Annual Meeting of the COMPLEX Doctoral School
UMONS

Self-segregation in heterogeneous metapopulation landscapes

Jean-François de Kemmeter – 15/11/2022

Joint work with Timoteo Carletti and Malbor Asllani

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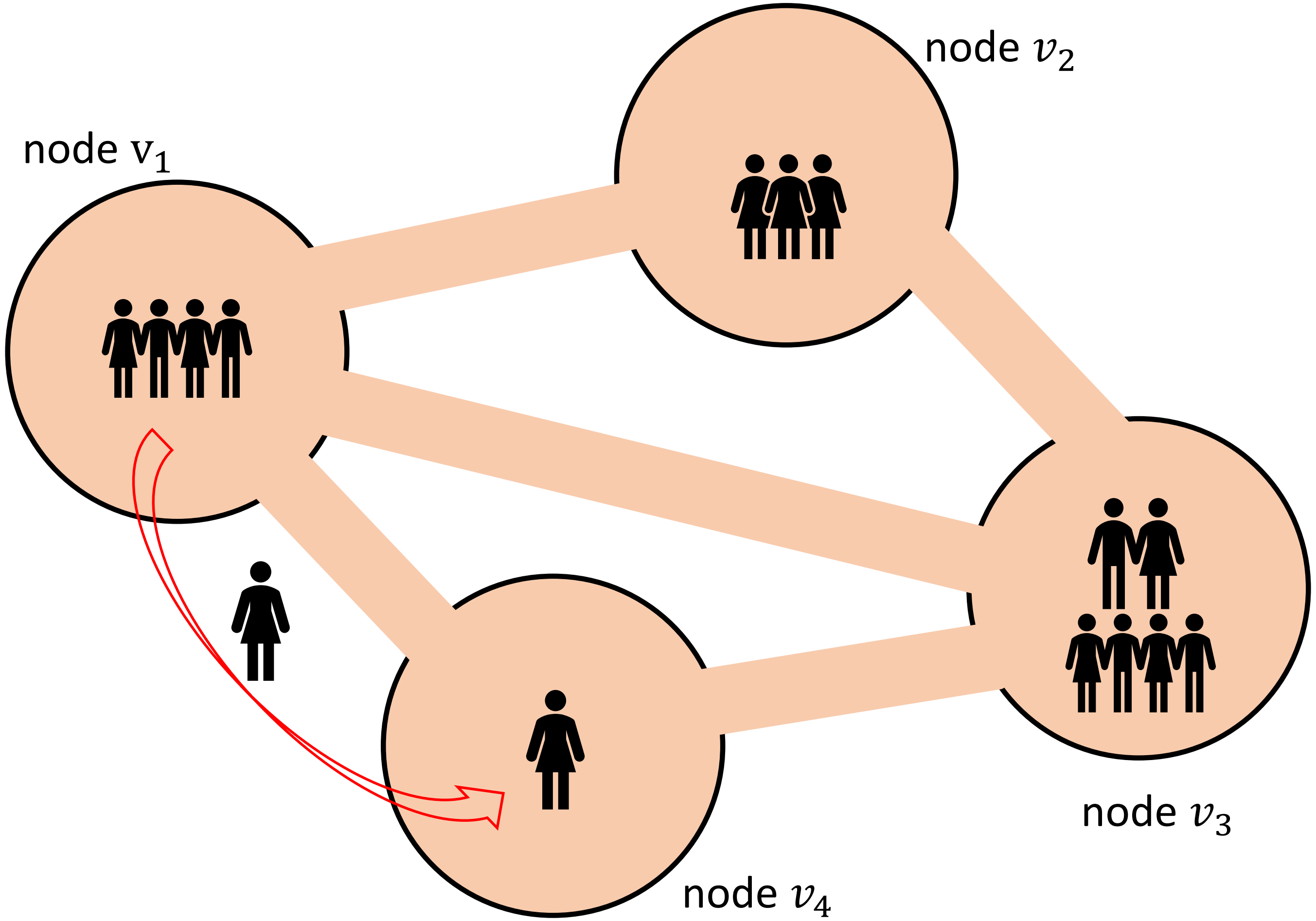
Plan

- Diffusion under crowded conditions: from the master equation to the deterministic limit
- Heterogeneous networks: emergence of “functional” communities
- Application to the habitat network of the Glanville fritillary butterfly
- Regular networks: symmetry breaking and emergence of empty nodes

