Interdisciplinary Training School for Doctoral students 2022

EFID - UAR

Social adoption on signed simplicial complexes

Jean-François de Kemmeter

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FACULTÉ DES SCIENCES

UNIVERSITÉ DE NAMUR

Social adoption on signed simplicial complexes

Ongoing research project with Luca Gallo*, Fabrizio Boncoraglio* and Timoteo Carletti°

*University of Catania °University of Namur

People interact pairwise or in groups of small size and (dis)trust each other

group interaction

pairwise interaction

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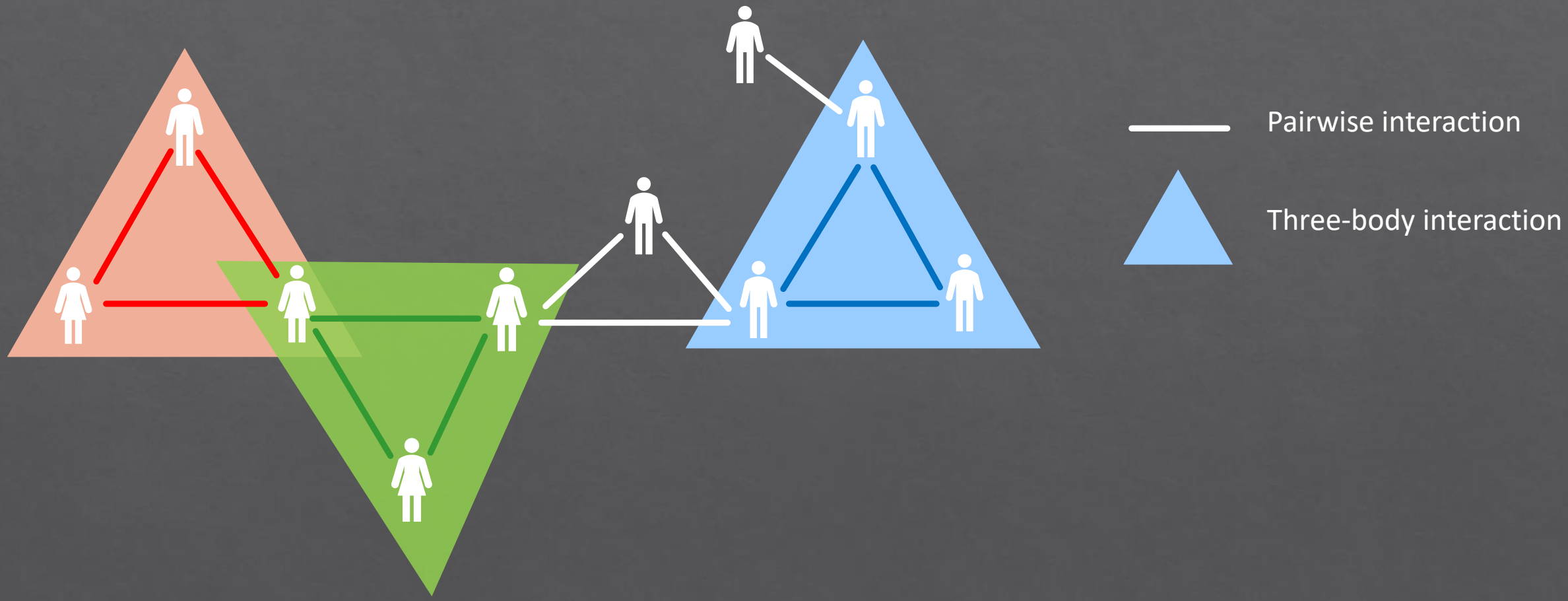
Simplicial complex



