CONFERENCE ON COMPLEX SYSTEMS
PALMA DE MALLORCA
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Social adoption on signed simplicial complexes

Jean-François de Kemmeter

18/10/2022

In a nutshell

People interact pairwise or in groups of small size and (dis)trust each other

group interaction

pairwise interaction

Iacopini, I., Petri, G., Barrat, A., & Latora, V. (2019). Simplicial models of social contagion. *Nature communications*, *10*(1), 1-9.

Joint work with...

Fabrizio Boncoraglio, Timoteo Carletti, Luca Gallo and Vito Latora

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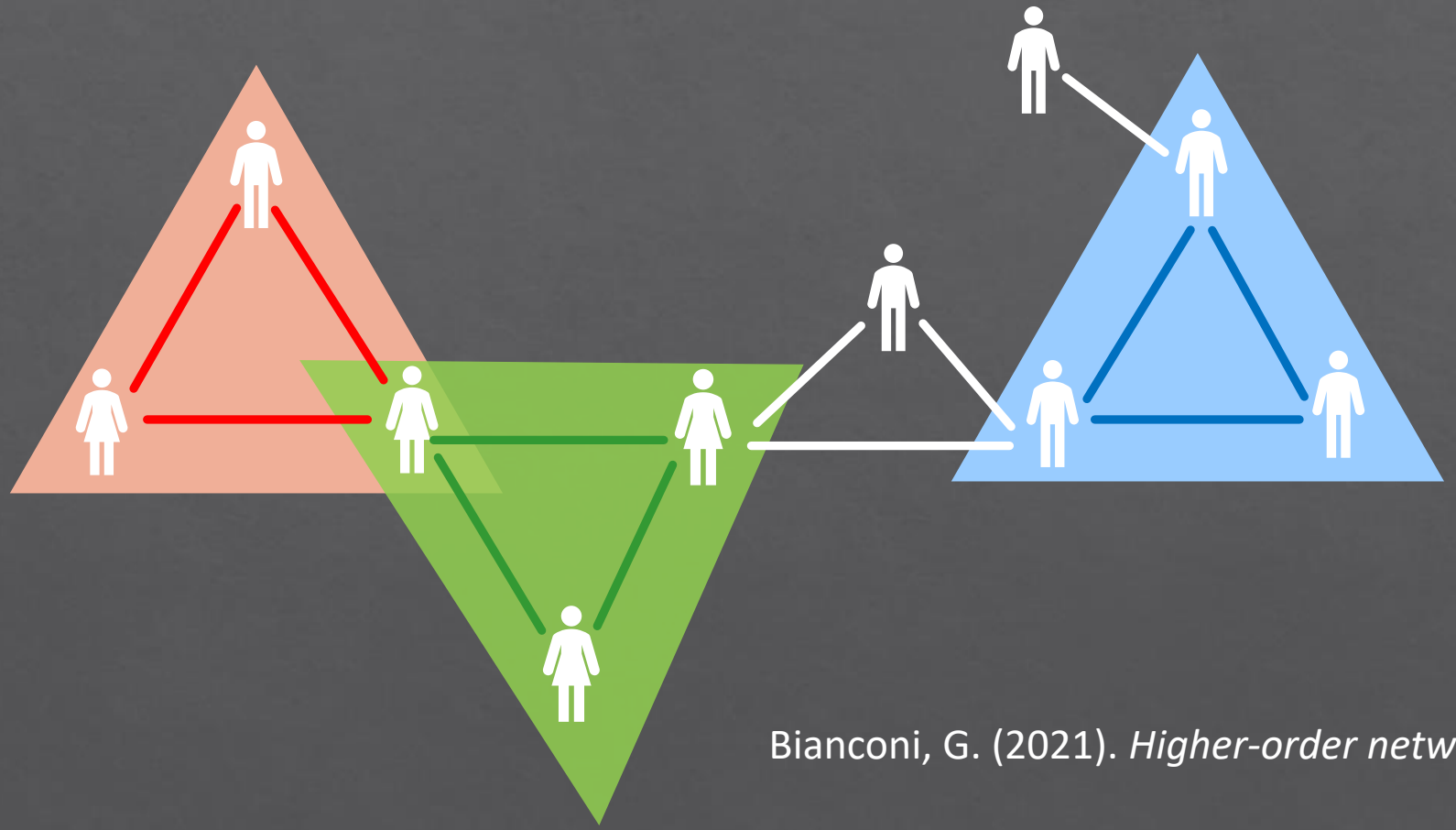
Summary

- How to model information spreading in social groups with (dis)trust ?
- Infection and recovery channels
- Mean-field limit and stochastic simulations
- Infection is a non-monotonous function of the amount of trust
- Social balance theory