Stochastic process



n_i : number of agents in node v_i $(i = 1, \cdots, \Omega)$

$\boldsymbol{n}(t) = (n_1(t), n_2(t), \cdots, n_\Omega(t))$

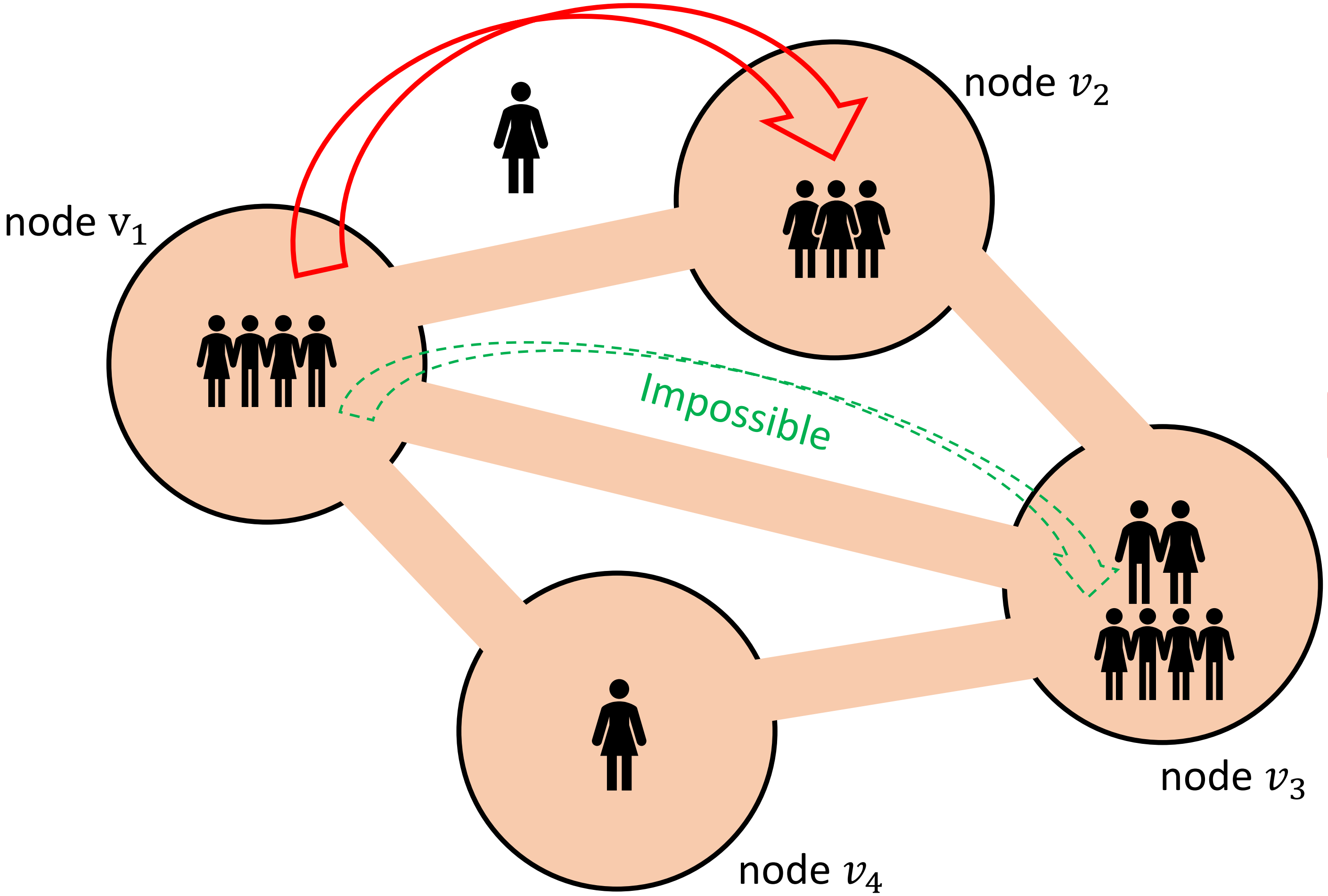
$P(\boldsymbol{n}, t)$: probability of observing state $\boldsymbol{n}$ at time t

Master equation

$$\frac{dP(\boldsymbol{n}, t)}{dt} = \sum_{\boldsymbol{n}' \neq \boldsymbol{n}} T(\boldsymbol{n}|\boldsymbol{n}')P(\boldsymbol{n}', t) - T(\boldsymbol{n}'|\boldsymbol{n})\, P(\boldsymbol{n}, t)$$

$T(\boldsymbol{n}'|\boldsymbol{n})$: transition probability from state $\boldsymbol{n}$ to state **n’**

Random walk in crowded conditions



Example : at most 6 agents per node

PHYSICAL REVIEW LETTERS **120**, 158301 (2018)

Hopping in the Crowd to Unveil Network Topology

Malbor Asllani,[1,*] Timoteo Carletti,[1] Francesca Di Patti,[2] Duccio Fanelli,[2] and Francesco Piazza[3]

PHYSICAL REVIEW RESEARCH **2**, 033012 (2020)

Nonlinear walkers and efficient exploration of congested networks

Timoteo Carletti,[1] Malbor Asllani,[2] Duccio Fanelli,[3] and Vito Latora[4,5,6]