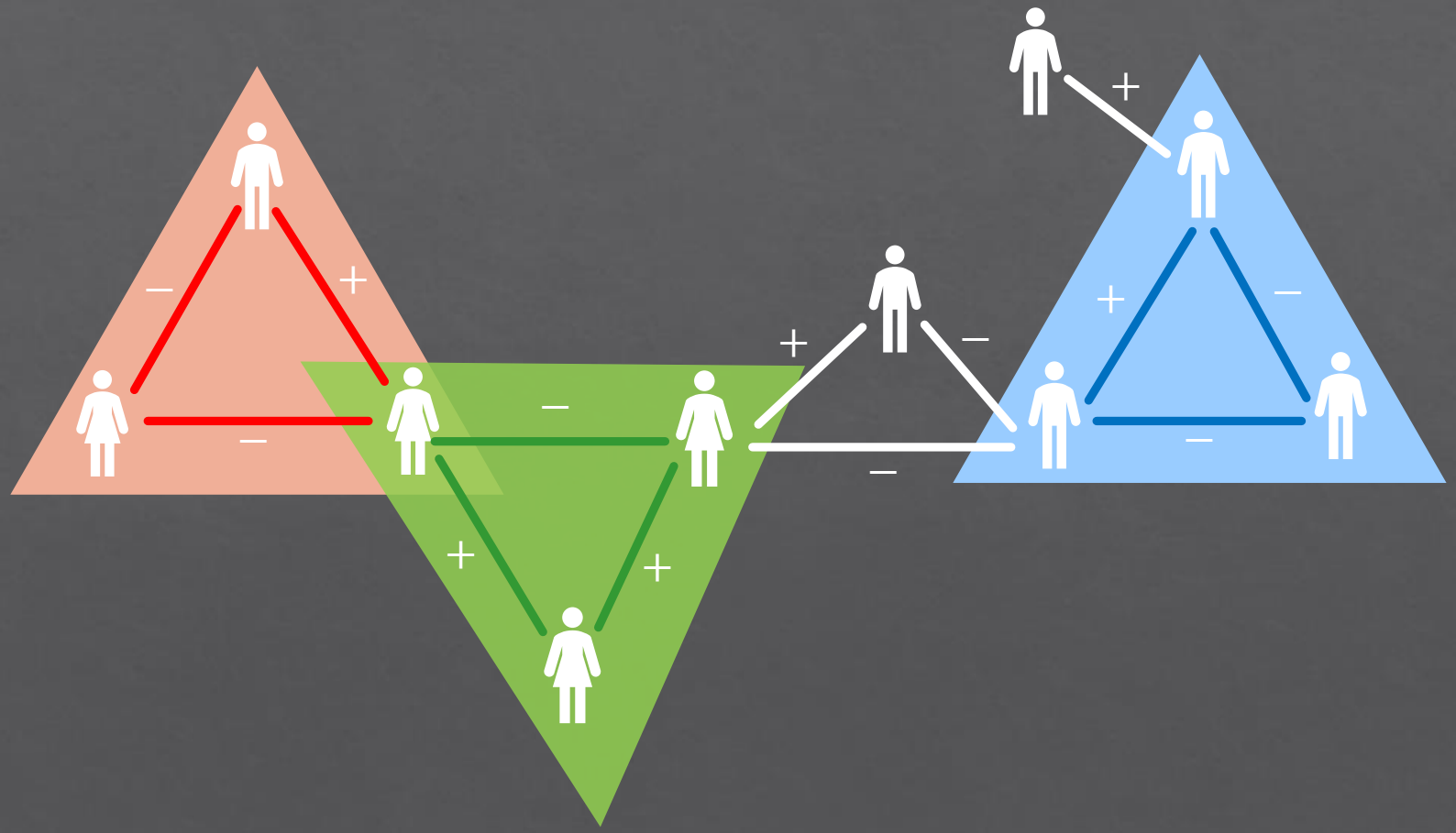
Bianconi, G. (2021). *Higher-order networks*. Cambridge University Press.

Trust and distrust relations

People trust or distrust each other

\+ Trust relation

− Distrust relation



Newman, M. (2018). *Networks*. Oxford university press.