Simplicial complex



Pairwise interaction

Three-body interaction

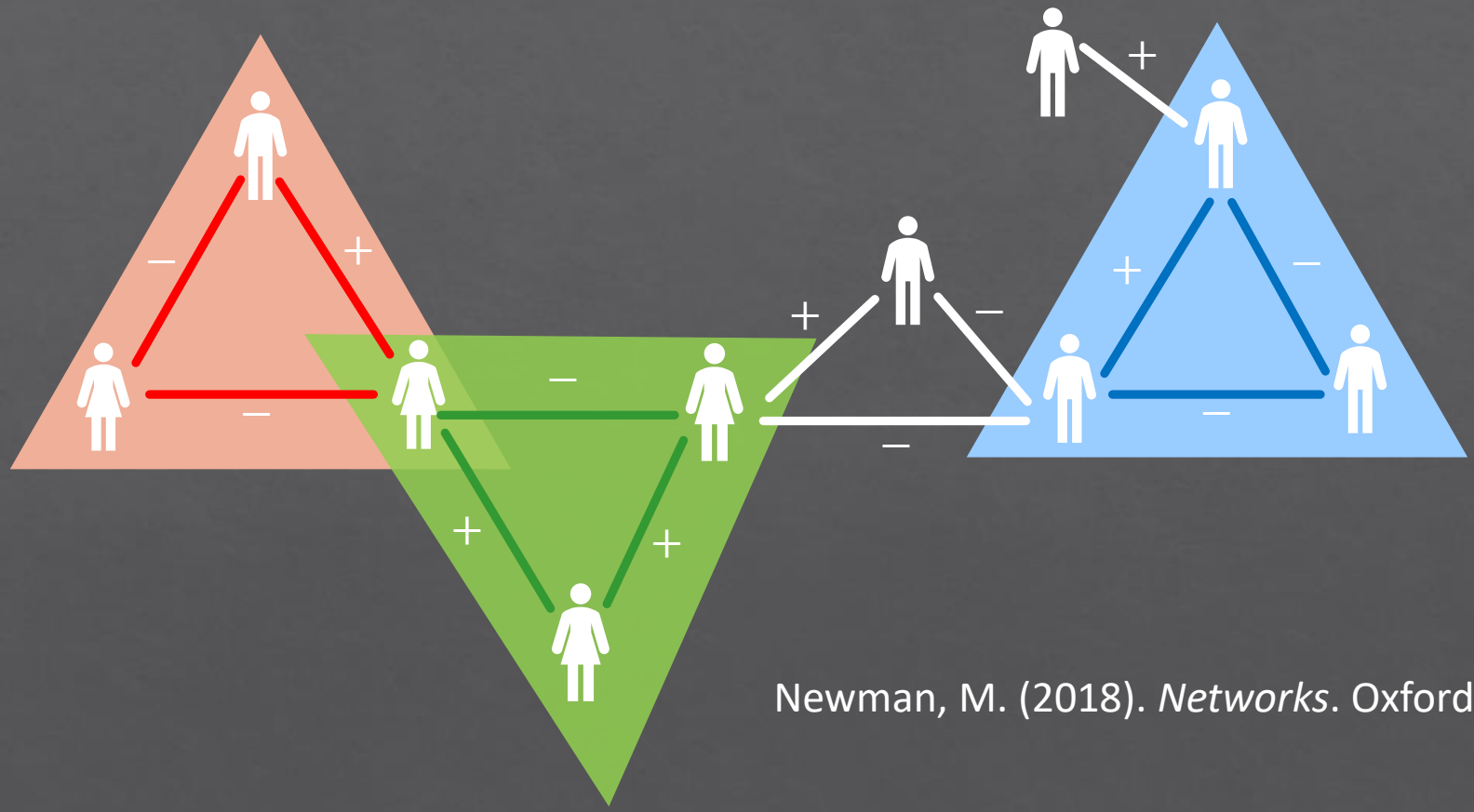
Bianconi, G. (2021). *Higher-order networks*. Cambridge University Press.

Signed simplicial complex

People trust or distrust each other

\+ Trust relation

– Distrust relation



Newman, M. (2018). *Networks*. Oxford university press.

Compartmental model

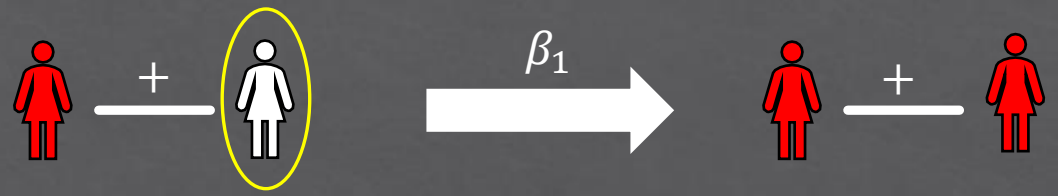
Two compartments

"Infected" individual
(has adopted the opinion)

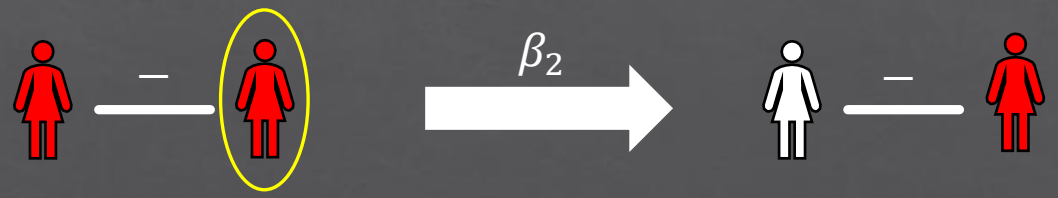
Susceptible individual
(has <u>not</u> adopted the opinion)