Limiting carrying capacity: at most N agents per node

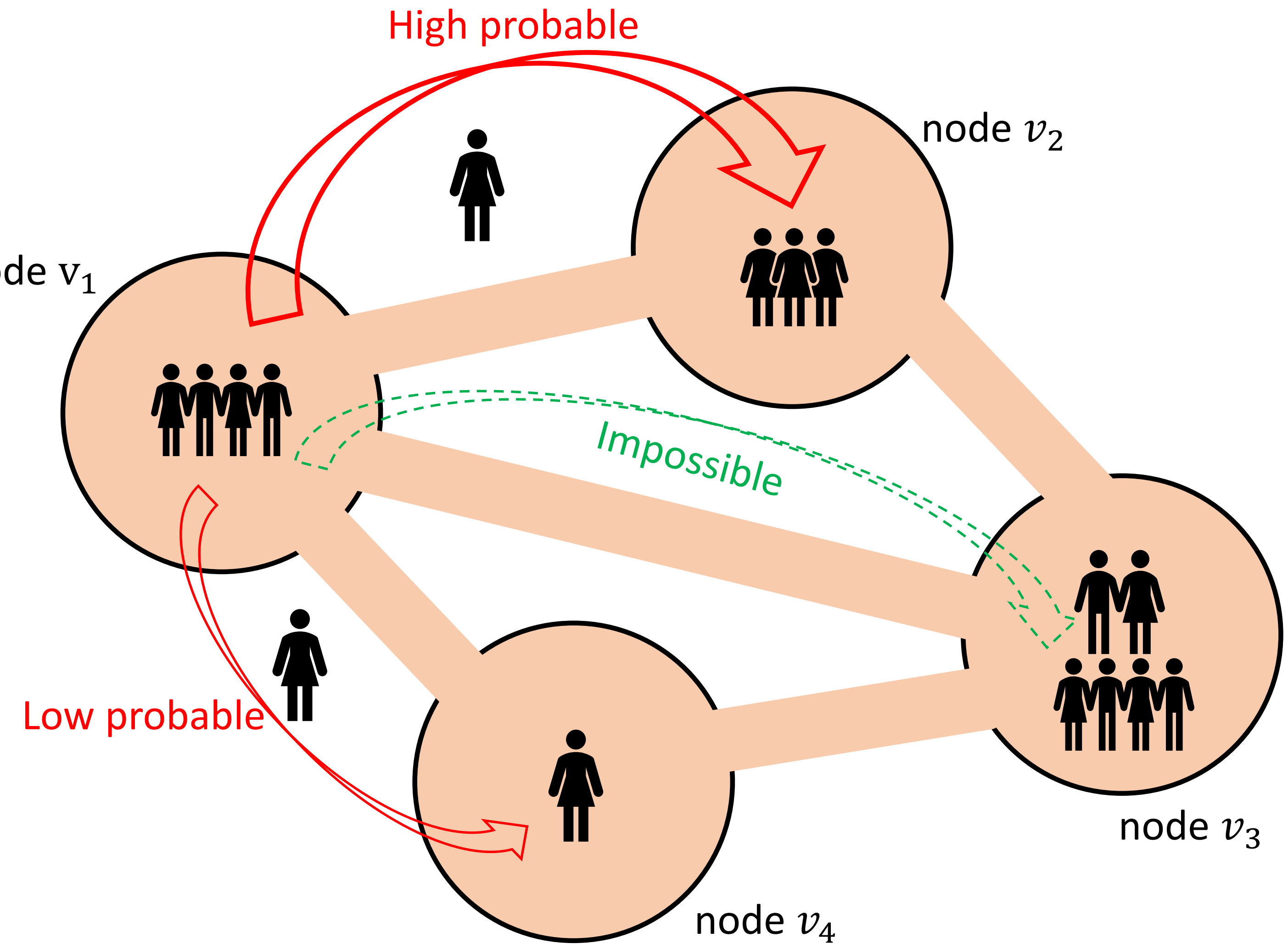
Willingness of agents to leave a node

$$T(n_i - 1, n_j + 1 | n_i, n_j) = \frac{A_{ij}}{k_i}\frac{n_i}{N}\, g\left(\frac{n_j}{N}\right)$$

Ability of agents to settle in a node

$$g(1) = 0, \text{ e.g. } g\left(\frac{n_j}{N}\right) = \left(\frac{N - n_j}{N}\right)^b$$

Self-recruitment process



PNAS

Larger groups are more successful in innovative problem solving in house sparrows

András Liker[1] and Veronika Bókony

$$T\left(n_i - 1, n_j + 1 \middle| n_i, n_j\right) = \frac{A_{ij}}{k_i}\frac{n_i}{N}\, g\left(\frac{n_j}{N}\right)$$

Ability of agents to settle in a node

$$g\left(\frac{n_j}{N}\right) = \left(\frac{n_j}{N}\right)^a \left(\frac{N - n_j}{N}\right)^b$$

Group cooperation   crowding

Mean-field equations

$$\rho_i \coloneqq \lim_{N\to+\infty}\left\langle\frac{n_i}{N}\right\rangle \Rightarrow \dot{\rho}_i = \sum_{j=1}^{\Omega}\left[-\frac{A_{ij}}{k_i}\rho_i g(\rho_j) + \frac{A_{ji}}{k_j}\rho_j g(\rho_i)\right] = \sum_{j=1}^{\Omega} L_{ij}\left[\rho_j g(\rho_i) - \frac{k_j}{k_i}\rho_i g(\rho_j)\right]$$