Compartmental model

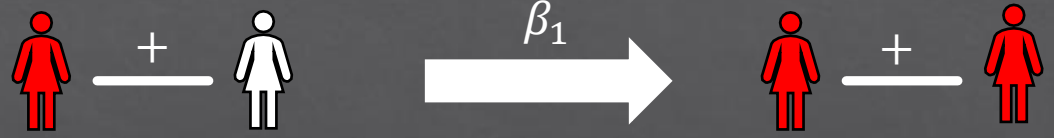
Two compartments

"Infected" individual

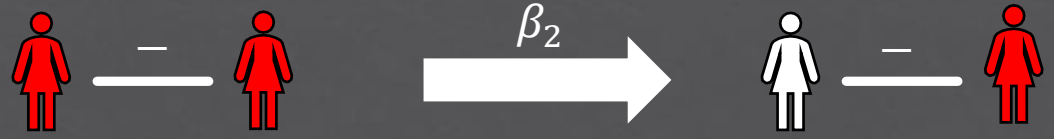
Susceptible individual

Node recovery

Pairwise infection



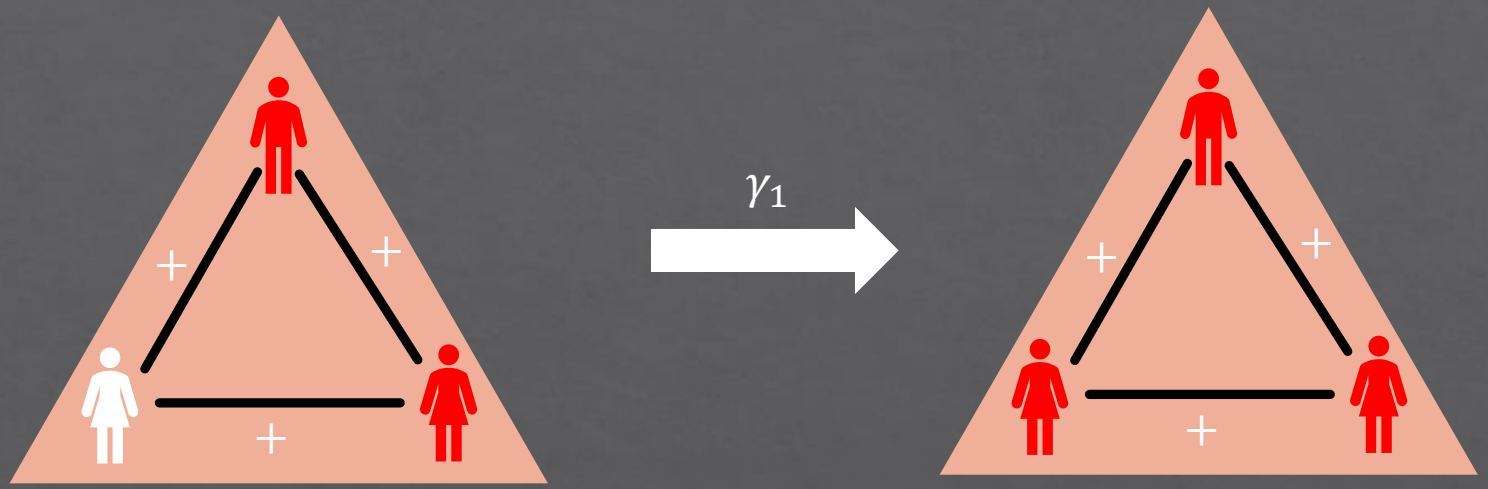
Pairwise recovery



Iacopini, I., Petri, G., Barrat, A., & Latora, V. (2019). Simplicial models of social contagion. *Nature communications*, *10*(1), 1-9.