Node recovery

Pairwise infection

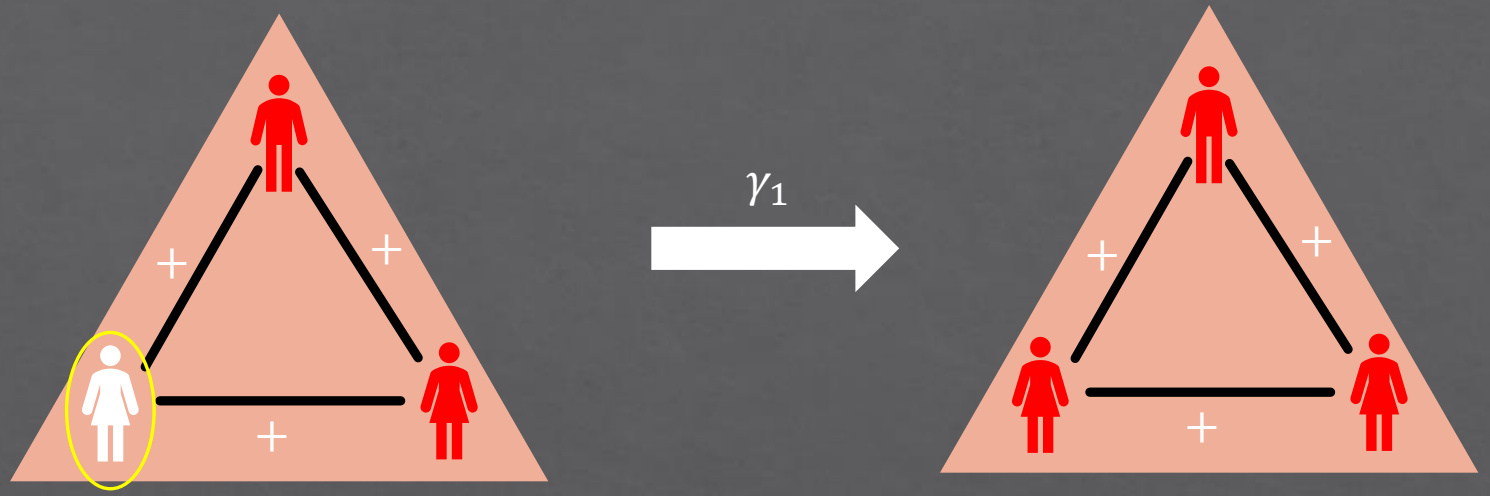


Pairwise recovery



Three-body infections

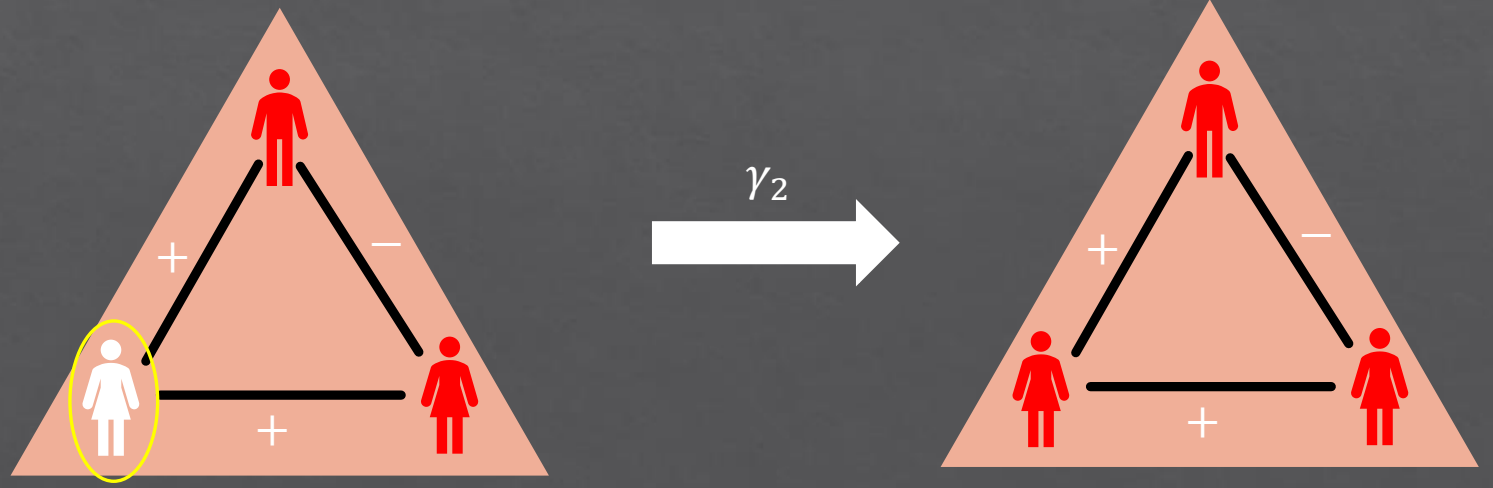
Three-body infection



Susceptible individual

"Infected" individual

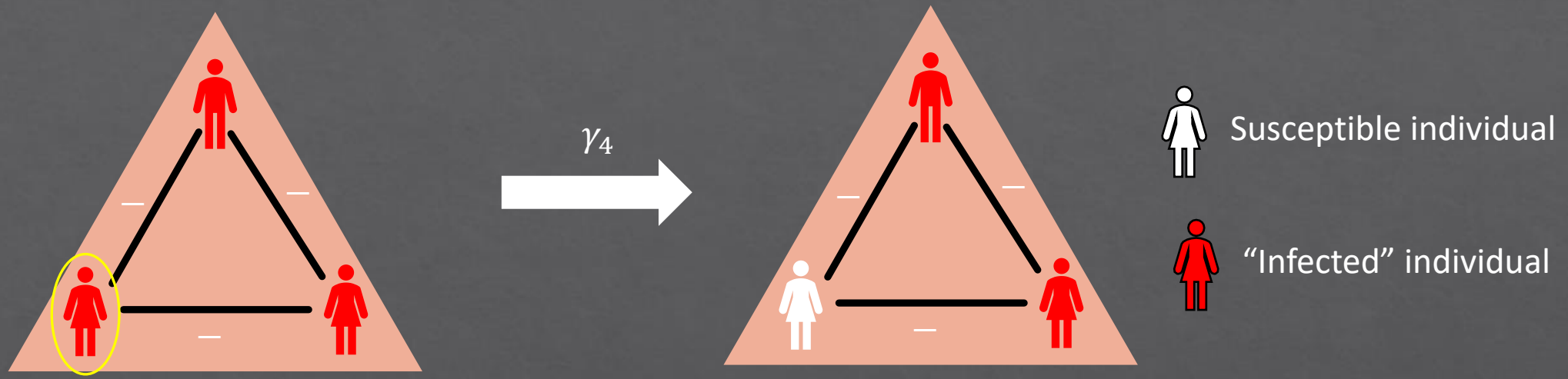
Three-body infection



$\gamma_2 < \gamma_1$

Three-body recovery

Three-body recovery



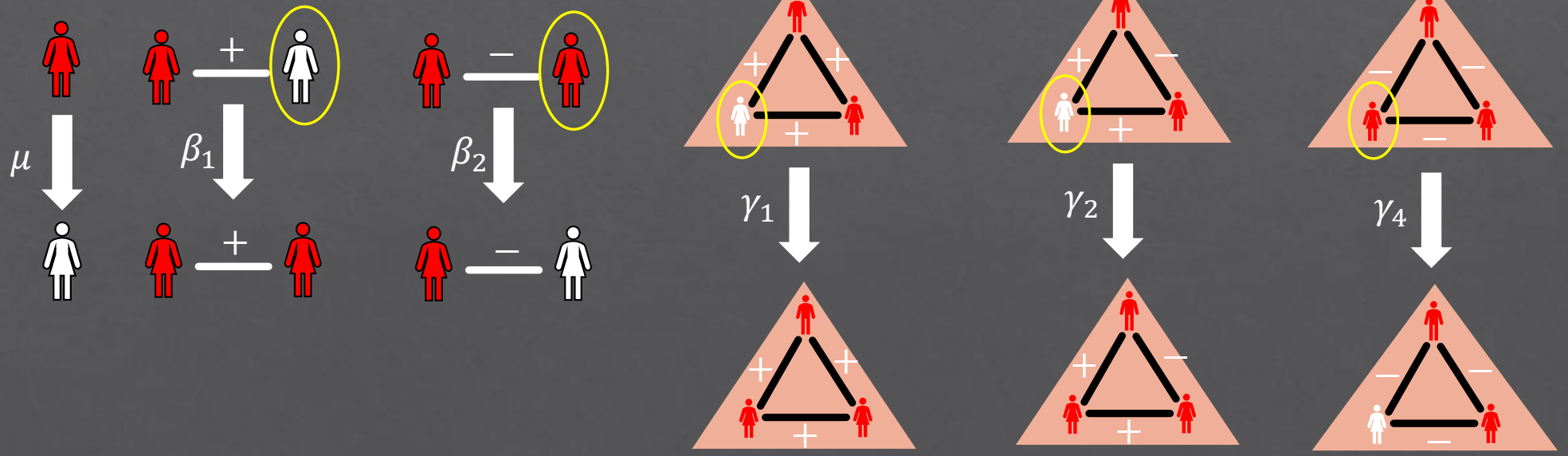