Random-walk Laplacian

$$L_{ij} = \frac{A_{ij}}{k_j} - \delta_{ij}$$

Mass conservation

$$\sum_i \dot{\rho}_i = 0$$

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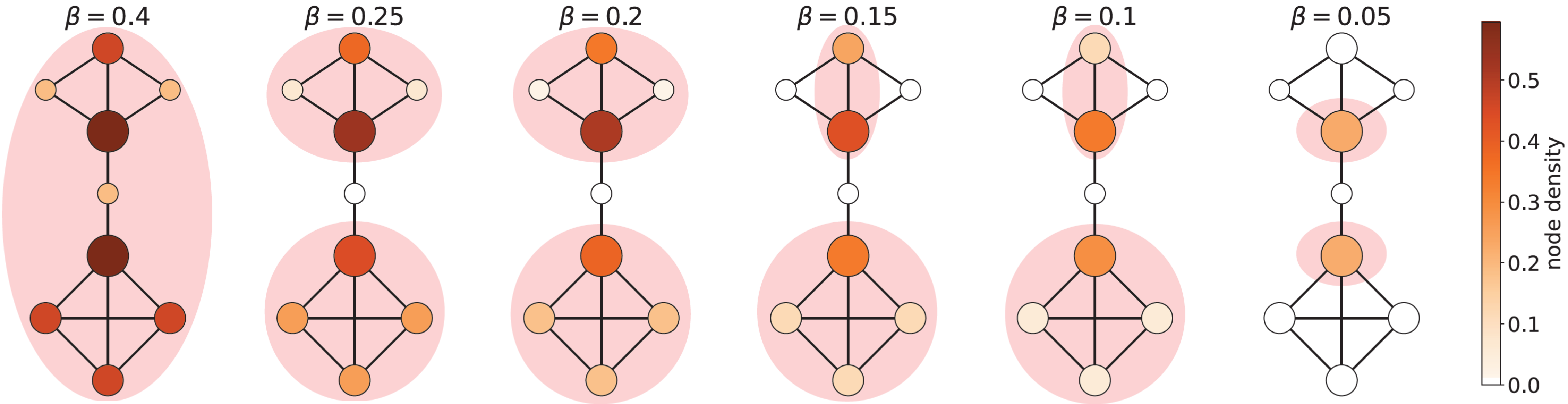
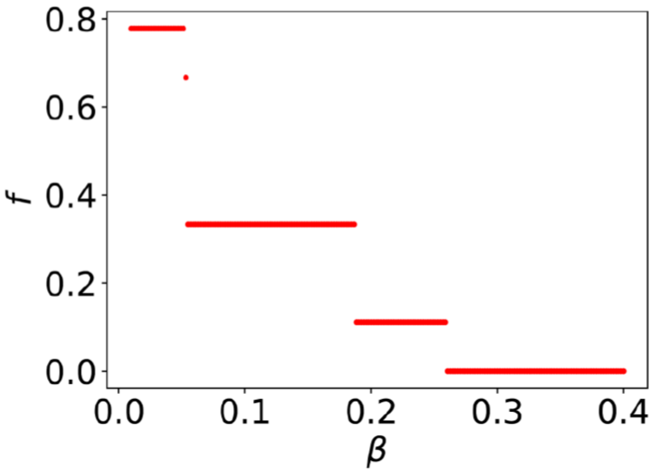
Self-segregation in heterogeneous metapopulation landscapes

Jean-François de Kemmeter [a,*], Timoteo Carletti [a], Malbor Asllani [b,c]

Fraction of empty nodes and quantization





Initial conditions : $\rho_i(0) = \beta \ \forall i$

f : fraction of empty nodes

$$g(x) = x(1-x)$$

$$\Downarrow$$

Stationary solutions $\rho_i^\infty = 1 - \frac{C_\gamma}{k_i}$ (γ^{th}cluster)

Application: Glanville fritillary butterfly

Glanville fritillary butterfly (Melitaea cinxia). From [1]

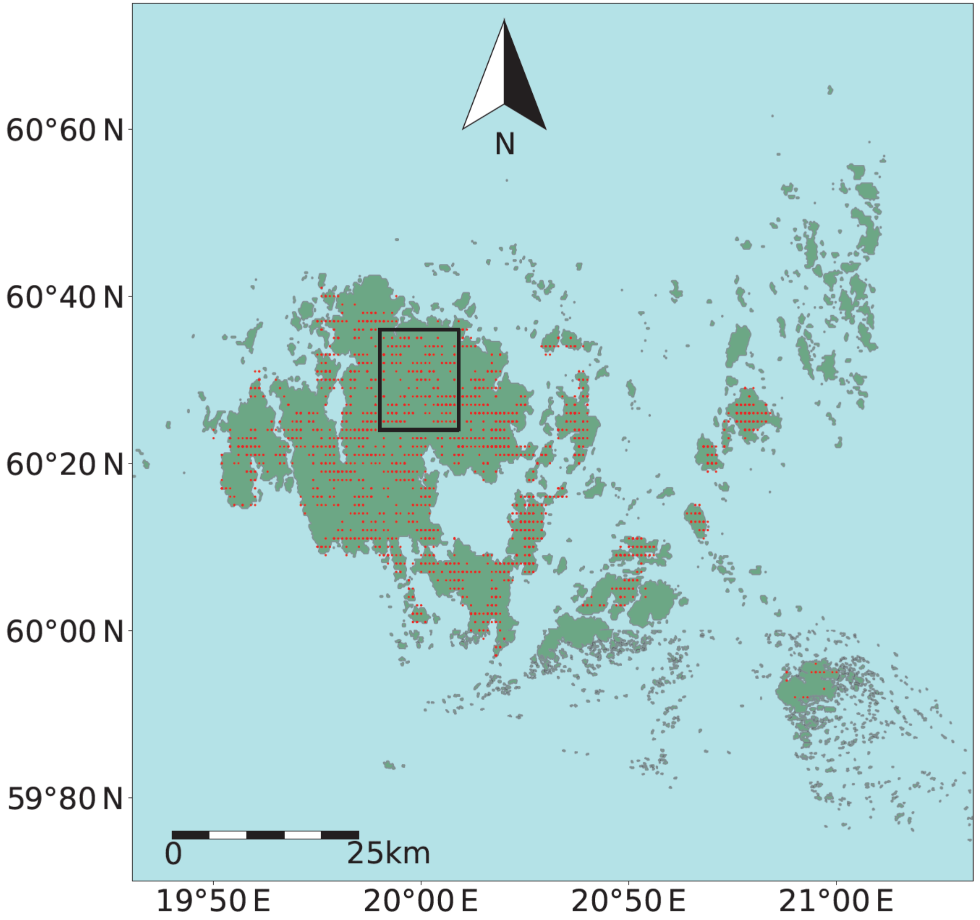
4500 suitable habitat patches [2]
15-20 % of occupied patches [2]
Mean dispersal distance ≈ 1km [3]