Compartmental model

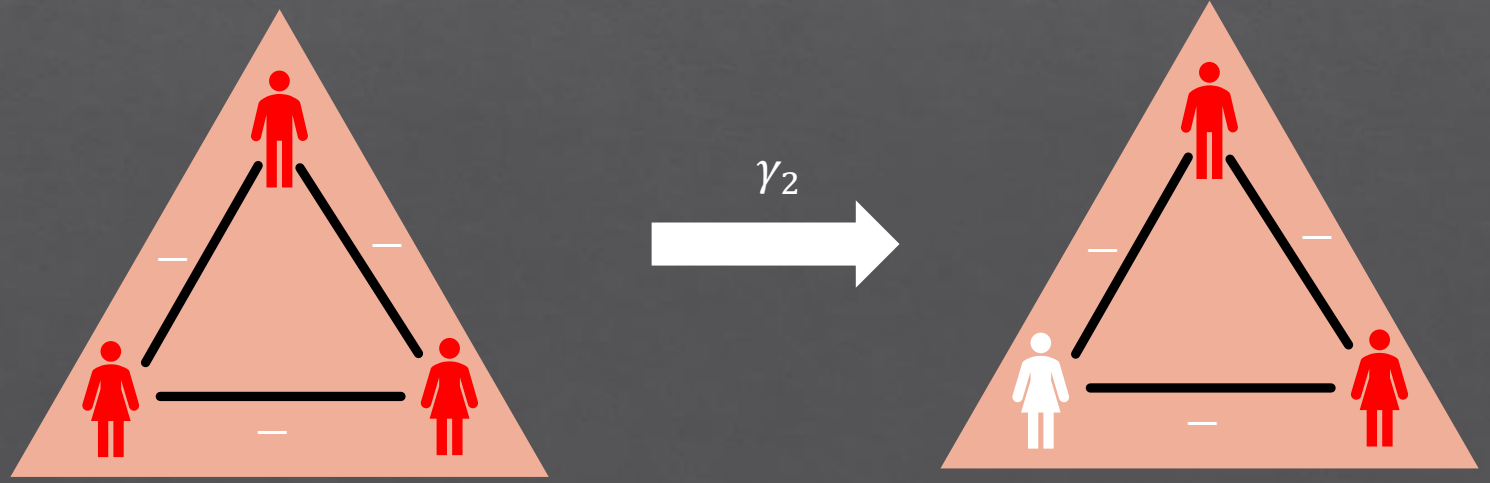
Three-body infection



Susceptible individual

"Infected" individual

Three-body recovery



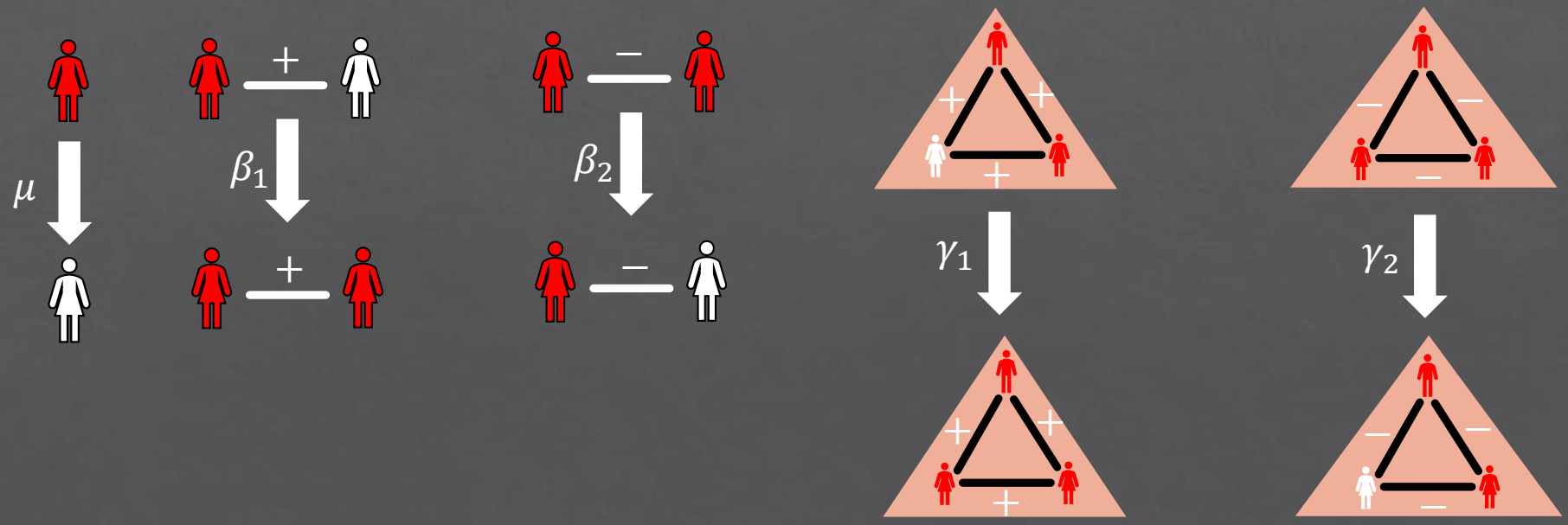
Mean-field equations

$\langle k \rangle$ ≔ average node degree

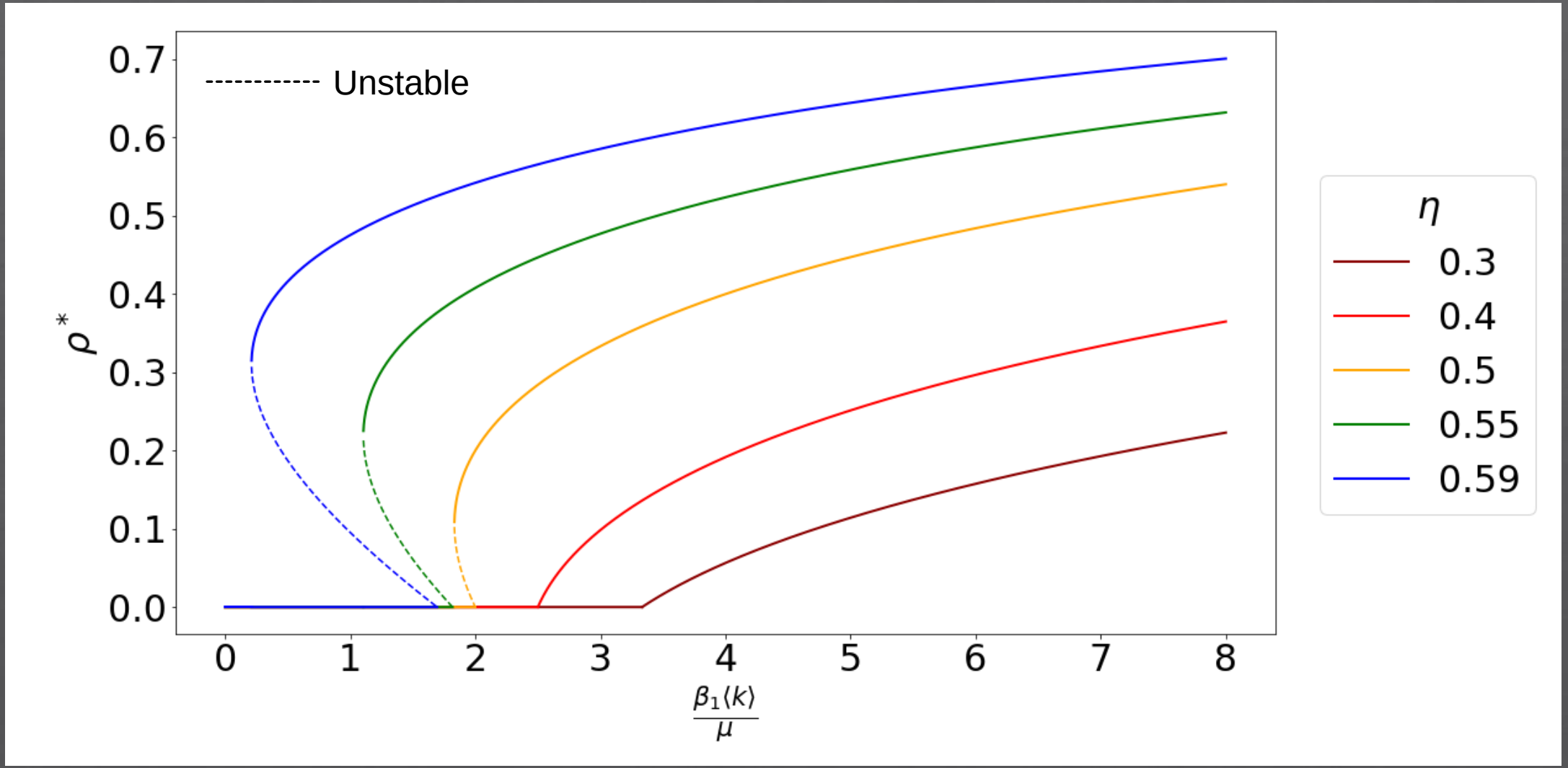
$\langle k_\Delta \rangle$ ≔ average number of three-body interactions in which a node takes part

ρ ≔ fraction of infected nodes.

$$\frac{d\rho}{dt} = -\mu\rho + \beta_1\langle k\rangle\eta\rho(1-\rho) - \beta_2\langle k\rangle(1-\eta)\rho^2 + \gamma_1\langle k_\Delta\rangle\eta^3\rho^2(1-\rho) - \gamma_2\langle k_\Delta\rangle(1-\eta)^3\rho^3$$