Mean-field limit

ρ ≔ fraction of infected nodes

$l_+(l_-)$ ≔ mean number of trust (distrust) links incident to a node

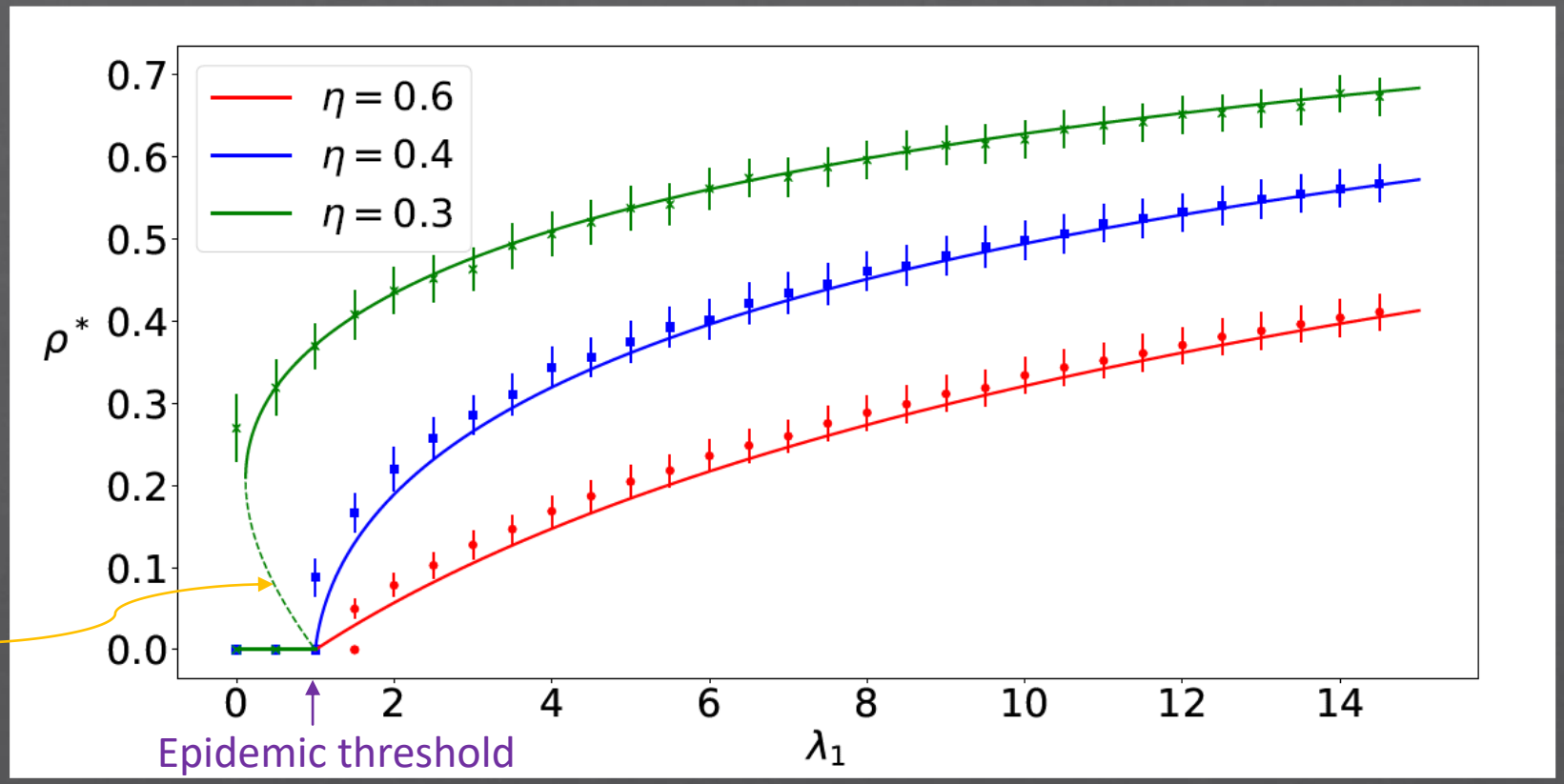
τ_i ≔ mean number of triangles with i "+" signs incident to a node

$$\frac{d\rho}{dt} = -\mu\rho \;+\; \beta_1 l_+ \rho(1-\rho) \;-\; \beta_2 l_- \rho^2 \;+\; \gamma_1\tau_3\,\rho^2(1-\rho) \;+\; \gamma_2\tau_2\,\rho^2(1-\rho) \;-\; \gamma_4\tau_0\,\rho^3$$



