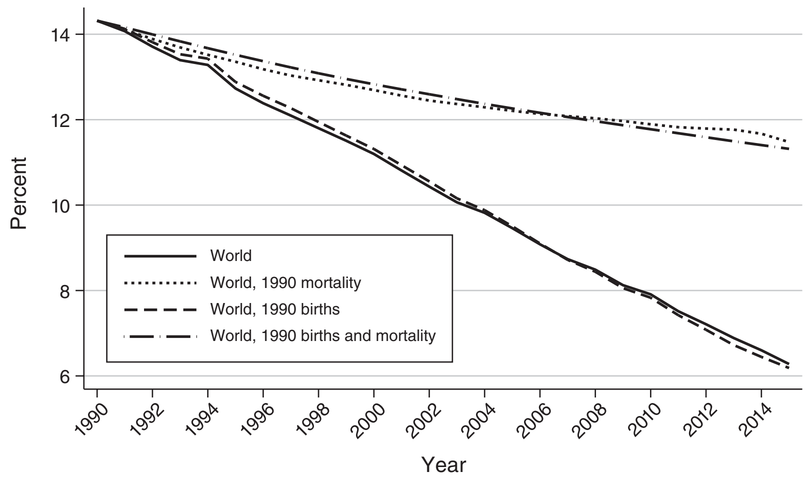

Figure 7. Counterfactual Worlds: Evolution of GD's Life-Span Deprivation Component, with $\hat{a} = 50$

Individual countries are often compared and ranked according to their level of deprivation (Høyland, Moene, and Willumsen 2012). On the basis of income deprivation (starting from the least poor), Gabon is nineteenth and Morocco twenty-first in 1993. If one uses total deprivation (GD) instead, Gabon is thirty-eighth and Morocco thirty-second. That's a difference of 9 ranks out of 113 countries. Table 4 decomposes the sources of this re-ranking in 1993. While both countries have a similar level of alive deprivation (with a value of HC around 5.5 percent), mortality rates are much higher in Gabon. Thus, in 1993, life expectancy at birth in Morocco is 67 years, against 59 years in Gabon.

More generally, some countries are actually much more deprived than originally thought on the basis of HC, and the use of GD leads to substantial re-rankings across countries. The average difference in ranking when applying GD instead of HC across all the 113 countries analyzed is equal to 4.1 ranks throughout the period. In particular, countries of the ex-USSR and a few African countries are much more deprived than indicated by the HC ratio, while the ranks of Latin American countries improve substantially. As the relative importance of the life-span component increases, these re-rankings have become larger over time, from 4.4 in 1990 to 4.9 in 2015. The largest re-ranking is that of Azerbaijan, which loses 26 ranks in 2010 and 2014: while alive deprivation almost disappeared in Azerbaijan, its population still faces relatively large levels of life-span deprivation.

The Evolution of Deprivation.—GD can also be used to assess the evolution of deprivation in a given country. Consider the cases of the Comoros and Botswana. Figure 9 illustrates the contrasting evolutions of HC and GD in the case of the Comoros for the period 1990–2015 and Botswana during the 1990s.

According to HC, alive deprivation increased in the Comoros by 60 percent between 1990 and 2015. However, over the same period, GD fell by 23 percent.

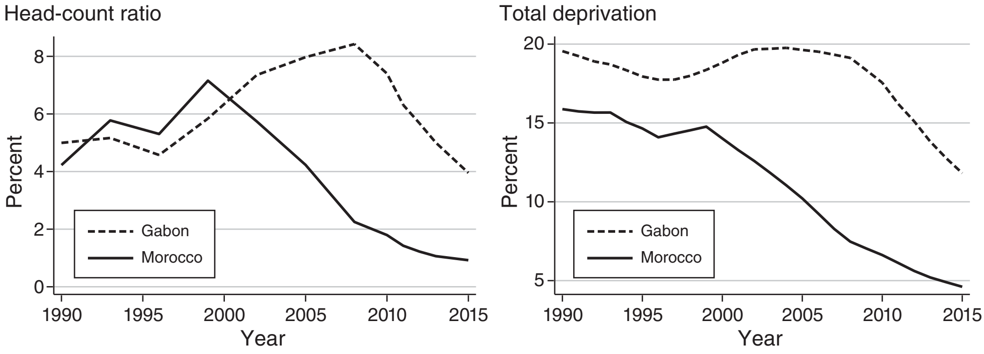

Figure 8. Examples of Re-rankings: Gabon and Morocco. HC and GD with $\hat{a} = 50$