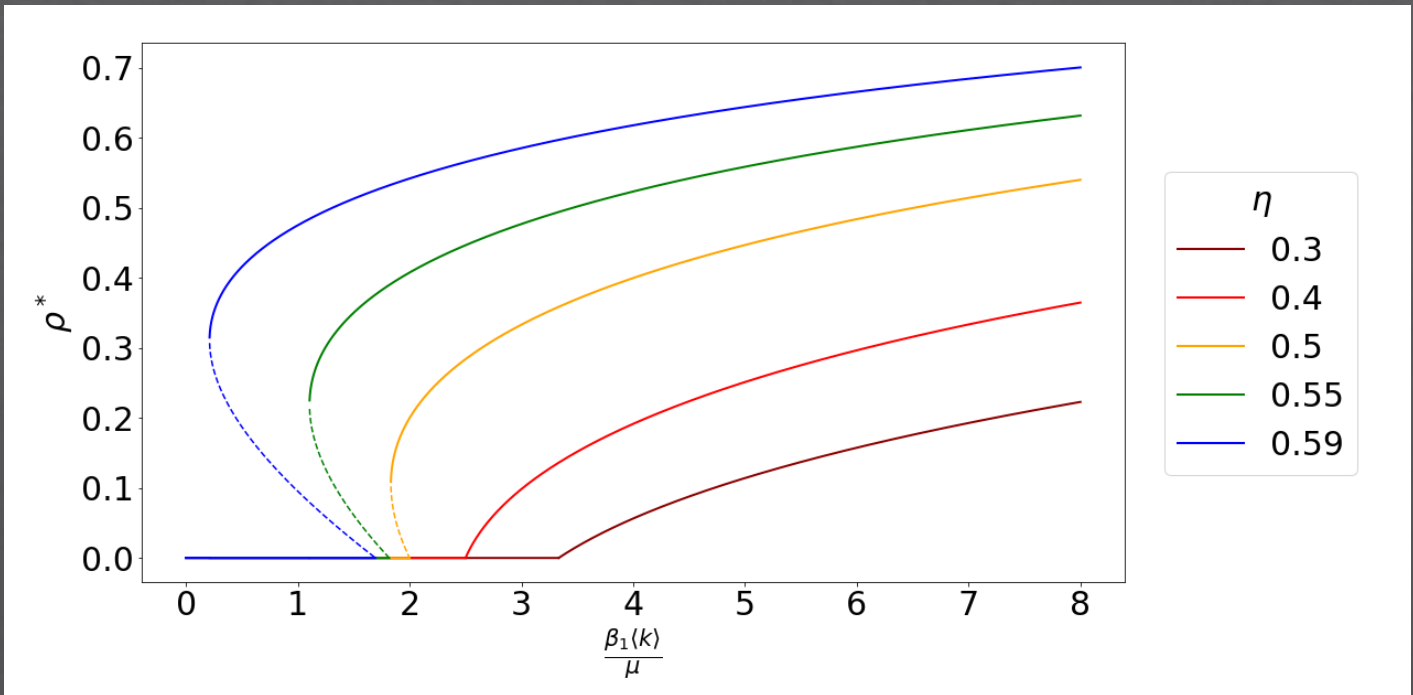
Phase diagram and bistability



$\langle k\rangle = 40\,;\langle k_\Delta\rangle = 12\,;\mu = 0{,}05\,;\beta_2 = 0{,}005\,;\gamma_1 = 0{,}15\,;\gamma_2 = 0{,}1$

Phase diagram and bistability

$$\frac{d\rho}{dt} = -\mu\rho + \beta_1\langle k\rangle\eta\rho(1-\rho) - \beta_2\langle k\rangle(1-\eta)\rho^2 + \gamma_1\langle k_\Delta\rangle\eta^3\rho^2(1-\rho) - \gamma_2\langle k_\Delta\rangle(1-\eta)^3\rho^3$$