Mean-field solutions and stochastic simulations

Construction of the simplicial complex: 2-simplicies (“full triangles”) inherit signs from the pairwise interactions



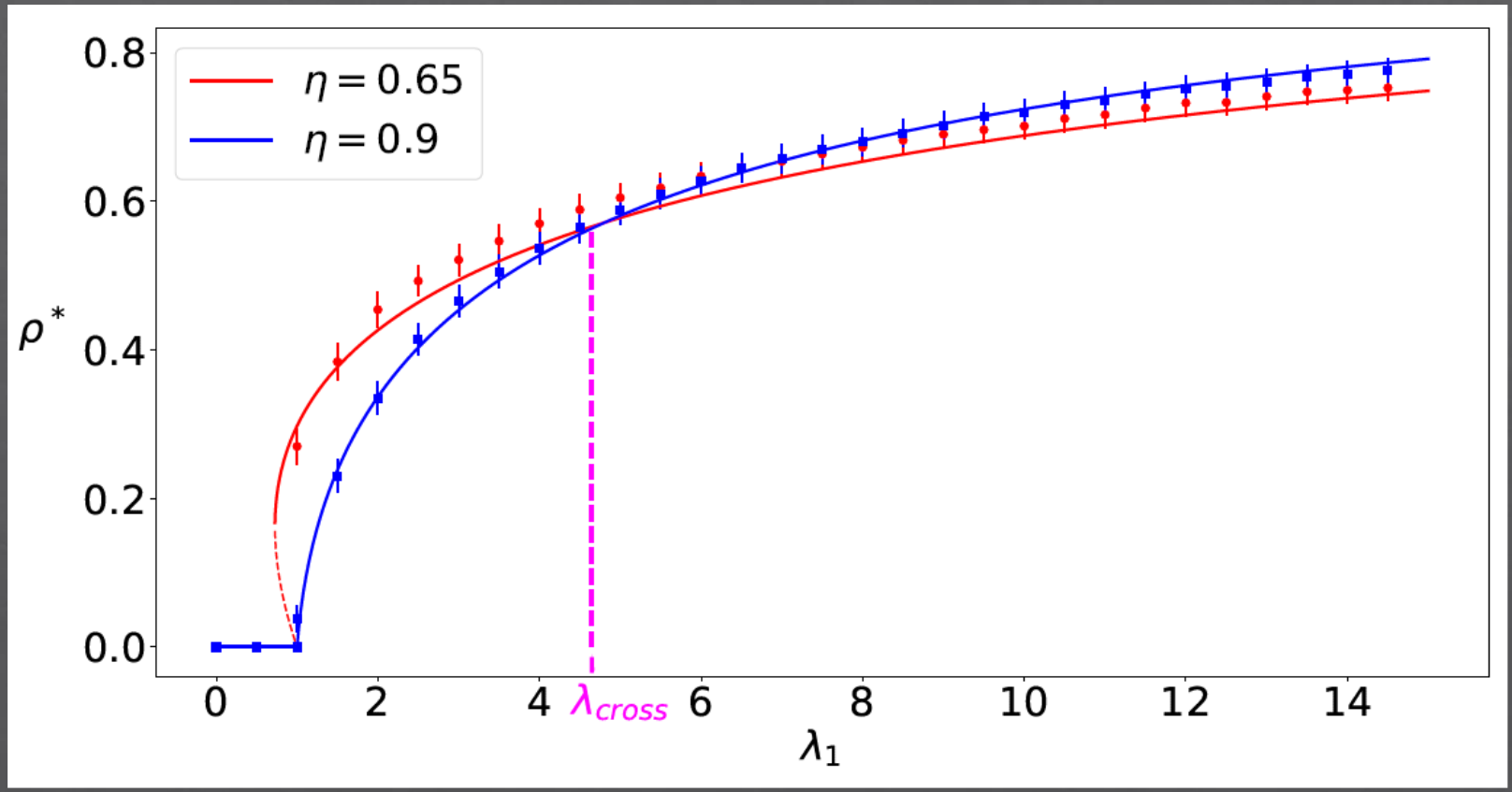
Unstable

$$\lambda_1 = \frac{\beta_1 l_+}{\mu}$$

$N = 500\,, \langle k \rangle = 60\,, \langle k_\Delta \rangle = 10\,, \mu = 0.07\,, \beta_2 = 0{,}04\,, \gamma_1 = 0.3\,, \gamma_2 = 0.2\,, \gamma_4 = 0.2$