Table 4—An Example of Re-ranking: Decomposition of GD in Gabon and Morocco in 1993

	Unit	Gabon	Morocco	Computation
Living population	Person-years (thousand)	1,043	26,449	Source: GBD (2017)
HC	Percent	5.2	5.8	Source: PovCalNet
Alive deprivation (PYAD)	Person-years (thousand)	54	1,534	Living population × HC
Life-span deprivation (PYPL)	Person-years (thousand)	174	3,100	See equation (3)
Deprived population	Person-years (thousand)	228	4,634	PYAD + PYPL
Reference population	Person-years (thousand)	1,217	29,549	Living population + PYPL
GD_1	Percent	17.9	15.7	$\frac{\textit{Deprived Population}}{\textit{Reference Population}}$

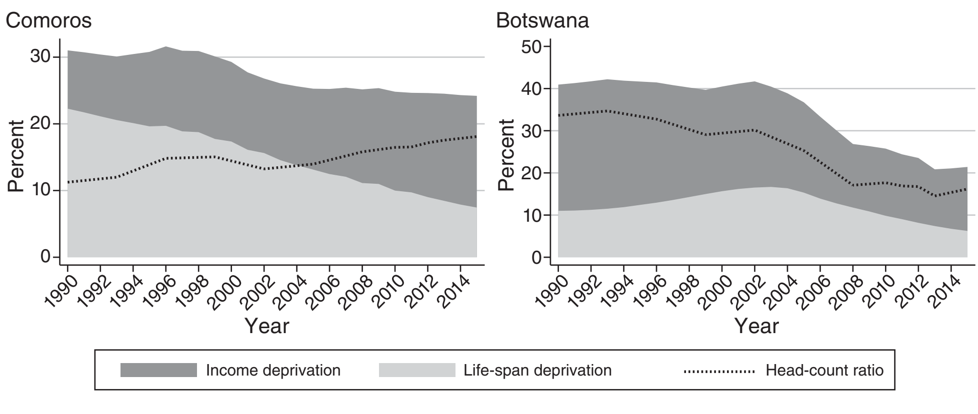

Figure 9. Differences in Trends between GD and HC: Comoros and Botswana. HC and GD with $\hat{a} = 50$

Focusing on alive deprivation hides the large progress made in life-span deprivation: while the number of PYADs increased more rapidly than the population, the number of PYPLs decreased drastically during the period. Overall, the total number of person-years lost to both types of deprivation remained constant, while total population increased by about 50 percent, leading to the observed fall in GD. In the case of Botswana, HC decreased by 12 percent between 1990 and 2001, while GD remained roughly stable (+0.7 percent). This is due to the large increase in mortality rates that followed the spread of the HIV epidemics.

Comoros and Botswana are two selected examples of the opposite diagnoses one can draw when measuring deprivation using GD instead of the HC ratio. How often do these opposite diagnostics arise in the last 25 years? In Figure 10, we plot the ratio of the value of GD in year t relative to its value in $t-5$ for each country in our sample against that for HC. As indicated by the figure, overall, the two measures generally agree. For most countries and periods, a decrease (increase) in HC is accompanied by a decrease (increase) in GD. Note that the relation between the two measures is flatter than the 45º line, which indicates that HC varied more than GD, owing to the fact that more progress was made against alive deprivation than against life-span deprivation between 1990 and 2015. However, the two measures do not always agree, as attested by the large number of points located in the northwest and in the southeast quadrants. These points represent 8.4 percent of the comparisons made: in these cases, the diagnostic of deprivation based on income contradicts the one based on total deprivation. Note that this result relies on the conservative assumption $\gamma = 1$, as this percentage goes up to 26.9 percent when the value of γ tends to infinity. Finally, given the increasing importance of life-span deprivation in total deprivation, these reversals are bound to be much more frequent in the future.