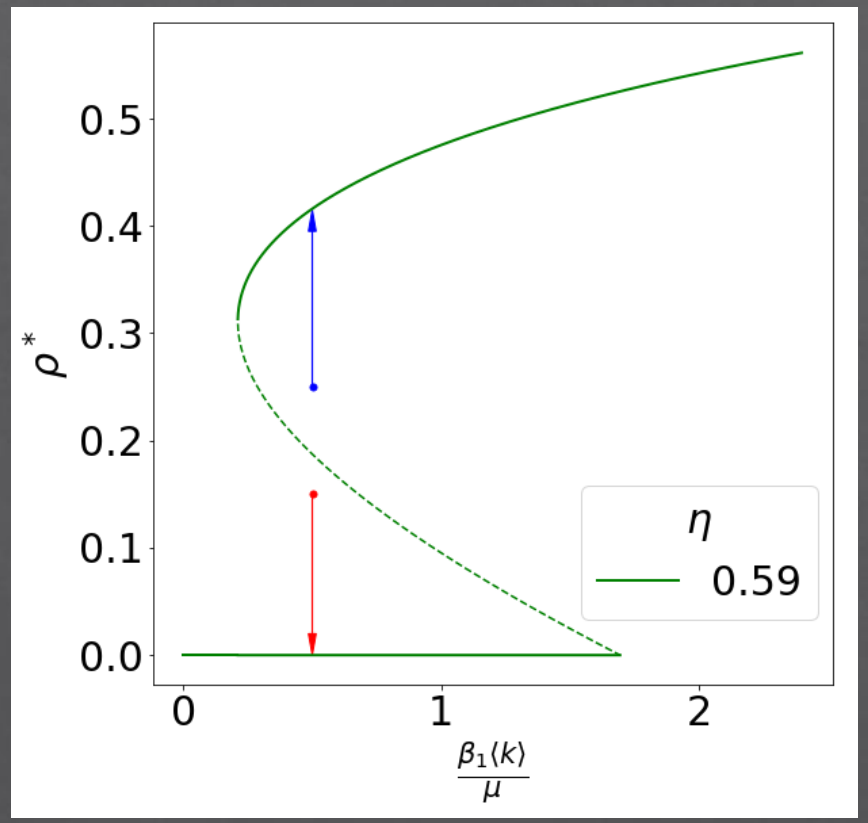


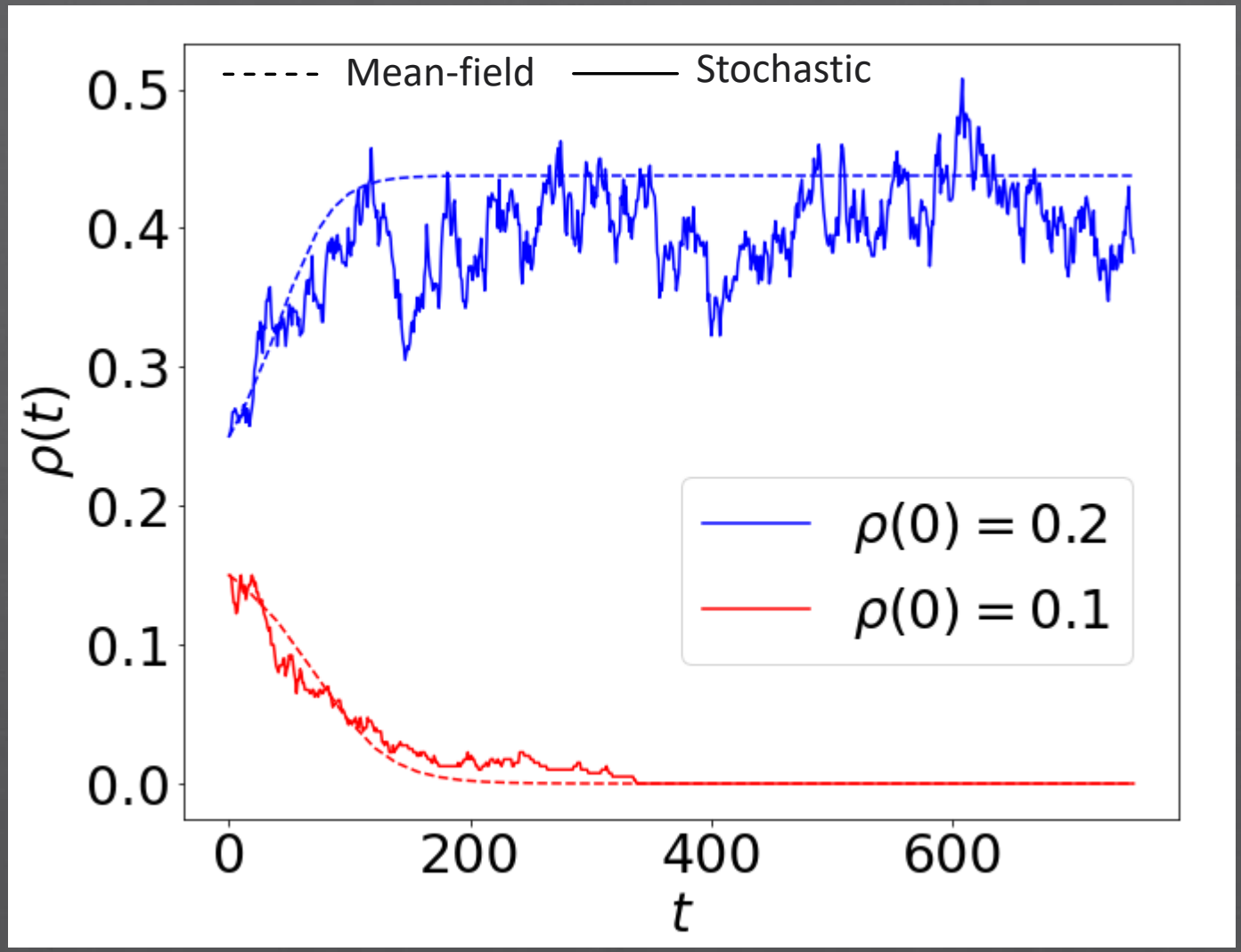
$\rho \approx 0$ ➡ $\frac{d\rho}{dt} \approx -\mu\rho + \beta_1\langle k\rangle\eta\rho$