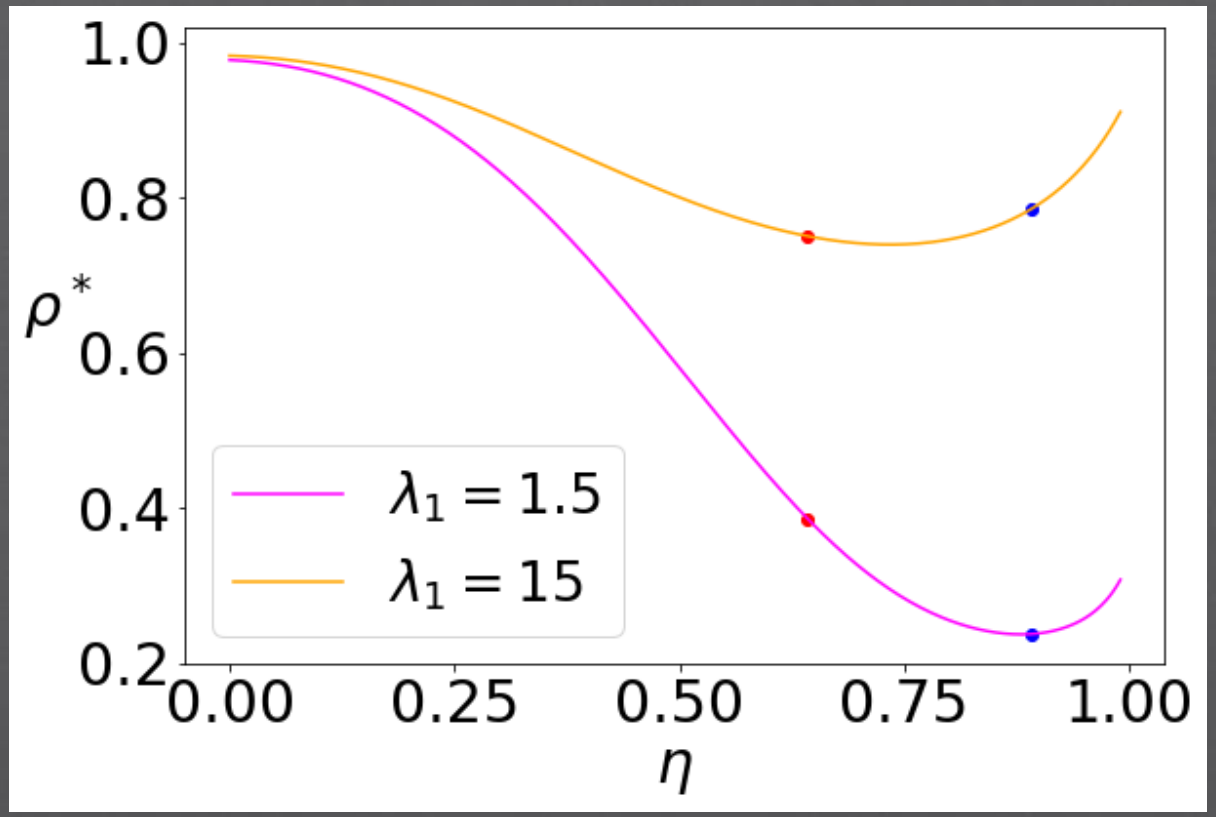
Non-monotonous behaviour of the infection with respect to the amount of distrust

Construction of the simplicial complex: 2-simplices (“full triangles”) inherit signs from the pairwise interactions



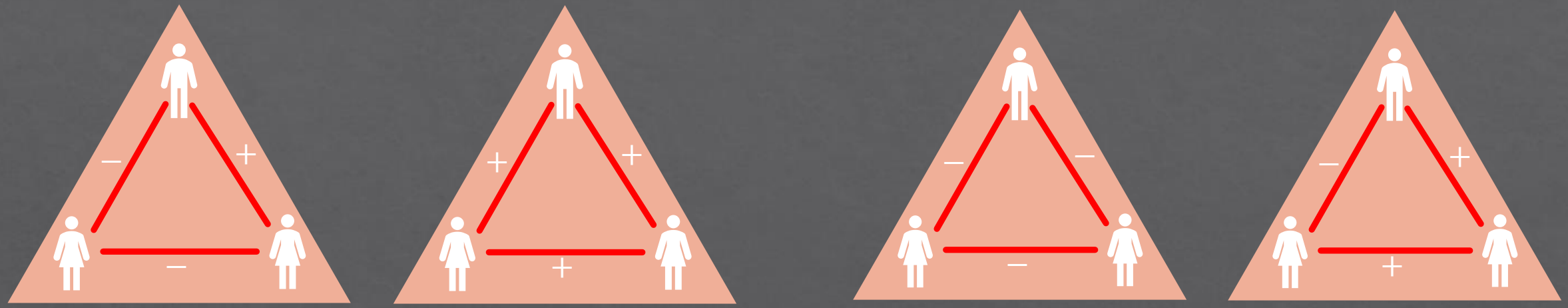
$N = 500\,, \langle k \rangle = 60\,, \langle k_\Delta \rangle = 10\,, \mu = 0.07\,, \beta_2 = 0\,, \gamma_1 = 0.3\,, \gamma_2 = 0.2\,, \gamma_4 = 0.3$

Non-monotonous behaviour of the infection with respect to the amount of distrust



$N = 500\,, \langle k \rangle = 60\,, \langle k_\Delta \rangle = 10\,, \mu = 0.07\,,$
$\beta_2 = 0\,, \gamma_1 = 0.3\,, \gamma_2 = 0.2\,, \gamma_4 = 0.3$

Structural balance theory



Balanced triangles

Unbalanced triangles

[1] Heider, F. (1946). Attitudes and cognitive organization. *The Journal of psychology*, *21*(1), 107-112.

[2] Cartwright, D., & Harary, F. (1956). Structural balance: a generalization of Heider's theory. *Psychological review*, *63*(5), 277.

[3] Szell, M., Lambiotte, R., & Thurner, S. (2010). Multirelational organization of large-scale social networks in an online world. *PNAS*, *107*(31), 13636-13641.