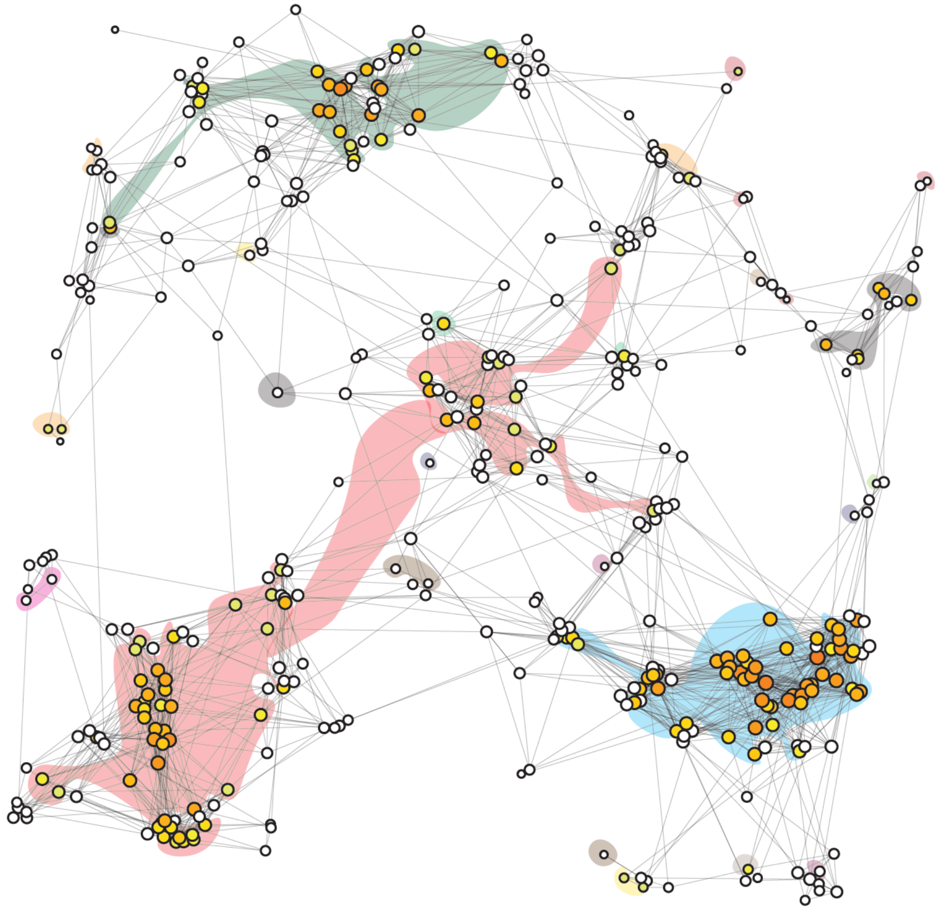
[1] https://butterfly-conservation.org/butterflies/glanville-fritillary

[2] Opedal, Øystein H., et al. "Host-plant availability drives the spatiotemporal dynamics of interacting metapopulations across a fragmented landscape." *Ecology* 101.12 (2020): e03186.

[3] Fountain, Toby, et al. "Inferring dispersal across a fragmented landscape using reconstructed families in the Glanville fritillary butterfly." *Evolutionary Applications* 11.3 (2018): 287-297.

Application: Glanville fritillary butterfly





Random geometric network with an exponential dispersal kernel $f(r) = \lambda e^{-\lambda r}$ with $\lambda = 0{,}65$
Average node density: $\beta = 0{,}08$

The case of regular networks

$$\dot{\rho}_i = \sum_{j=1}^{\Omega} L_{ij}\left[\rho_j g(\rho_i) - \rho_i g\left(\rho_j\right)\right] \text{ with } L_{ij} = \frac{A_{ij}}{k} - \delta_{ij}$$

Stability of the homogeneous fixed point ?