➡ $\rho^* > 0$ if $-\mu + \beta_1\langle k\rangle\eta > 0$

$\langle k\rangle = 40\,;\langle k_\Delta\rangle = 12\,;\mu = 0{,}05\,;\beta_2 = 0{,}005\,;\gamma_1 = 0{,}15\,;\gamma_2 = 0{,}1$

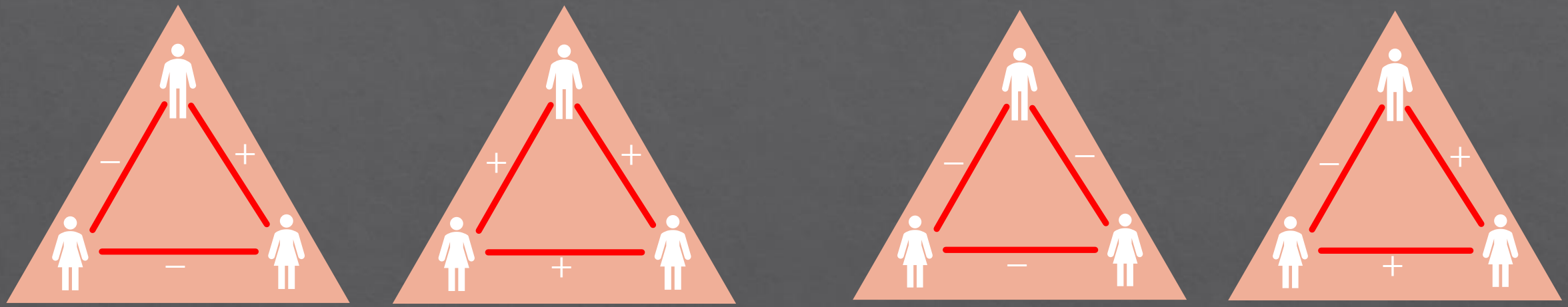
Stochastic simulations and bistability





$\langle k\rangle = 40\,;\langle k_\Delta\rangle = 12\,;\mu = 0{,}05\,;\beta_2 = 0{,}005\,;\gamma_1 = 0{,}15\,;\gamma_2 = 0{,}1\,;\eta = 0{,}65\,;\beta_1 = 0{,}002$ (400 nodes)

Structural balance theory



Balanced triangles

Unbalanced triangles

[1] Heider, F. (1946). Attitudes and cognitive organization. *The Journal of psychology*, *21*(1), 107-112.

[2] Cartwright, D., & Harary, F. (1956). Structural balance: a generalization of Heider's theory. *Psychological review*, *63*(5), 277.

[3] Szell, M., Lambiotte, R., & Thurner, S. (2010). Multirelational organization of large-scale social networks in an online world. *PNAS*, *107*(31), 13636-13641.

Thank you !