Social balance theory

τ_i ≔ mean number of triangles with i "+" signs incident to a node

$$\tau_0 = p_0 \eta^3 \langle k_\Delta \rangle$$

$$\tau_1 = 3 p_1 \eta^2 (1-\eta) \langle k_\Delta \rangle$$

$$\tau_2 = 3 p_2 \eta (1-\eta)^2 \langle k_\Delta \rangle$$

$$\tau_3 = p_3 (1-\eta)^3 \langle k_\Delta \rangle$$

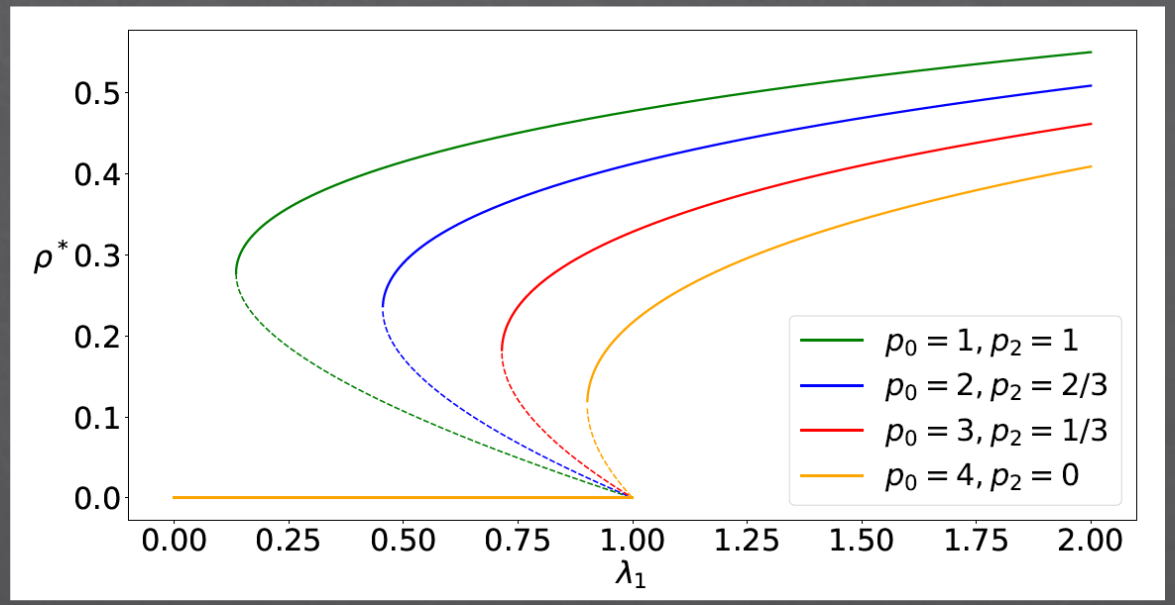
with $\tau_0 + \tau_1 + \tau_2 + \tau_3 = \langle k_\Delta \rangle$

⟹ $p_0 = 0 = p_2$: only balanced (full) triangles
$p_1 = 0 = p_3$: only unbalanced (full) triangles

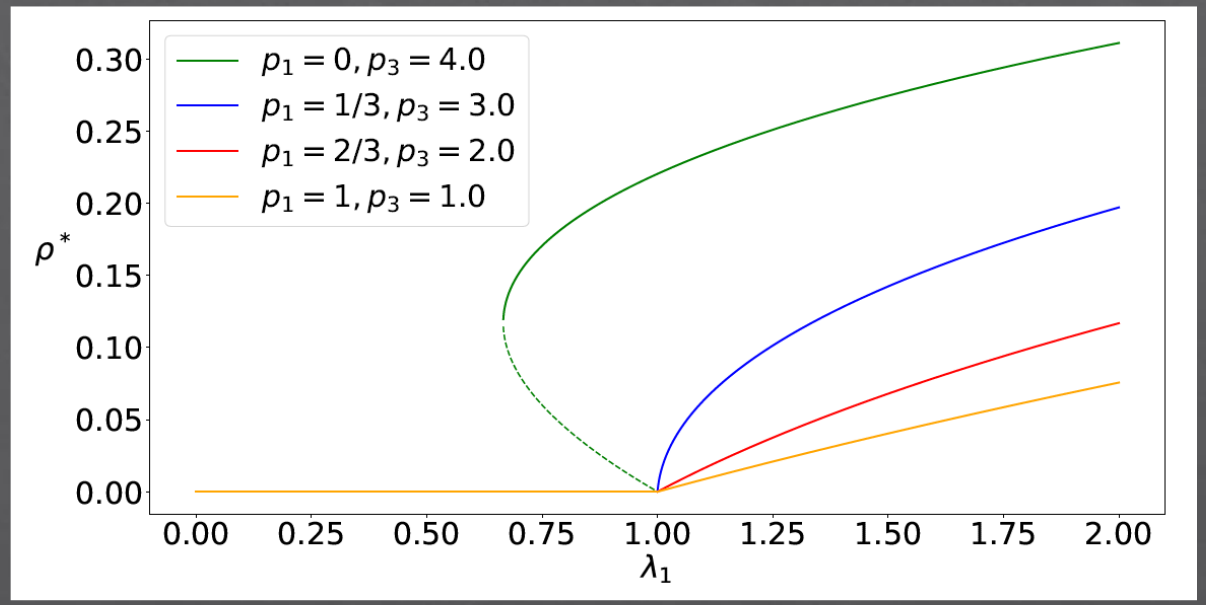
Construction of the simplicial complex:

We assign to each link a "−" sign with a probability η and a "+" sign otherwise.
We assign to each 2-simplex (i.e. "full triangle") i positive signs with a probability $\frac{\tau_i}{\langle k_\Delta \rangle}$

Social balance theory



$\langle k \rangle = 30$, $\langle k_\Delta \rangle = 15$, $\eta = 0.5$, $\mu = 0.05$, $\beta_2 = 0.01$,
$\gamma_1 = 0.15$, $\gamma_2 = 0.1$, $\gamma_4 = 0.05$



$\langle k \rangle = 30$, $\langle k_\Delta \rangle = 15$, $\eta = 0.5$, $\mu = 0.05$, $\beta_2 = 0.06$,
$\gamma_1 = 0.15$, $\gamma_2 = 0.05$, $\gamma_4 = 0.05$

Conclusion

- We have investigated how social groups and (dis)trust among agents influence the spread of an opinion, idea,…
- We compared stochastic simulations (Gillespie) with the mean-field predictions
- The infection can be a non-monotonous function of the amount of trust
- A biased distribution of edge signs can increase the infection and induce bistability compared to the reshuffled case

Thank you for your attention !