V. Concluding Remarks

Most measures of poverty or deprivation ignore premature mortality. In this paper, we propose two measures of "total deprivation" that combine meaningfully information on income poverty and early mortality in a population, by adding time units spent in income poverty and time units of life lost due to premature mortality. This additive approach follows from the mutually exclusive nature of the two dimensions considered, poverty and premature death. We characterize our proposed measures, show that they satisfy a number of desirable properties, and contrast their implications with existing indexes. Among the two measures we propose here, the generated deprivation index is probably the most relevant, given the data available. It is based on current mortality, as measured by the number of years prematurely lost by individuals dying in the current period. It captures how much deprivation has been generated in a given year, which makes it more sensitive to contemporaneous changes in the society.

Our aggregation method allows placing an explicit and meaningful lower bound on the normative trade-off (the weight γ) between premature mortality and poverty. This lower bound is based on the view that being prematurely dead is no better than being in alive deprivation ($\gamma \geq 1$). Using this conservative approach, we estimate that about one-third of total deprivation worldwide is generated by premature mortality and two-thirds by income poverty. Given the overall decline in income poverty worldwide, the importance of life-span deprivation is bound to increase over time, which justifies the systematic inclusion of life-span deprivation into deprivation measures. At the country level, ignoring premature mortality leads to biased evaluations in the level and in the evolution of deprivation.

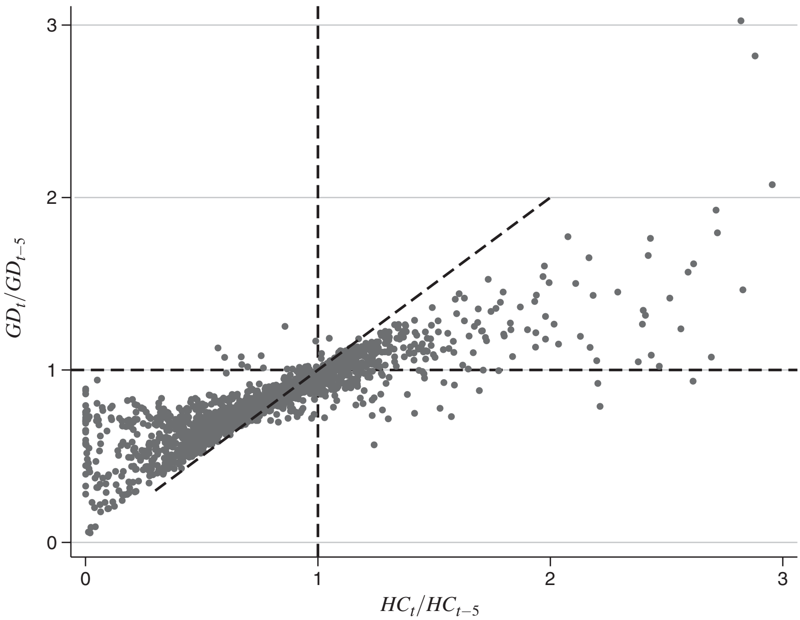

Figure 10. Deprivation Trends. HC and GD with $\hat{a} = 50$, t to $(t-5)$ Ratios

Note: Only observations with $HC_t/HC_{t-5} < 3$ are reported.

Our indexes provide a new lens through which to approach the trade-off between saving lives and the cost of doing so. To analyze such a trade-off, economists typically resort to measuring the value of life, as implicitly revealed by policy choices or legislative measures (e.g., Viscusi 1993). This concept is, however, often incomprehensible in larger audiences, which prevents critical public debates from taking place. The indexes we propose do not formulate this trade-off in money terms but in the number of years spent in poverty that can be generated to save one year of life. We hope this alternative formulation will prove less controversial, thereby allowing a more peaceful deliberation.

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