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Theoretical condition for transparency in mesoporous layered optical media: Application to switching of hygrochromic coatings

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Mesoporous Bragg stacks are able to change color upon infiltration or displacement of liquid compounds inside their porous structure. Reversible switching from transparency to coloration offers additional functionality. Based on Bruggeman’s effective medium theory, we derive a transparency master equation, which is valid for bilayers of arbitrary host materials and pore-filling compounds. The transparency condition fixes pore volume fractions such that the effective refractive index is homogenized through the bilayer, hence, through arbitrary layered optical media built from this bilayer. This general concept is applied to the case of switching of hygrochromic coatings made of mesoporous mixed oxide Bragg stacks. © 2014 AIP Publishing LLC.

Hygrochromic materials are porous materials whose color changes upon either infiltration or displacement of liquid compounds inside their porous structure. Remarkably, nature provides various organic templates of such materials, which in turns give inspiration for, e.g., novel humidity sensors. Generally, an intricate ordered porous multilayer structure, either exposed or buried, is involved with specific photonic crystal periodicity in one, two, or three dimensions. Not only water infiltration but also vapor condensation inside accessible pores of exposed structures can produce color changes as a result of the shift of the Bragg diffraction peak induced by refractive index modulation. Based on this principle, chemical sensors made of dielectric mesoporous Bragg stacks were exploited for organic vapor detection. The design of one-dimensional photonic crystals, so called Bragg stacks, is based on the Bragg diffraction law: 

\[ \lambda_b = 2N \lambda_{eff} \]

\( \lambda_b \) is the Bragg peak wavelength, \( \lambda_{eff} \) is the effective refractive index of a bilayer, \( N \) is the spatial period, i.e., bilayer thickness. By suitably selecting layer thicknesses, porosities, and host materials with adequate refractive indexes, the Bragg peak is positioned in the visible range, producing a specific color, which then varies upon liquid infiltration or displacement. Although this recipe has been applied many times, it has only been adapted recently for switching from transparency to coloration by tailoring porosities of adjacent layers.

The effective permittivity of a bilayer, noted \( \varepsilon_{eff} \), is the solution of

\[ f_p \frac{\varepsilon_p - \varepsilon_{eff}}{\varepsilon_{eff} + (\varepsilon_p - \varepsilon_{eff})L} + f_h \frac{\varepsilon_h - \varepsilon_{eff}}{\varepsilon_{eff} + (\varepsilon_h - \varepsilon_{eff})L} = 0, \]

where \( f_p \) is the pore (host material) volume fraction, \( \varepsilon_p = \sqrt{n_p^2} \) is the refractive index of the material filling the pores (air: \( n_p = 1.00 \) and water: \( n_p = 1.33 \) ), \( \varepsilon_h = \sqrt{n_h^2} \) is the host refractive index (TiO\(_2\): \( n_h = 2.50 \) and SiO\(_2\): \( n_h = 1.50 \) ), and \( f \) is the depolarization factor. The explicit solution of this second degree equation is (assuming spherical pores, i.e., \( L = 1/\beta \))

\[ \varepsilon_{eff} = \frac{1}{4} \left( \beta + \sqrt{\beta^2 + 8k_0 f_p} \right), \]

where \( \beta = [(3f_h - 1)\varepsilon_h + (3f_p - 1)\varepsilon_p] \). When considering an arbitrary bilayer, noted \( (L_1, L_2) \), transparency requires a
theoretical condition that is defined by the relationship \( f_{p,2} \equiv \zeta(f_{p,1}) \) such that \( \varepsilon_{\text{eff},1} = \varepsilon_{\text{eff},2} \). If this condition is fulfilled, any multilayer stack (not necessarily periodic) made of \( L_1 \) and \( L_2 \) layers behaves effectively as a homogeneous medium. If \( L_1 \) and \( L_2 \) host materials are optically transparent (dielectrics), the whole stack remains transparent to light, assuming constant refractive indexes over the wavelength range of interest. Mathematically, the transparency condition can be derived by writing (1) for layers \( L_1 \) and \( L_2 \) and then equating both expressions. By introducing the dimensionless variable \( u = \varepsilon_{\text{eff}}/\varepsilon \) and the function \( g(u) = (1 - u)/(1 + (1/L - 1)u) \), the transparency condition can be expressed as

\[
f_{p,2} = f_{p,1} g(u_{p,1}) - g(u_{h,1}) + g(u_{h,2}) - g(u_{p,2}).
\]

(3)

Although the relationship \( f_{p,2} = \zeta(f_{p,1}) \) is nonlinear (arguments of \( g \) functions actually depend on \( f_{p,1} \) or \( f_{p,2} \)), we will see hereafter that transparency master curves are well approximated by linear dependencies in practical cases.