Let $\rho_i = \beta + \delta_i$ with $\beta \in (0,1)$ and $\delta_i = o(1)$

$\Longrightarrow$ $\dot{\delta} = J\delta$ with $J = \underbrace{[g(\beta) - \beta g'(\beta)]}_{\xi(\beta)}L$

Semi-definite negative matrix

$\Longrightarrow$ Stability condition: $\xi(\beta) > 0$

Ring

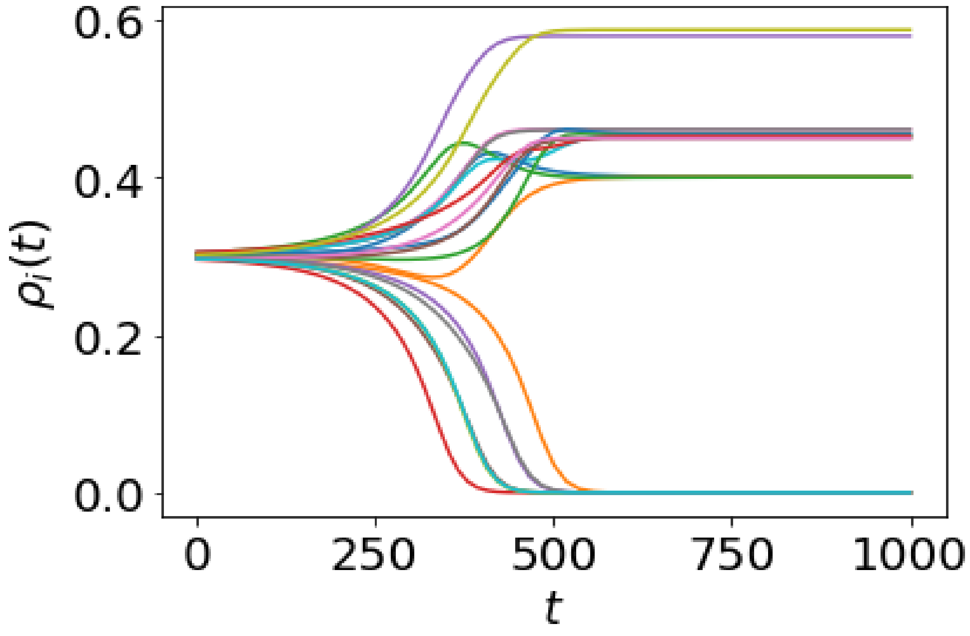
$$g(x) = x^a(1-x)^a \, , a \in \mathbb{R}^+$$

If $a \leq 1$: the homogeneous state is always stable

If $a > 1$: the homogeneous state is stable $\Longleftrightarrow \beta > \dfrac{a-1}{2a-1}$

$a = 2 \Longrightarrow$ Homogeneous state unstable if $\beta < \frac{1}{3}$

Ring network ($\beta = 0.3$; $\Omega = 20$)