Appendix: stationary densities

$$\rho_0^* = 0 \quad \text{or} \quad \rho_\pm^* = \frac{\Lambda - \lambda_1 - \lambda_2 \pm \sqrt{(\Lambda - \lambda_1 - \lambda_2)^2 - 4(\Lambda + \Lambda_4)(1 - \lambda_1)}}{2(\Lambda + \Lambda_4)}$$

with $\Lambda = \frac{\gamma_1 \tau_3}{\mu} + \frac{\gamma_2 \tau_2}{3\mu}$, $\Lambda_4 = \frac{\gamma_4 \tau_1}{3\mu}$, $\lambda_1 = \frac{\beta_1 l_+}{\mu}$, $\lambda_2 = \frac{\beta_2 l_-}{\mu}$

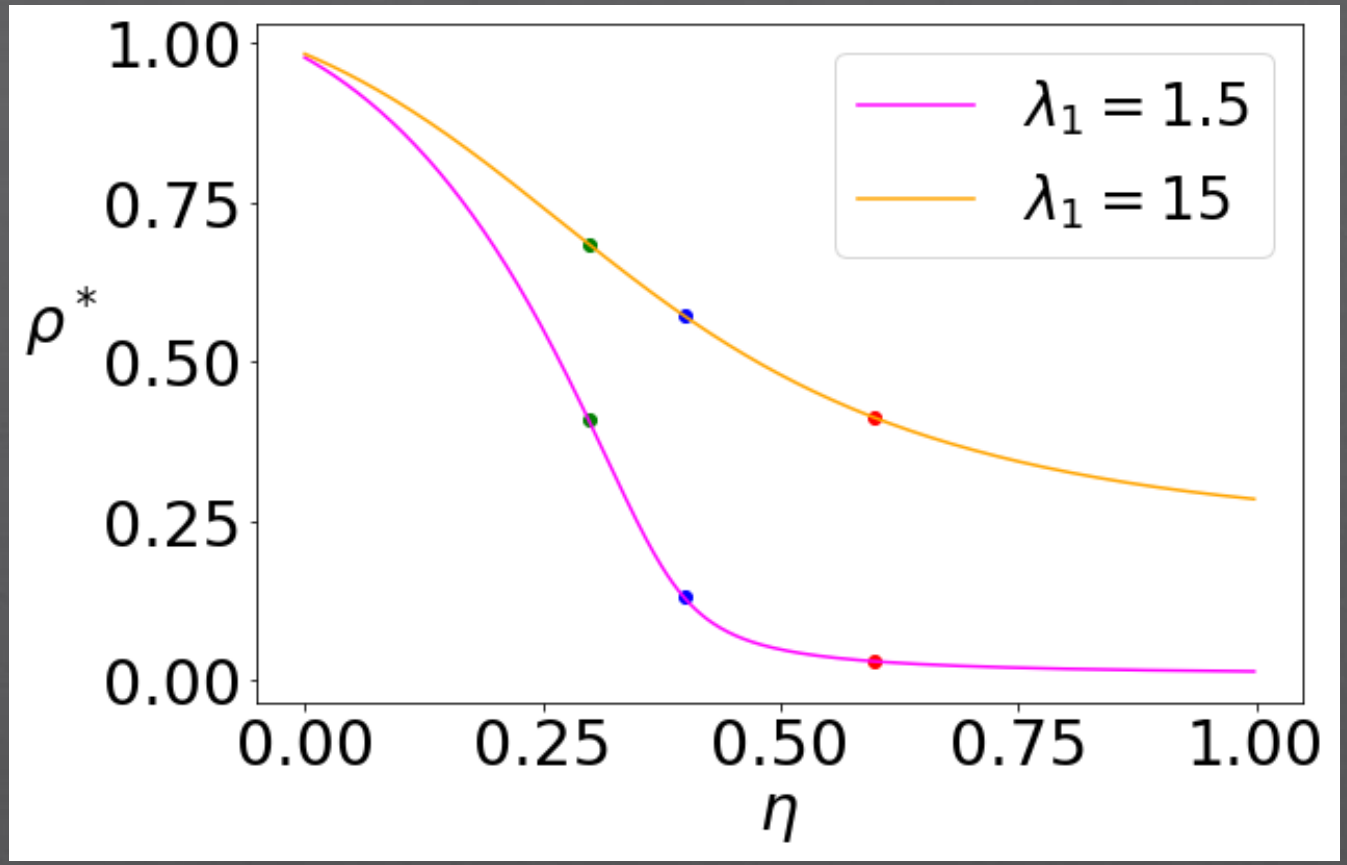
$$\lambda_{crit} = -\left[\sqrt{\lambda_2 + \Lambda + \Lambda_4} - \sqrt{\Lambda + \Lambda_4}\right]^2 + 1$$

$\lambda_1 < \lambda_{crit}$	$\lambda_{crit} < \lambda_1 < 1$ $\Lambda < 1 + \lambda_2$	$\lambda_{crit} < \lambda_1 < 1$ $\Lambda > 1 + \lambda_2$	$\lambda_1 \geq 1$
ρ_0^* (stable)	ρ_0^* (stable)	$\rho_0^* = 0$ and ρ_+^* (stable) ρ_-^* (unstable)	ρ_+^* (stable) ρ_0^* (unstable)

Bistability

jean-francois.dekemmeter@unamur.be

Appendix: infection as a function of the distrust



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$\beta_2 = 0{,}04\,, \gamma_1 = 0.3\,, \gamma_2 = 0.2\,, \gamma_4 = 0.2$

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