Assuming that pores can be either empty or completely filled with liquid (water), four configurations of the photonic crystal unit cell, i.e., \((L_1, L_2)\) bilayer, exist a priori for which (3) can be satisfied (Fig. 1). At first glance, it appears easier to achieve transparency with empty pores in high-index \( L_1 \) layers and liquid-filled pores in low-index \( L_2 \) layers. Indeed, this configuration (Fig. 1, II) helps decreasing (increasing) effective index in \( L_1 \) (\( L_2 \)) layers, hence, matching both indexes at some point. However, transparency can still be achieved in three other configurations by playing on porosities. Since the porosity can never be higher than unity, it may nevertheless happen that no combination of porosities exists allowing to achieve transparency: this is the case, for example, with water-filled pores in \( L_1 \) layer as far as \( f_{p,2} > 0.33 \) (Fig. 1, III). When the transparency state is defined by empty pores in both layers (Fig. 1, I) and in the limit (non physical) case where \( f_{p,2} = 1 \), one finds \( f_{p,1} = 1 \). This is logical since \( L_2 \) material, in this limit case, is actually air void and the only possibility to match the index in \( L_1 \) material is to have air void as well. A similar argument (with liquid replacing air) applies to the transparency state defined by liquid-filled pores in both layers (Fig. 1, IV). On the other hand, when the transparency state is defined by empty pores in \( L_1 \) layer and liquid-filled pores in \( L_2 \) layer and in the same limit case as above \( f_{p,2} = 1 \), we find that \( f_{p,1} < 1 \). Again, this is logical since \( L_2 \) material, in this case, is actually liquid and matching the liquid index (1.33 for water) can be obtained using a sufficiently large but less than unity fraction of empty pores in \( L_1 \) material.

Let us, now apply this general concept to the switching behavior of a mesoporous Bragg stack of \( N \) bilayers \((L_1, L_2)\). We first assume that the transparent state \( (\Psi_T) \) is the one for which all pores in both \( L_1 \) and \( L_2 \) layers are empty, say \( f_{p,1} = 0.72 \) and \( f_{p,2} = 0.30 \) (Fig. 1, I). In practice, the \( \Psi_T \) state could be realized upon dry environment. The effective refractive index, common to both \( L_1 \) and \( L_2 \) layers, is then calculated from (1): \( n_{\text{eff},1} = n_{\text{eff},2} = 1.34 \). Thanks to this index matching, the reflectance (transmittance) of Bragg stacks made of such layers is low (high) and does not depend on the number of bilayers: the stacks behave as effective transparent slabs on glass substrate (Fig. 2(a)). Weak spectral oscillations are due to Fabry-Perot interferences associated with the total (stack) thickness and therefore, depend on the number of bilayers. Now, without changing porosities, we assume that all pores are filled with liquid, hence, forcing the system to switch to the colored state \( (\Psi_C) \). In practice, the \( \Psi_C \) state could be realized upon liquid infiltration through the porous network. Because pores are now filled with liquid, the index

![Image](image_url)

**FIG. 1.** Transparency master curves corresponding to mesoporous TiO\(_2\)(\(L_1\))/SiO\(_2\)(\(L_2\)) bilayers with pores (circles) either empty (white) or filled with water (blue).

**FIG. 2.** Theoretical reflectance (solid curves) and transmittance (dashed curves) spectra of mesoporous TiO\(_2\)/SiO\(_2\) Bragg stacks on glass substrate (\(n_{\text{glass}} = 1.51\)). Layer thickness: 65 nm (TiO\(_2\)) and 100 nm (SiO\(_2\)). Number of bilayers: \(N = 3\) (blue and red curves) and \(N = 6\) (violet and brown curves). Blue and violet (red and brown) curves correspond to (a) pore filled with water (air) in both TiO\(_2\) and SiO\(_2\) layers or (b) pore filled with water (air) in TiO\(_2\) layers and air (water) in SiO\(_2\) layers. Pore volume fractions: SiO\(_2\): \(f_{p,1} = 0.30\) (a) and (b), TiO\(_2\): \(f_{p,1} = 0.72\) (a) or 0.65 (b).
The matched effective index is now \( n_{\text{eff},2} = 1.45 \) and empty pores in high-index oxides do not result here in a shift of the Bragg wavelength (change of color) but in switching from transparency to coloration. Only recently, we reported on the fabrication of mesoporous Bragg stacks that were able to switch from transparency to coloration by infiltration of water.\(^{13}\) The present work on mesoporous TiO_2/SiO_2 Bragg chemical sensors,\(^{5–11}\) shows up and its strength increases with the number of bilayers (Fig. 2(a)). If the Bragg resonance is tuned in the negative index contrast appears: \( fp \) being the opposite configuration. For example, keeping \( fp,2 = 0.30 \) as above, we must now take \( fp,1 = 0.65 \) (Fig. 1, II). The matched effective index is now \( n_{\text{eff},1} = n_{\text{eff},2} = 1.45 \). As before, the Bragg stacks in \( \Psi_T \) state respond as effective transparent slabs though with reduced Fabry-Perot fringes due to the better index matching between the slab and the substrate (Fig. 2(b)). Upon switching from \( \Psi_T \) to \( \Psi_C \) states, the Bragg resonance is significantly stronger than before, for the same number of bilayers. This is due to the fact that higher index contrast \( (n_{\text{eff},1} = 1.70 \text{ versus } n_{\text{eff},2} = 1.34) \) occurs with liquid-filled pores in \( L_1 \) layers and empty pores in \( L_2 \) layers. With \( fp,1 \) and \( fp,2 \) being related by (3), the transmittance contrast \( (\Delta T = T^{\Psi_T} - T^{\Psi_C} \text{ at } \lambda \approx \lambda_B) \) of a \( N \)-bilayer Bragg stack can be calculated as a function of the pore volume fraction in one layer, e.g., \( fp,2 \) (Fig. 3). If, the \( \Psi_T \) state is chosen to be the configuration in which all pores of both layers are empty (Fig. 1, I), the contrast decreases with increasing \( fp,2 \) values.