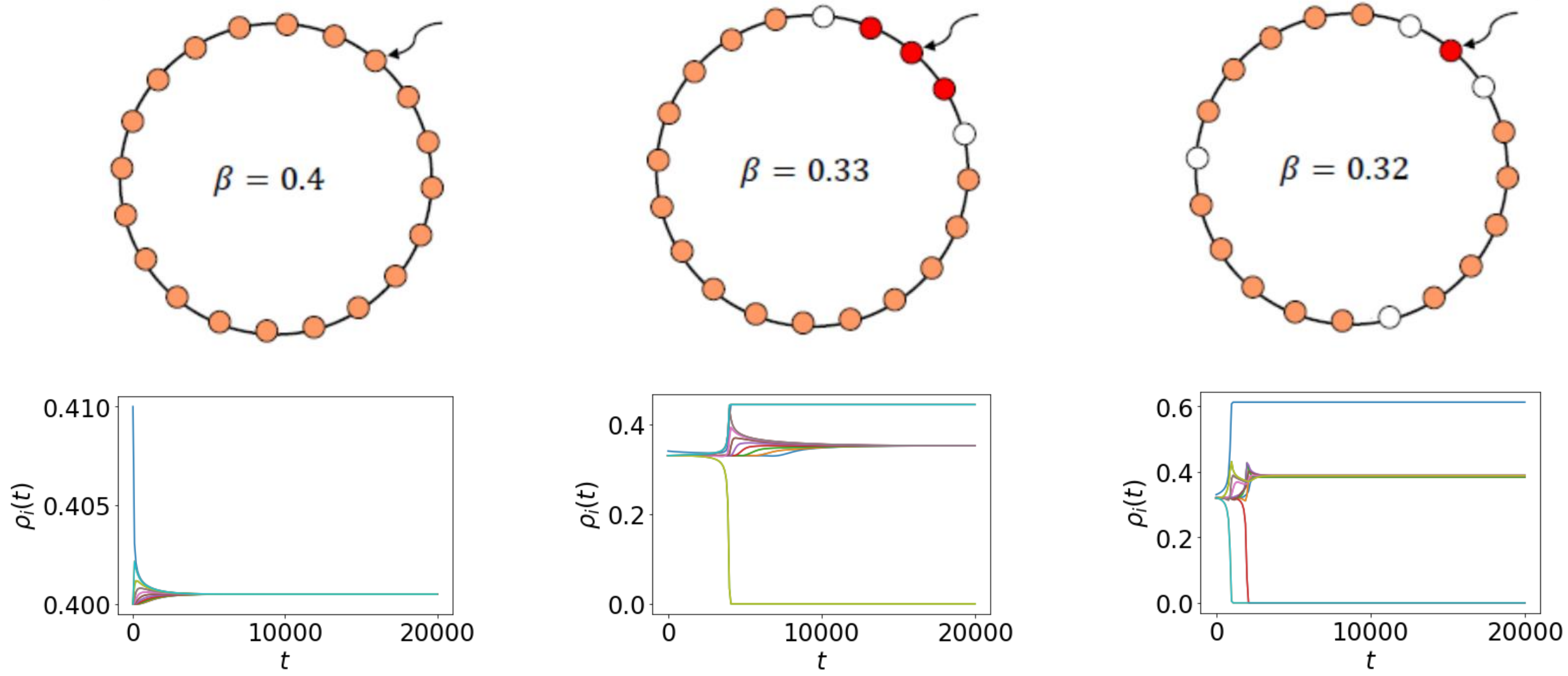


Ring

$\rho_i(0) = \beta + 0.01\delta_{i,1}\ (i = 1, \cdots, 20)$

$g(x) = x^2(1-x)^2$

perturbation



Ring

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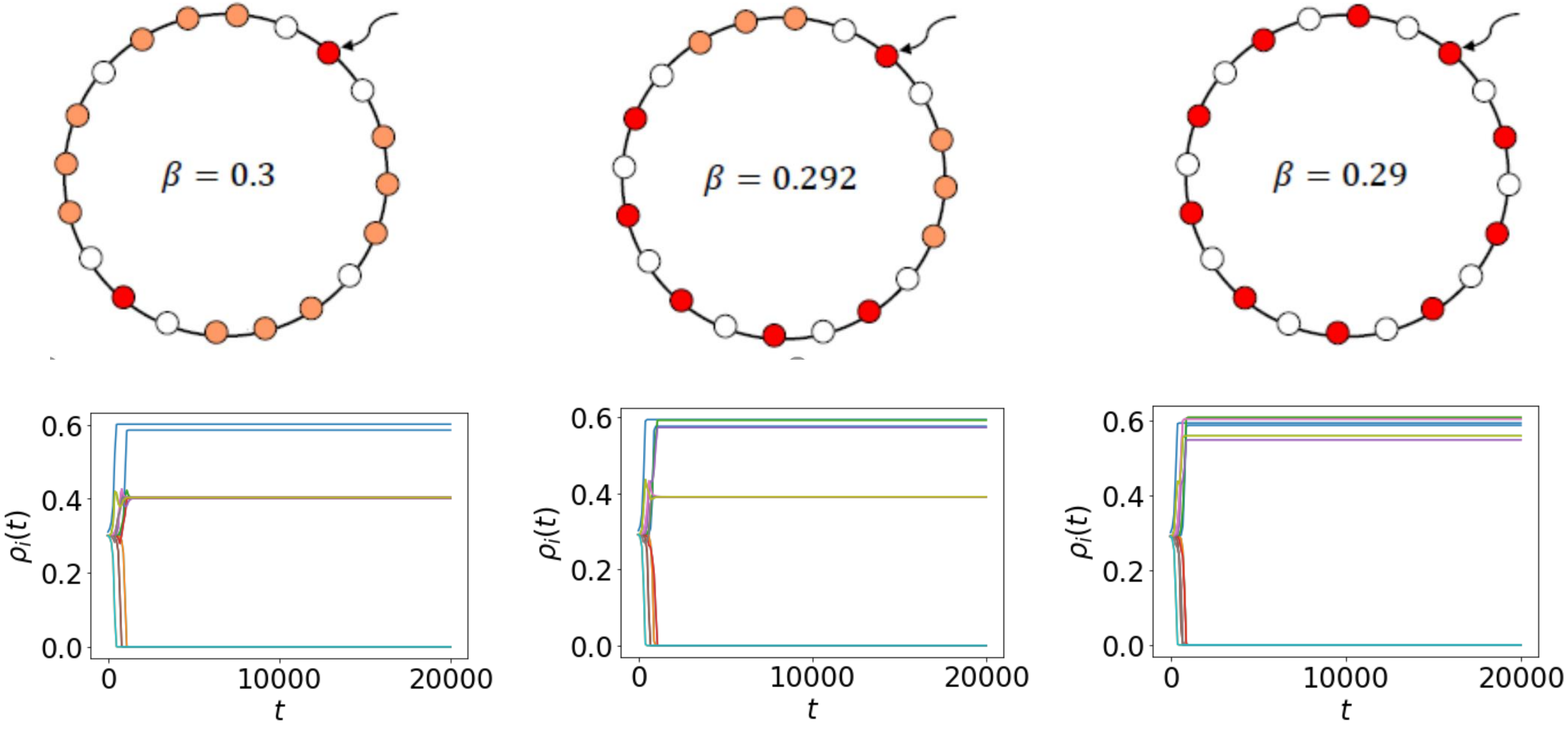
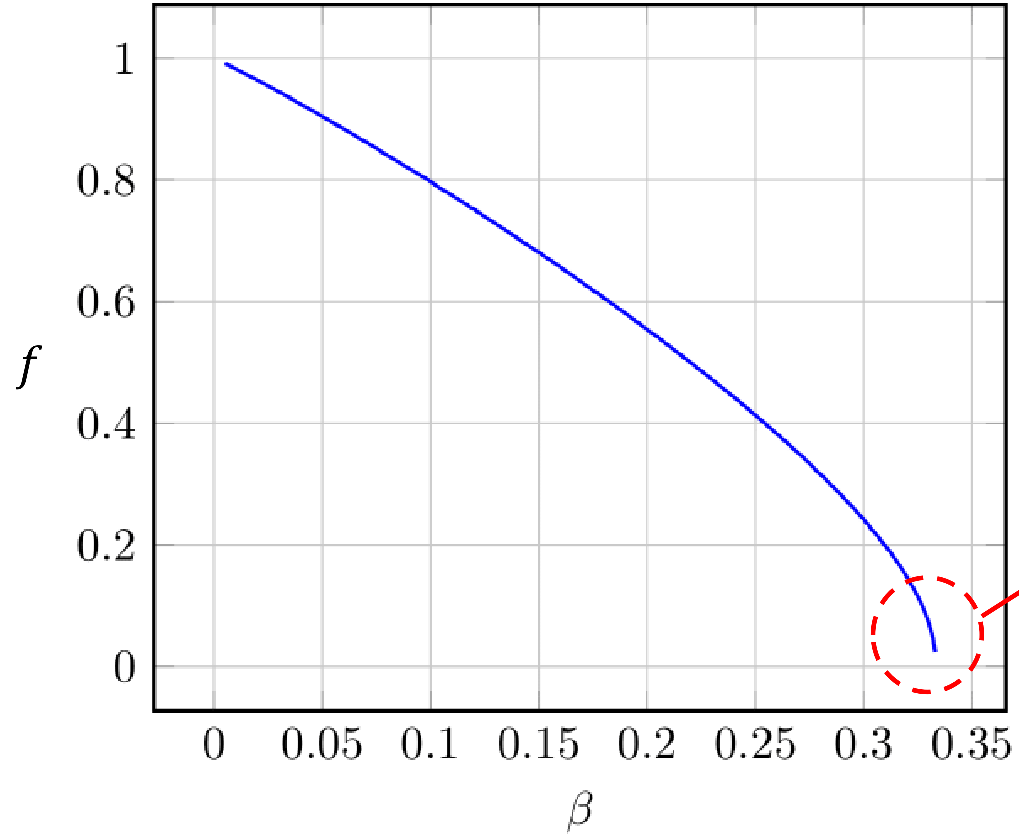
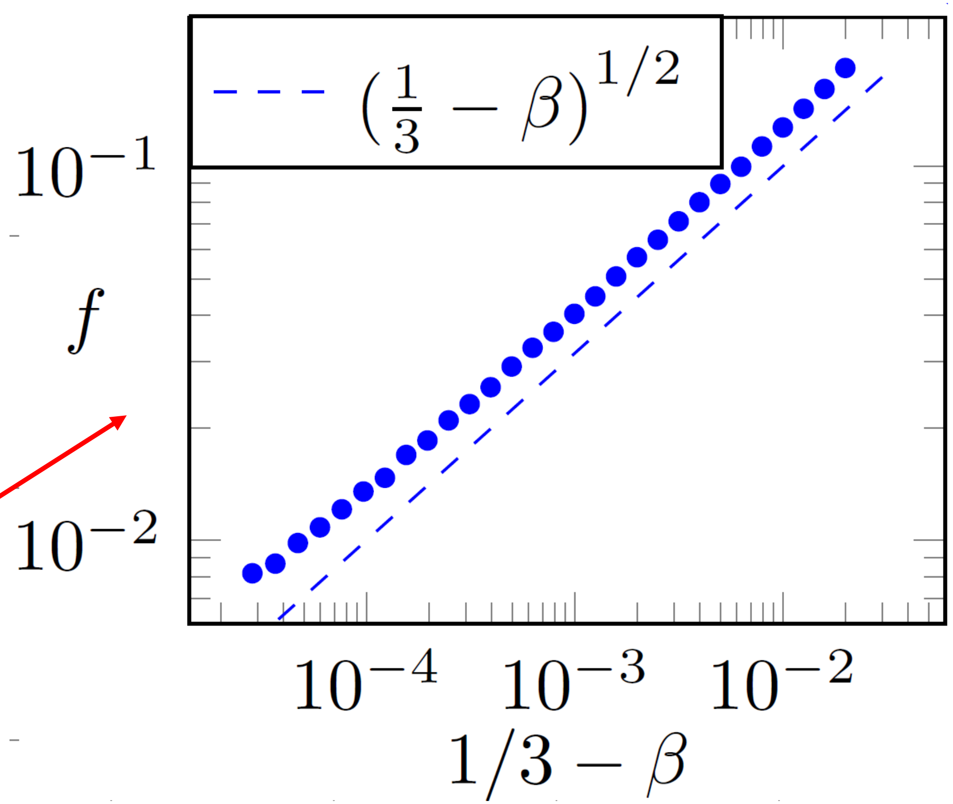


Complete graph





5000 nodes ; $g(x) = x^2(1-x)^2$; $\rho_i(0) \in \left[\beta - \frac{\delta}{2}, \beta + \frac{\delta}{2}\right]$ with $\delta = 10^{-5}$

Conclusion

We have investigated a class of nonlinear diffusion processes on top of networks that take into account the social behavior of agents and the limiting carrying capacities of the nodes.

The process can lead to the emergence of vacant nodes that separate the mass into distinct clusters; it reproduces the vacant patches found in the habitat network of the Glanville fritillary butterfly species in the Åland islands.

When taken place on top of regular networks, the homogeneous fixed point can become unstable as the average node density is reduced and empty nodes can emerge.

Thank you for your attention !

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