The fact that the contrast drops to zero when \( fp,2 = 1.0 \) is consistent with previous discussion on the corresponding transparency master curve. On the other hand, if the \( \Psi_T \) state is chosen to be the configuration with empty pores in \( L_1 \) layers and liquid-filled pores in \( L_2 \) layers (Fig. 1, II), the contrast increases with increasing \( fp,2 \) values. The fact that higher contrasts can be achieved in this case is consistent with our previous discussion on transparency master curves. In both cases, higher contrasts can be achieved by increasing the number of bilayers.

In practical devices, it might seem extremely difficult to localize the liquid in the pores of either \( L_1 \) or \( L_2 \) layer and to displace it from one layer to the other. Solutions can, however, be found by combining surface chemistry, fluidics, and electro-wetting. Chemical surface functionalization can be used to make pores hydrophilic or hydrophobic in specific layers, and capillary attraction due to controlled pore size variations between adjacent layers can be used to promote infiltration of the liquid across the whole layer stack.\(^{12,13}\) Electro-wetting can be used to force displacement of the liquid across adjacent layers.\(^{17}\)

Hygrochromic coatings (deposited on glass substrate) consisting of Bragg stacks \((N = 3 \text{ bilayers})\) of mesoporous oxides \((L_1 \text{ layers: } x\text{TiO}_2 (1-x)\text{Al}_2\text{O}_3, \text{ } L_2 \text{ layers: } \text{SiO}_2)\) were fabricated by a modified sol-gel method according a procedure described previously.\(^{13}\) The molar ratio \( x \) of mixed oxide layers was adjusted in order to fulfill the transparency condition in dry conditions (empty pores in all layers) with available porogen agents.\(^{13}\) Almost perfect matching between measured effective indexes was obtained using \( x = 0.6 \) (\( x = 0.5 \) used previously): \( n_{\text{eff},1} = 1.32 \) and \( n_{\text{eff},2} = 1.31 \). Using these values, porosities equal to \( fp,1 = 0.68 \) and \( fp,2 = 0.37 \) were deduced from (2) with \( n_{p,1} = 2.13, n_{p,2} = 1.50, \) and \( n_{p,1} = n_{p,2} = 1.0 \). The corresponding point \((fp,1,fp,2)\) lay close to the transparency master curve (Fig. 1, I). The evolution of the color of a half-coated glass sample during successive wetting and drying steps is shown in Fig. 4. Initially, i.e., in dry conditions, the coated surface is almost transparent (Fig. 4, middle left photograph). The faint greenish color is due to slight effective index mismatch, enhanced by iridescence at non zero viewing angle. As a drop of water slips on

![FIG. 3. Transmittance contrast \( \Delta T \) upon switching from transparent to colored states as function of pore volume fraction \( fp,2 \). Transparent state \( \Psi_T \) and colored state \( \Psi_C \) configurations are defined in Fig. 1. Pore volume fraction \( fp,1 \) is fixed by the corresponding transparency master curve. Bragg layer stack parameters are the same as in Fig. 2. Number of bilayers: \( N = 3 \) (dashed curves) and \( N = 6 \) (solid curves).]{fig9}

![FIG. 4. Evolution of the color of a glass slide coated (half of the surface) with mesoporous oxide Bragg layer stack during successive wetting and drying steps.]{fig10}
the surface, the color immediately changes to orange-yellow under the streak left by the drop (Fig. 4, left top). As the water drop is further spread on the surface with a cotton stick (not shown), the orange-yellow color becomes uniform on the coated surface (Fig. 4, top right). After the sample is dried for a short time (1 min), the color fades to a faint green (Fig. 4, middle right). Transparency is not fully restored because some pores are still filled with water due to the short drying time. Re-wetting of the surface (Fig. 4, bottom) causes a color to appear again, which is slightly more yellow than during the first wetting. This result indicates the possibility to vary slightly the color by playing on the pore filling fraction. Finally, after drying for a longer time (5 min), transparency is restored. Note that the color could be more saturated by increasing the number of bilayers (here, \( N = 3 \)).

In conclusion, we theoretically derived the general condition for achieving transparency in mesoporous layered optical media and we applied it to the switching of hygrochromic coatings from transparency to Bragg-type coloration upon displacement of water in the porous structure. The proposed concept is believed to find applications in various sensing applications and smart privacy glass windows.

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